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非阿贝尔规范场场量二次变换为零对群参数的限制

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摘　要

本文导出了非阿贝尔规范场场量二次变换为零的充要条件；还自然地导致群参数及费米场场量的二次变换为零.

（一）

众所周知，非阿贝尔规范场有效拉氏函数密度 $\mathscr{L}_{\text{eff}}$ 的 BRS 变换，具有幂零性，即二次 BRS 变换为零，这时的 $\mathscr{L}_{\text{eff}}$ 已是规范固定的[1]. 那么使非阿贝尔规范场拉氏函数密度 $\mathscr{L}$ 不变的规范变换，在 $\mathscr{L}$ 的规范尚未固定时，场量的二次变换为零的条件是什么呢？

非阿贝尔规范场的拉格朗日函数密度为：

$$\mathscr{L}=-\frac{1}{4}\left(\partial_\mu A_\nu^a(x)-\partial_\nu A_\mu^a(x)+gf^{abc}A_\mu^b(x)A_\nu^c(x)\right)^2+\psi^+\gamma_4(\gamma_\mu\partial_\mu+m),\tag{1}$$

式中 $A_\mu^a(x)$ 为规范场，a、b、c 为内部空间分量指标，ψ 为费米场，g 为耦合常数，x 为时空坐标. 在变换：

$$\delta\psi=-i\frac{\lambda^a}{2}\theta^a(x)\psi(x),\tag{2}$$

$$\delta\bar{\psi}=i\bar{\psi}\frac{\lambda^a}{2}\theta^a(x),\tag{3}$$

$$\delta A_\mu^a(x)=-\frac{1}{g}D_\mu^{ab}\theta^b(x)\tag{4}$$

下，$\mathscr{L}$ 不变[2]. 式中 $\theta(x)$ 为群参数，

$$D_\mu^{ab}=\delta^{ab}\partial_\mu-gf^{abc}A_\mu^c(x).\tag{5}$$

现在来研究非阿贝尔规范场场量的二次规范变换为零即 $\delta\delta A_\mu^a(x)=0$ 时，对群参数 $\theta(x)$ 的限制.

由(4)式：

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$$\delta\delta A_\mu^a(x) = -\frac{1}{g}\delta(D_\mu^{ab}\theta^b(x))$$

$$= -\frac{1}{g}[(\delta D_\mu^{ab})\theta^b(x) + D_\mu^{ab}\delta\theta^b(x)]. \tag{6}$$

$$\begin{aligned}(\delta D_\mu^{ab})\theta^b(x) &= [\delta(\delta^{ab}\partial_\mu - gf^{abc}A_\mu^c(x))]\theta^b(x)\\ &= -gf^{abc}(\delta A_\mu^c(x))\theta^b(x)\\ &= f^{abc}[D_\mu^{cd}\theta^d(x)]\theta^b(x)\\ &= f^{abc}[(\delta^{cd}\partial_\mu - gf^{cde}A_\mu^e(x))\theta^d(x)]\theta^b(x)\\ &= f^{abc}[\partial_\mu\theta^c(x)]\theta^b(x) - gf^{abc}f^{cde}A_\mu^e(x)\theta^d(x)\theta^b(x).\end{aligned} \tag{7}$$

(7) 式右端的第一项

$$f^{abc}[\partial_\mu\theta^c(x)]\theta^b(x) = \frac{1}{2}f^{abc}\partial_\mu(\theta^c(x)\theta^b(x)) - \frac{1}{2}f^{abc}\theta^c(x)\partial_\mu\theta^b(x) + \frac{1}{2}f^{abc}(\partial_\mu\theta^c(x))\theta^b(x). \tag{8}$$

将 (7) 式第二项的求和指标作替代：$b\to d$，$d\to b$，可化为

$$\begin{aligned}f^{abc}f^{cde}A_\mu^e(x)\theta^d(x)\theta^b(x) &= f^{adc}f^{cbe}A_\mu^e(x)\theta^b(x)\theta^d(x)\\ &= -f^{adc}f^{ceb}A_\mu^e(x)\theta^b(x)\theta^d(x),\end{aligned}$$

所以

$$\begin{aligned}f^{abc}f^{cde}A_\mu^e(x)\theta^d(x)\theta^b(x) &= \frac{1}{2}f^{abc}f^{cde}A_\mu^e(x)\theta^d(x)\theta^b(x)\\ &\quad - \frac{1}{2}f^{adc}f^{ceb}A_\mu^e(x)\theta^b(x)\theta^d(x)\\ &= \frac{1}{2}(f^{abc}f^{cde} + f^{adc}f^{ceb})A_\mu^e(x)\theta^d(x)\theta^b(x)\\ &\quad - \frac{1}{2}f^{adc}f^{ceb}A_\mu^e(x)(\theta^b(x)\theta^d(x) + \theta^d(x)\theta^b(x))\\ &= -\frac{1}{2}f^{aec}f^{cbd}A_\mu^e(x)\theta^d(x)\theta^b(x)\\ &\quad - \frac{1}{2}f^{adc}f^{ceb}A_\mu^e(x)(\theta^b\theta^d + \theta^d\theta^b).\end{aligned} \tag{9}$$

已利用了

$$f^{abc}f^{cde} + f^{adc}f^{ceb} + f^{aec}f^{cbd} = 0.$$

(8) 式右端第一项减去 (9) 式右端第一项乘以 g 得：

$$\begin{aligned}&\frac{1}{2}f^{abc}\partial_\mu(\theta^c(x)\theta^b(x)) + \frac{g}{2}f^{aec}f^{cbd}A_\mu^e(x)\theta^d(x)\theta^b(x)\\ &= \frac{1}{2}\delta^{ac}\partial_\mu(f^{cbd}\theta^d(x)\theta^b(x)) + \frac{g}{2}f^{aee}f^{cdb}A_\mu^e(x)\theta^b(x)\theta^d(x)\\ &= -(\delta^{ac}\partial_\mu - gf^{ace}A_\mu^e(x))\left(\frac{1}{2}f^{cbd}\theta^b(x)\theta^d(x)\right)\\ &= -(D_\mu^{ac})\delta\theta^c(x) = -(D_\mu^{ab})\delta\theta^b(x).\end{aligned} \tag{10}$$

已取

$$\delta\theta^c(x)=\frac{1}{2}f^{cbd}\theta^b(x)\theta^d(x), \tag{11}$$

即已假定群参数 $\theta^c(x)$ 按 (11) 式变换.

将 (8)、(9) 和 (10) 式代入 (7) 式得:

$$(\delta D_\mu^{ab})\theta^b(x)=-D_\mu^{ab}\delta\theta^b(x)+\frac{g}{2}f^{ade}f^{ceb}A_\mu^c(x)[\theta^b(x)\theta^d(x)+\theta^d(x)\theta^b(x)]$$
$$+\frac{1}{2}f^{abc}[(\partial_\mu\theta^c(x))\theta^b(x)-\theta^c(x)\partial_\mu\theta^b(x)],$$

即

$$\delta(D_\mu^{ab}\theta^b(x))=\frac{g}{2}f^{ade}f^{ceb}A_\mu^c(x)[\theta^b(x)\theta^d(x)+\theta^d(x)\theta^b(x)]$$
$$+\frac{1}{2}f^{abc}[(\partial_\mu\theta^c(x))\theta^b(x)-\theta^c(x)\partial_\mu\theta^b(x)]. \tag{12}$$

由于 f^{abc}、$A_\mu^a(x)$、$\theta^a(x)$ 都不等于零，所以要 $\delta\delta A_\mu^a(x)=0$，即 $\delta(D_\mu^{ab}\theta^b(x))=0$，除 $\theta^a(x)$ 需满足 (11) 式外,必须取

$$\theta^b(x)\theta^d(x)+\theta^d(x)\theta^b(x)=0, \tag{13}$$

和

$$\frac{1}{2}f^{abc}[(\partial_\mu\theta^c(x))\theta^b(x)-\theta^c(x)\partial_\mu\theta^b(x)]=0. \tag{14}$$

而 (14) 式即

$$\partial_\mu\left(\frac{1}{2}f^{abc}\theta^c(x)\theta^b(x)\right)-f^{abc}\theta^c(x)\partial_\mu\theta^b(x)$$
$$=-\partial_\mu\delta\theta^a(x)-f^{abc}\theta^c(x)\partial_\mu\theta^b(x)=0,$$

所以

$$\delta(\partial_\mu\theta^a(x))=-f^{abc}\theta^c(x)\partial_\mu\theta^b(x)=f^{abc}\theta^b(x)\partial_\mu\theta^c(x). \tag{14'}$$

将 (11) 式对时空求导数得:

$$\partial_\mu\delta\theta^a(x)=\partial_\mu\left(\frac{1}{2}f^{abc}\theta^b(x)\theta^c(x)\right)$$
$$=\frac{1}{2}f^{abc}(\partial_\mu\theta^b(x))\theta^c(x)-\frac{1}{2}f^{abc}(\partial_\mu\theta^c(x))\theta^b(x)$$
$$=f^{abc}(\partial_\mu\theta^b(x))\theta^c(x).$$

于是 (14) 式可由 (11) 式对时空求导数得到,运算中已用了 (13) 式.

因此,要 $\delta\delta A_\mu^a(x)=-\frac{1}{g}\delta(D_\mu^{ab}\theta^b(x))=0$，必须有

$$\delta\theta^a(x)=\frac{1}{2}f^{abc}\theta^b(x)\theta^c(x), \tag{11}$$

及

$$\theta^b(x)\theta^d(x)+\theta^d(x)\theta^b(x)=0; \tag{13}$$

反之，如 $\theta^b(x)$ 服从 (11) 和 (13) 式，而 (11) 式对时空求导便得 (14) 式,所以由 (12)

知，$\delta(D_\mu^{ab}\theta^b(x))=0$，因 $\delta\delta A_\mu^a(x)=-\frac{1}{g}\delta(D_\mu^{ab}\theta^b(x))$，所以 $\delta\delta A_\mu^a(x)=0$.

（二）

现在来证明 $\delta\delta\theta^a(x)=0$.

$$\delta\delta\theta^a(x)=\frac{1}{2}f^{abc}\delta(\theta^b\theta^c)=\frac{1}{2}f^{abc}[(\delta\theta^b)\theta^c+\theta^b\delta\theta^c]$$

$$=\frac{1}{2}f^{abc}\left[\frac{1}{2}f^{bde}\theta^d\theta^e\theta^c+\theta^b\frac{1}{2}f^{cde}\theta^d\theta^e\right]=0,$$

已利用了 $f^{acb}=-f^{abc}$ 及(13)式.

再来证明 $\delta\delta\bar{\phi}=0$.

$$\delta\delta\bar{\phi}=\delta\left(i\bar{\phi}\frac{\lambda^a}{2}\theta^a\right)=\delta\bar{\phi}i\frac{\lambda^a}{2}\theta^a+i\bar{\phi}\frac{\lambda^a}{2}\delta\theta^a=0.$$

因

$$\delta\bar{\phi}i\frac{\lambda^a}{2}\theta^a=\left(i\bar{\phi}\frac{\lambda^a}{2}\theta^a\right)i\frac{\lambda^b}{2}\theta^b=-\bar{\phi}\frac{1}{2}\left(\frac{\lambda^a}{2}\frac{\lambda^b}{2}-\frac{\lambda^b}{2}\frac{\lambda^a}{2}\right)\theta^a\theta^b$$

$$=-i\bar{\phi}\frac{1}{2}f^{abc}\frac{\lambda^c}{2}\theta^a\theta^b=-i\bar{\phi}\frac{\lambda^a}{2}\delta\theta^a.$$

同理可证：$\delta\delta\phi=0$.

（三）

由以上讨论可知，当而且仅当 $\theta^a(x)$ 和 $\delta\theta^a(x)$ 满足(13)、(11)式时，才有 $\delta\delta A_\mu^a(x)=0$，而且自然有 $\delta\delta\theta^a(x)=0$，一定导致 $\delta\delta\bar{\phi}=\delta\delta\phi=0$，即

$$\delta^2(A_\mu^a(x)、\theta^a(x)、\bar{\phi}(x)、\phi(x))=0. \tag{15}$$

但(11)和(13)式还可化简．由(2)、(3)和(4)知 $\theta^a(x)$、$\theta^b(x)$为对易量（$\because A_\mu^a(x)$为对易量，ϕ、$\bar{\phi}$ 为反对易量），所以(13)式成为

$$2\theta^b(x)\theta^d(x)=0,$$

又由(2)、(3)和(4)式知 $\theta^b(x)\neq0$，因而 $\theta^d(x)\neq0$，所以只有取 $\theta^b(x)$ 是反对易的无穷小量 $\xi(x)$ 和反对易的鬼场 $C^a(x)$ 的乘积即

$$\theta^b(x)=\xi(x)C^b(x) \tag{16}$$

时，才能有

$$2\theta^b(x)\theta^d(x)=2\xi(x)C^b(x)\xi(x)C^d(x)=0,$$

而且有

$$\delta\theta^a(x)=\frac{1}{2}f^{abc}\xi(x)C^b(x)\xi(x)C^c(x)=0, \tag{11'}$$

因 $\xi^2(x)=0$．这样 $\delta\delta A_\mu^a(x)=0$ 的充要条件就变为(11′)和(16)式．而表示规范变换及群参数变换幂零性的(15)式仍成立.

如 ξ 与时空坐标 x 无关时，则有

$$\theta^a(x) = \xi C^a(x) \tag{17}$$

和

$$\delta\theta^a(x) = \xi\delta C^a(x) = \frac{1}{2} f^{abc}\xi C^b(x)\xi C^c(x).$$

即

$$\delta C^a(x) = -\frac{\xi}{2} f^{abc} C^b(x) C^c(x), \tag{18}$$

所以得到了鬼场的变换规律；而 (15) 式成为

$$\delta^2(A^a_\mu(x), \psi(x), \bar{\psi}(x), C^a(x)) = 0. \tag{19}$$

因而 (1) 式在变换

$$\delta\psi = -i\frac{\lambda^a}{2}\xi(x)C^a(x)\psi(x) \tag{2'}$$

$$\delta\bar{\psi} = i\bar{\psi}\frac{\lambda^a}{2}\xi(x)C^a(x) \tag{3'}$$

$$\delta A^a_\mu(x) = -\frac{1}{g} D^{ab}_\mu(\xi(x)C^b(x)) \tag{4'}$$

下，或在变换

$$\delta\psi = -i\frac{\lambda^a}{2}\xi C^a(x)\psi(x) \tag{2''}$$

$$\delta\bar{\psi} = i\bar{\psi}\frac{\lambda^a}{2}\xi C^a(x) \tag{3''}$$

$$\delta A^a_\mu(x) = -\frac{\xi}{g} D^{ab}_\mu C^b(x) \tag{4''}$$

下，保持不变，并有规范变换的幂零性.

如在 (1) 式中引入规范固定项 $\mathscr{L}_f$ 和鬼粒子项 $\mathscr{L}_g$，并将 (19) 式与 BRS 变换的幂零性比较，可知

$$\delta(\mathscr{L}_f + \mathscr{L}_g) = 0$$

的变换完全由反鬼粒子的变换 $\delta C^{a+}(x)$ 决定.

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Constraint on the Parameter of Group By Two Order Transformation of Field Variable Being Equal to Zero in Non-Abelian Gauge Field

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ABSTRACT

A sufficient and necessary condition that two order transformation of field variable in non-abelian gauge field is equal to zero is derived. Two order transformation of both the parameter of Group and field variable of Fermi fields are necessarily equal to zero.