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Nonperturbative Quark Propagator and Study of Nontrivial Q^2-Dependence in Nucleon Structure Functions

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In consideration of the lowest-order nonperturbative effect due to the quark condensate $\langle\bar{q}q\rangle$ and the gluon condensate $\langle GG\rangle$ on the quark propagator, we calculate the QCD nonperturbative quark propagator under the chain approximation. Using the obtained nonperturbative quark propagator, we analyze the nonperturbative effect in the nucleon structure function and show the nontrivial Q^2-dependence in the nucleon structure function.

Key words: quark condensate, nonperturbative quark propagator, nucleon structure function.

1. INTRODUCTION

Within the framework of the quark parton model (QPM) [1], deep inelastic lepton-nucleon scattering is viewed simply as an incoherent sum of elastic scatterings of leptons on "quasi-free" quarks. The structure function could therefore be expressed as a sum of parton momentum distributions weighted by the square of the charge of corresponding partons (quarks plus antiquarks). This is a good description for the large momentum transfer deep inelastic process. In order to test this in experiments, various sum rules were put forward. One of them which corresponds to the

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nonpolarized lepton-nucleon deep-inelastic scattering process is the Gottfried sum rule defined as [2]

$$S_G = \int_0^1 \frac{dx}{x} [F_2^p(x) - F_2^n(x)] , \tag{1}$$

which would be $S_G = \frac{1}{3}$ in the QPM [2]. But the NMC Collab. found that $S_G = 0.235 \pm 0.026$ [3], which deviates from that expected by the quark parton model. In addition, the Ellis-Jaffe sum rule is defined as [4]

$$\Gamma_1^p = \int dx g_1^p(x) , \tag{2}$$

where

$$g_1^p(x) = \frac{1}{2} \left[\frac{4}{9} \Delta u(x) + \frac{1}{9} \Delta d(x) + \frac{1}{9} \Delta s(x) \right] , \tag{3}$$

Δu, Δd, and Δs are the spin fractions of the u, d, and s quarks, respectively (normalized to 1). The experimental value of Γ_1^p also deviates from that expected by the QPM [5]. The experimental data of both the Gottfried sum and the Ellis-Jaffe sum are less than those expected by the QPM. Many studies have been made for explaining this deviation [6,7]. In this paper, we investigate the nontrivial Q^2-dependence of the nucleon structure function. In consideration of the lowest-order nonperturbative effect due to the quark condensate $\langle \bar{q}q \rangle$ and gluon condensate $\langle GG \rangle$ on the quark propagator, we calculate the QCD nonperturbative quark propagator under the China approximation. Using the obtained nonperturbative quark propagator, we analyze the nonperturbative effect in the nucleon structure function and show the nontrivial Q^2-dependence in the nucleon structure function. Next, the nontrivial Q^2-dependences of the Gottfried sum and the Ellis-Jaffe sum are discussed. Finally, the discussion and summary are given.

2. NONPERTURBATIVE QUARK PROPAGATOR

Let us start with writing the free quark propagator

$$i [S_F(x - y)]_{ij}^{\alpha\beta} = \langle 0 | T q_i^\alpha(x) \bar{q}_j^\beta(y) | 0 \rangle , \tag{4}$$

where α, β are color indices and i, j are Dirac spinor indices. The free quark propagator can be expressed in momentum space as

$$S_F^{-1}(p) = \not{p} - m_L , \tag{5}$$

where m_L is the perturbative (current) quark mass which can be neglected in the large momentum transfer process. Either the experiment of the NMC ($Q^2 = 4$ GeV^2) for measuring the Gottfried sum or the experiment ($Q^2 = 10.7$ GeV^2) done by Adams [5] for determining the Ellis-Jaffe sum are performed at the medium energy region. In this region, the nonperturbative effect induced by the QCD vacuum cannot be underestimated. Generally, attention is paid to the effect of the condensates of quarks and gluons on the quark and gluon propagators [8,9]. In this paper, we only consider the contributions of the lowest dimensional condensates, i.e., dimension-3 quark condensate [see Fig. 1(b)] and dimension-4 gluon condensate [see Fig. 1(c)]. In the calculation, the QCD sum rule method [10] is used to consider both perturbative effect and nonperturbative effect in the operator product expansion (OPE) where the nonperturbative effect is expressed by the nonzero average expectation of the composed operators such as $\langle \bar{q}q \rangle$ and $\langle GG \rangle$. It is generally accepted that these nonzero vacuum expectations reflect the nonperturbative properties of the physical vacuum. The lowest-order quark and gluon condensates are generally regarded as parameters in the OPE and the higher dimension condensates are neglected.

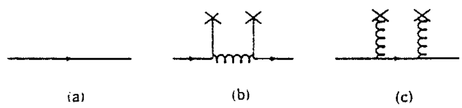

Fig. 1
The nonperturbative quark propagator including:
(a) The perturbative free quark propagator, (b) Lowest-order correction due to the nonvanishing value of $\langle \bar{q}q \rangle$, (c) Lowest-order correction due to the nonvanishing value of $\langle GG \rangle$.

To calculate the nonperturbative quark propagator, we first consider the nonperturbative vacuum expectation value (VEV) of two-gluon fields. In the calculation, the fixed point gauge is taken, i.e.,

$$x^{\mu} B_{\mu}(x) = 0, \tag{6}$$

and then,

$$B_{\mu}(x) = \frac{1}{2} x^{\nu} G^{a}_{\nu\mu}(0) + \frac{1}{1!3} x^{\nu} x^{\rho} \partial_{\nu} G^{a}_{\rho\mu}(0) + \frac{1}{n!(n+2)} x^{\nu} x^{\nu_1} \cdots x^{\nu_n} \partial_{\nu_1} \cdots \partial_{\nu_n} G^{a}_{\nu\mu}(0) + \cdots, \tag{7}$$

obviously,

$$B_{\mu}(0) \equiv 0. \tag{8}$$

Introducing covariant differential

$$D_{\mu} = \partial_{\mu} - igB^{a}_{\mu} t^{a}, \tag{9}$$

where $t^{a} = \frac{\lambda^{a}}{2}$ $(a = 1, 2, \cdots, 8)$ are the Gell-Mann matrices, then Eq.(9) can be re-expressed as

$$\begin{aligned} B^{a}_{\mu}(x) &= \frac{1}{2} x^{\lambda} G^{a}_{\lambda\mu}(0) + \frac{x^{\lambda} x^{\sigma}}{3} [D_{\sigma}(0), G^{a}_{\lambda\mu}(0)] \\ &+ \frac{x^{\lambda} x^{\sigma} x^{\tau}}{8} \{D_{\sigma}(0), [D_{\tau}(0), G^{a}_{\lambda\mu}(0)]\} + \cdots, \end{aligned} \tag{10}$$

In the following calculation, only the first term in the expansion of $B^{a}_{\mu}(x)$ is taken, i.e., we assume that the $O(|x|^2)$ terms are small enough to be neglected for short distances $|x|$. So one can obtain

$$\begin{aligned} \langle 0 | B^{b}_{\mu}(y) B^{c}_{\nu}(z) | 0 \rangle_{\text{NP}} &= \frac{1}{4} y^{\lambda} z^{\rho} \langle 0 | G^{b}_{\lambda\mu}(0) G^{c}_{\rho\nu}(0) | 0 \rangle + \cdots \\ &= \frac{1}{4} \frac{1}{96} y^{\lambda} z^{\rho} (g_{\lambda\rho} g_{\mu\nu} - g_{\lambda\nu} g_{\mu\rho}) \langle GG \rangle + \cdots, \end{aligned} \tag{11}$$

where

$$\langle GG \rangle = \langle 0 | G^{a}_{\lambda\mu}(0) G^{a}_{\lambda\mu}(0) | 0 \rangle. \tag{12}$$

By using the similar method to the expansion of two-gluon fields, the nonperturbative VEV of two-quark fields can be expressed as

$$\begin{aligned} &\langle 0 | :\bar{q}^{\alpha}_{r}(y) q^{\beta}_{n}(z): | 0 \rangle_{\text{NP}} \\ &= \langle 0 | :\bar{q}^{\alpha}_{r}(0) q^{\beta}_{n}(0): | 0 \rangle \\ &- (y^{\lambda} - z^{\lambda}) \langle 0 | :\bar{q}^{\alpha}_{r} [D_{\lambda}(0) q_{n}(0)]^{\beta}: | 0 \rangle \\ &+ \left[\frac{y^{\lambda} y^{\tau}}{2} + \frac{z^{\lambda} z^{\tau}}{2} - y^{\lambda} z^{\tau} \right] \langle 0 | :\bar{q}^{\alpha}_{r}(0) [D_{\lambda}(0) D_{\tau}(0) q_{n}(0)]^{\beta}: | 0 \rangle + \cdots, \end{aligned} \tag{13}$$

where

$$\langle 0|:\bar{q}_r^\alpha(0)\,q_n^\beta(0):|0\rangle = \left(\frac{\delta^{\alpha\beta}}{3}\right)\left(\frac{\delta_{nr}}{4}\right)\langle 0|:\bar{q}_j^\omega(0)\,q_j^\omega(0):|\rangle \equiv \frac{\delta^{\alpha\beta}}{3}\,\delta_{nr}\,\frac{\langle\bar{q}q\rangle}{4}, \tag{14}$$

with

$$\langle\bar{q}q\rangle \equiv \langle 0|:\bar{q}_j^\omega(0)\,q_j^\omega(0):|0\rangle, \tag{15}$$

In Eq.(14), color superscript indices (α, β) will always lead to a trivial factor $\frac{\delta^{\alpha\beta}}{3}$; and spinor indices (r, n) to $\frac{\delta_{nr}}{4}$. For the case in which the momentum transfer is large enough, or the distances $|y|$ and $|z|$ are small enough, the first two lowest terms would be taken and the higher order terms can be neglected. Now, we express the VEV of the second term in Eq.(13) as:

$$\langle 0|:\bar{q}_r^\alpha[D_\lambda(0)\,q_n(0)]^\beta:|0\rangle = K(\gamma_\lambda)_{nr}, \tag{16}$$

In order to determine the specified form of K, we contract $(\gamma_\lambda)_{nr}$ into both sides of Eq.(16) and introduce the motion equation

$$\not{D}q = -imq, \tag{17}$$

and then

$$(-im)\,\frac{\delta^{\alpha\beta}}{3}\langle\bar{q}q\rangle = 16K, \tag{18}$$

Therefore

$$\langle 0:|\bar{q}_r^\alpha[D_\lambda(0)\,q_n(0)]^\beta:|0\rangle = \frac{-im\delta^{\alpha\beta}}{48}\langle\bar{q}q\rangle(\gamma_\lambda)_{nr}, \tag{19}$$

Substituting Eq.(14) and Eq.(19) into Eq.(13), one finds

$$\langle 0|:q_n^\alpha(y)\,\bar{q}_r^\beta(z):|0\rangle_{\rm NP} = -\frac{\delta^{\alpha\beta}\langle\bar{q}q\rangle}{12}\left\{1-\frac{im[\gamma\cdot(y-z)]}{4}\right\}_{nr}, \tag{20}$$

Now let us calculate the contributions of the lowest quark and gluon condensates to the quark propagator. The modification of the quark propagator in Fig. 1(b) can be expressed as:

$$\begin{aligned} {\rm i}[S_{\rm F}^{(2)}(p)]_{kl}^{\rho\sigma} = &-\frac{g_s^2}{4}\int {\rm d}^4x\,{\rm e}^{{\rm i}p\cdot x}\int {\rm d}^4y\int {\rm d}^4z \\ &\times[\langle 0|Tq_k^\rho(x)\,\bar{q}_i^\tau(y)|0\rangle_{\rm pert}\,\gamma_{in}^\mu\,\lambda_{\tau\alpha}^b \\ &\times\langle 0|:q_n^\alpha(y)\,\bar{q}_r^\beta(z):|0\rangle_{\rm NP}\,\gamma_{rj}^\nu\,\lambda_{\beta\omega}^c\,\langle 0|Tq_j^\omega(z)\,\bar{q}_l^\sigma(0)|0\rangle_{\rm pert} \\ &\times\langle 0|TB_\mu^b(y)\,B_\nu^c(z)|0\rangle_{\rm pert}], \end{aligned} \tag{21}$$

where the normal product results from the time-order product expansion of quark fields.

$$Tq(x)\,\bar{q}(y) = \langle 0|Tq(x)\,\bar{q}(y)|0\rangle_{\rm pert} + :q(x)\,\bar{q}(y):, \tag{22}$$

where the first term on the right-hand side gives the usual free quark propagator, while the second term no longer vanishes since the nonperturbative structure of the QCD vacuum. The perturbative time-order product in Eq.(21) corresponds to the perturbative propagator in the ordinary perturbative theory, i.e.,

$$\langle 0 \mid T q_k^\rho(x)\, \bar{q}_l^\tau(y) \mid 0 \rangle_{\text{pert}} = \mathrm{i}\delta^{\rho\tau} \int \frac{\mathrm{d}^4 q}{(2\pi)^4}\, \mathrm{e}^{-\mathrm{i}q\cdot(x-y)} \left[\frac{\not{q}_{kl} + m_{\mathrm{L}}\delta_{kl}}{q^2 - m_{\mathrm{L}}^2} \right], \tag{23}$$

and

$$\langle 0 \mid T B_\mu^b(y)\, B_\nu^c(z) \mid 0 \rangle_{\text{pert}} = \mathrm{i}\delta^{bc} \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\, \mathrm{e}^{-\mathrm{i}k\cdot(y-z)} \left[-\frac{g_{\mu\nu}}{k^2} + \frac{\xi k_\mu k_\nu}{k^4} \right], \tag{24}$$

where ξ is the gauge parameter. In order to complete the integral in Eq.(21), we write the factor $\mathrm{e}^{\mathrm{i}p\cdot x}$ as $\mathrm{e}^{\mathrm{i}p\cdot(x-y)}\,\mathrm{e}^{\mathrm{i}p\cdot(y-z)}\,\mathrm{e}^{\mathrm{i}p\cdot z}$ and change integration variables from $[\mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{d}^4z]$ to $[\mathrm{d}^4(x-y)\,\mathrm{d}^4(y-z)\,\mathrm{d}^4z]$. Integrating Eq.(21) one finds

$$\begin{aligned} \mathrm{i}[S_{\mathrm{F}}^{(2)}(p)]_{kl}^{\rho\sigma} = &\left[\frac{\not{p}+m_{\mathrm{L}}}{p^2-m_{\mathrm{L}}^2} \right]_{ki} \left[\frac{\mathrm{i}g_s^2 \lambda_{\rho\alpha}^b \lambda_{\beta\sigma}^b}{4(2\pi)^4} \right] \gamma_{in}^\mu \\ &\times \left\{ \int \mathrm{d}^4(y-z) \int \mathrm{d}^4k \langle 0 | : q_n^\alpha(y)\bar{q}_r^\beta(z) : | 0 \rangle_{\mathrm{NP}}\, \mathrm{e}^{\mathrm{i}(p-k)\cdot(y-z)} \right. \\ &\times \left. \left[-\frac{g_{\mu\nu}}{k^2} + \frac{\xi k_\mu k_\nu}{k^4} \right] \gamma_{rj}^\nu \right\} \left[\frac{\not{p}+m_{\mathrm{L}}}{p^2-m_{\mathrm{L}}^2} \right]_{jl}. \end{aligned} \tag{25}$$

The full quark propagator $S_{\mathrm{F}}(p)$ is related to its self-energy via the relationship

$$\begin{aligned} S_{\mathrm{F}}(p) &\equiv [\not{p} - m_{\mathrm{L}} - \Sigma(p)]^{-1} \\ &= (p - m_{\mathrm{L}})^{-1} + (\not{p} - m_{\mathrm{L}})^{-1}\Sigma(p)(\not{p} - m_{\mathrm{L}})^{-1} + \cdots \\ &= (\not{p} - m_{\mathrm{L}})^{-1} + S_{\mathrm{F}}^{(2)}(p) + \cdots, \end{aligned} \tag{26}$$

The self-energy corresponding to Eq.(25) is

$$\begin{aligned} \sum\nolimits_{tu}^{\rho\sigma}(p) &\equiv (\not{p} - m_{\mathrm{L}})_{tk}\,[S_{\mathrm{F}}^{(2)}(p)]_{kl}^{\rho\sigma}(\not{p} - m_{\mathrm{L}})_{lu} \\ &= \frac{g_s^2}{4(2\pi)^4} \lambda_{\rho\alpha}^b \lambda_{\beta\sigma}^b \gamma_{tn}^\mu \int \mathrm{d}^4(y-z) \int \frac{\mathrm{d}^4k}{k^4} \\ &\times \langle 0 | : q_n^\alpha(y)\,\bar{q}_r^\beta(z) : | 0 \rangle_{\mathrm{NP}}\, \mathrm{e}^{\mathrm{i}(p-k)\cdot(y-z)} \\ &\times [-g_{\mu\nu}k^2 + \xi k_\mu k_\nu]\,\gamma_{r\mu}^\nu. \end{aligned} \tag{27}$$

Substituting the VEV of Eq.(20) into the above equation and by using the identity

$$\begin{aligned} &\int \mathrm{d}^4(y-z) \int \mathrm{d}^4k\, [\gamma\cdot(y-z)]^j\, \mathrm{e}^{\mathrm{i}(p-k)\cdot(y-z)} f(k) \\ &= \left[-\mathrm{i}\gamma\cdot\frac{\partial}{\partial p} \right]^j \int \mathrm{d}^4(y-z) \int \mathrm{d}^4k\, \mathrm{e}^{\mathrm{i}(p-k)\cdot(y-z)} f(k) \\ &= (2\pi)^4 \left[-\mathrm{i}\gamma\cdot\frac{\partial}{\partial p} \right]^j f(p), \end{aligned} \tag{28}$$

we finally obtain

$$\sum^{(b)}(p)=\frac{g_s^2\langle\bar{q}q\rangle}{9p^2}[(4-\xi)-(1-\xi)m\not{p}/p^2], \tag{29}$$

where the index (b) means this contribution is due to Fig. 1(b). Similarly, we can derive the self-energy, due to Fig. 1(c),

$$\sum^{(c)}(p)=\frac{g_s^2\langle GG\rangle m_L(p^2-m_L\not{p})}{12(p^2-m_L^2)^3}. \tag{30}$$

Considering the contributions from Figs. 1(b)-1(c), the inverse of the quark propagator can be expressed as

$$\begin{aligned} S_F^{-1}(p)=\not{p}&\left[1+\frac{g_s^2\langle\bar{q}q\rangle(1-\xi)m}{9p^4}+\frac{g_s^2\langle GG\rangle m_L^2}{12(p^2-m_L^2)^3}\right]\\ &-\left[m_L+\frac{g_s^2\langle\bar{q}q\rangle(4-\xi)}{9p^2}+\frac{g_s^2\langle GG\rangle m_L p^2}{12(p^2-m_L^2)^3}\right], \end{aligned} \tag{31}$$

It is necessary to emphasize that m coming from the equation of quark motion [Eq.(17)] includes the effect of the condensates of nonperturbative QCD and is different from the pure perturbative (current) quark mass m_L. In a common sense, the current quark mass can be neglected. This is equivalent to neglecting the contribution from the gluon condensate. So S_F^{-1} can be rewritten as

$$S_F^{-1}(p)=\not{p}-M(p), \tag{32}$$

with

$$M(p)=\frac{g_s^2\langle\bar{q}q\rangle}{9p^2}\left[(4-\xi)-\frac{(1-\xi)\not{p}m}{p^2}\right], \tag{33}$$

While requiring the OPE parameter mass to be the pole of the $\langle\bar{q}q\rangle$-corrected quark propagator, i.e.,

$$M(p)|_{\not{p}=m}=\frac{g_s^2\langle\bar{q}q\rangle}{3m^2}=m, \tag{34}$$

We can obtain the solution of m which is independent of the gauge parameter ξ from the above relation,

$$m=M(p)|_{\not{p}=m}=\left[\frac{4\pi\alpha_s(Q^2)\langle\bar{q}q\rangle}{3}\right]^{1/3}, \tag{35}$$

Thus, the $\langle\bar{q}q\rangle$-corrected quark propagator can be written as

$$S_F^{-1}(p)=\not{p}-\left[\frac{4\pi\alpha_s(Q^2)\langle\bar{q}q\rangle}{3}\right]^{1/3}, \tag{36}$$

where the strong coupling constant

$$\alpha_s(Q^2)=\frac{4\pi}{\beta_0\ln\frac{Q^2}{\Lambda^2}}, \tag{37}$$

with $\beta_0=11-\frac{2}{3}N_F$ for $N_F=3$ and the dimensional parameter $\Lambda=0.25$ GeV.

3. NONTRIVIAL Q^2-DEPENDENCE OF NUCLEON STRUCTURE FUNCTION

Consider the inclusive lepton-nucleon scattering

$$1 + \mathrm{N} \to 1' + \mathrm{X}, \tag{38}$$

The hadronic structure is entirely contained in the tensor $W_{\mu\nu}$

$$\begin{aligned} W_{\mu\nu} &= (2\pi)^3 \sum_X \langle P | J_\mu | X \rangle \langle X | J_\nu | P \rangle \delta^4 (P_X - P - q) \\ &= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) W_1 + \frac{1}{M^2} \left(P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left(P_\nu - \frac{P \cdot q}{q^2} q_\nu \right) W_2 , \end{aligned} \tag{39}$$

Here M is the mass of the nucleon. If $W_{\mu\nu}$ is given, we can extract W_1 and W_2 through the following formulas:

$$W_1 = \frac{1}{2} \left[C_2 - \left(1 - \frac{\nu^2}{q^2} \right) C_1 \right] \left[1 - \frac{\nu^2}{q^2} \right]^{-1}, \tag{40}$$

$$W_2 = \frac{1}{2} \left[3C_2 - \left(1 - \frac{\nu^2}{q^2} \right) C_1 \right] \left[1 - \frac{\nu^2}{q^2} \right]^{-2}, \tag{41}$$

with

$$C_1 = W^\mu_\mu, \tag{42}$$

and

$$C_2 = \frac{P^\mu P^\nu}{M^2} W_{\mu\nu} , \tag{43}$$

We assume that in the deep inelastic scattering process, the high-energy lepton interacts with the "quasi-free" quarks in the nucleon. However, the quarks are effected by the quark and gluon condensates in the QCD vacuum.

With the impulse approximation and the incoherence assumption, the one-parton contribution to $W_{\mu\nu}$ is

$$\begin{aligned} w_{\mu\nu} &= (2\pi)^3 \frac{1}{2} \sum_{s,s'} \sum_{p'} \langle p, s | J_\mu | p', s' \rangle \langle p', s' | J_\nu | p, s \rangle \delta^4 (p' - p - q) \\ &= e_i^2 \int \frac{d^3 p'}{2p'_0} \delta^4 (p' - p - q) \frac{1}{2} \mathrm{Tr} [\gamma_\mu (\not p' + m) \gamma_\nu (\not p + m)] , \end{aligned} \tag{44}$$

According to the trace theorem $w_{\mu\nu}$ can be also equivalently expressed as

$$w_{\mu\nu} = e_i^2 \int \frac{d^3 p'}{2p'_0} \delta^4 (p' - p - q) \frac{1}{2} \mathrm{Tr} [\gamma_\mu (\not p' - m) \gamma_\nu (\not p - m)] , \tag{45}$$

i.e.,

$$w_{\mu\nu} = e_i^2 \int \frac{d^3 p'}{2p'_0} \delta^4 (p' - p - q) \frac{1}{2} \mathrm{Tr} [\gamma_\mu S_F^{-1} (p') \gamma_\nu S_F^{-1} (p)] , \tag{46}$$

where $iS_F(p)$ is the quark propagator and e_i is the charge of the quark in unit of e. In order to reflect the effect of the quark and gluon condensates in the QCD vacuum on the quark state, we take S_F^{-1} in Eq.(46) as that in Eq.(36). We adopt the parton picture in which the parton 4-momentum is expressed as

$$p^{\mu} = yp^{\mu} \ (0 \leq y \leq 1). \tag{47}$$

By means of Eqs.(40)-(41), we obtain the contribution of quark i to the nucleon structure function $F_2 = \nu W_2$,

$$F_2^{(i)}(y) = 2Mx^2 e_i^2 \delta(y-x) R_{NP}(Q^2), \tag{48}$$

with Bjorken variable

$$x = \frac{Q^2}{2M\nu} \tag{49}$$

and

$$R_{NP}(Q^2) = 1 - \frac{4}{Q^2}\left(\frac{4\pi\alpha_s\langle\bar{q}q\rangle}{3}\right)^{2/3}. \tag{50}$$

Suppose that the nucleon state contains $f_i(y)$ dy parton states of the type i in the interval dy, then

$$F_2 = \sum_i \int_0^1 \mathrm{d}y f_i(y) F_2^{(i)}(y), \tag{51}$$

As for the normalization of $f_i(y)$, it is noteworthy that a parton state has $2p_0$ partons per unit volume, while a nucleon state has $\frac{P_0}{M}$ nucleons per unit volume. Therefore, in one nucleon, the number of partons of type i, in the interval dy must be multiplied by

$$\frac{2p_0}{(P_0/M)} = 2My, \tag{52}$$

i.e.,

$$q_i(y)\,\mathrm{d}y = 2My f_i(y)\,\mathrm{d}y. \tag{53}$$

where $q_i(y)$ is the quark parton distribution with the constraint of the parton flavor number conservation. Summing the contributions of all quarks in the nucleon, we obtain the structure function of the nucleon

$$F_2 = \sum_i q_i(x) x e_i^2 R_{NP}(Q^2) \equiv \sum_i \tilde{q}_i(x) x e_i^2, \tag{54}$$

where

$$\tilde{q}_i(x) \equiv q_i(x) R_{NP}(Q^2). \tag{55}$$

Obviously, $\tilde{q}_i(x)$ is different from $q_i(x)$ since $q_i(x)$ represents the probability distribution of quarks of type i and satisfies the parton flavor number sum rule, but $\tilde{q}_i(x)$ does not. From Eq.(54), we see that the structure function, with a reducing factor of $R_{NP}(Q^2)$, is no longer simply the sum of the parton momentum distributions multiplied by the square charge of corresponding partons (quarks plus antiquarks) in the nucleon at finite Q^2. This Q^2-dependence in the nucleon structure function is different from the normal one, so it is designated as the nontrivial Q^2-dependence.

4. NONTRIVIAL Q^2-DEPENDENCE OF NUCLEON STRUCTURE FUNCTION AND PARTON SUM RULES

4.1. Nontrivial Q^2-dependence in the Gottfried sum rule

Equation (54) indicates that the nontrivial Q^2-dependences in both neutron and proton structure functions are the same. To show explicitly the nontrivial Q^2-dependence in the nucleon structure function, we give the values of $R_{NP}(Q^2)$ for various Q^2 in Fig. 2. In the estimation of $R_{NP}(Q^2)$, we

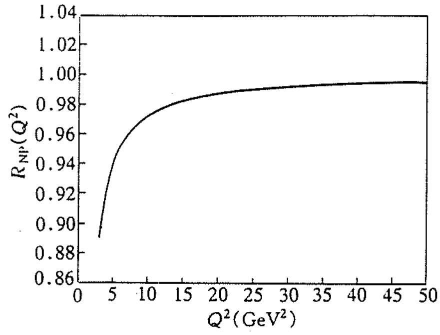

Fig. 2
The value of R_{NP} (Q^2) for various Q^2.

have taken the standard phenomenological value of the quark condensate $\langle \bar{q}q \rangle = -(0.25\text{GeV})^3$ obtained from the QCD sum rule [10].

If we adopt the assumption that sea quarks are SU (2) symmetry, then by using Eq.(54), the Gottfried sum rule can be re-expressed as

$$S_G = \frac{R_{NP}(Q^2)}{3} \int_0^1 [u_v(x) - d_v(x)]\,dx \\ = \frac{R_{NP}(Q^2)}{3}, \tag{56}$$

So the Gottfried sum rule has the same nontrivial Q^2-dependence as shown in Fig. 2 (except a factor $\frac{1}{3}$). In the small Q^2 region there is an obvious suppression compared to that of $\frac{1}{3}$, as expected by the quark-parton model. In our opinion, the suppression of the Gottfried sum rule is partly due to this nontrivial Q^2-dependence.

4.2. Nontrivial Q^2-dependence in the Ellis-Jaffe sum rule

The deviation of the Ellis-Jaffe sum rule from experimental data is mostly related to the nontrivial Q^2-dependence is the nucleon structure function. Now let us discuss qualitatively this problem by setting up the relation between the sum Γ_1^p of the Ellis-Jaffe sum rule and the sum S_G of the Gottfried sum rule.

As is well known, in the limit of $Q^2 = 0$, the axial couplings of the baryon octet are fairly described in terms of the valence quarks

$$u_v^{\uparrow} = 1 + F, \qquad u_v^{\downarrow} = 1 - F, \tag{57}$$

$$d_v^{\uparrow} = \frac{1 + F - D}{2}, \qquad d_v^{\downarrow} = \frac{1 - F + D}{2}. \tag{58}$$

The experimental values of F and D are [11]:

$$F = 0.46 \pm 0.01\,,\; D = 0.79 \pm 0.01 \tag{59}$$

For the sake of simplicity, we approximate the experimental values of F and D by fractions, i.e.,

$$F = \frac{1}{2}, \ D = \frac{3}{4}, \tag{60}$$

and then,

$$u_v^{\uparrow} = \frac{3}{2}, \ u_v^{\downarrow} = \frac{1}{2}, \tag{61}$$

$$d_v^{\uparrow} = \frac{3}{8}, \ d_v^{\downarrow} = \frac{5}{8}, \tag{62}$$

Assuming that the parton distribution at a given larger Q^2 is dependent of its value at $Q^2 = 0$, i.e.,

$$p(x) = \tilde{p}(x, p_v), \tag{63}$$

here $\tilde{p}$ describes the momentum smearing effect due to q-q interactions. For a given x, the smearing increases as a function of p_v. The larger the value of p_v is, the broader the range of the smearing of p_v. Similar to Ref. [12], we have,

$$u^{\downarrow}(x) = \frac{1}{2}[d^{\uparrow}(x) + d^{\downarrow}(x)] = \frac{1}{2}d(x) \tag{64}$$

and

$$\Delta u(x) = u^{\uparrow}(x) - u^{\downarrow}(x) = u(x) - d(x), \tag{65}$$

The above formula reflects the relationship between the polarized u-quark distributions $\Delta u(x)$ and the unpolarized distributions of $u(x)$ and $d(x)$. In consideration of the facts that $\Delta d_v(x) = \frac{1}{4}\Delta u_v$ itself is smaller than Δu_v, and the $\Delta u_v(x)$ term in $g_1^p(x)$ is multiplied by an additional factor 1/4, the contribution of $\Delta u_v(x)$ to $g_1^p(x)$ can be neglected. Without considering the contribution of the polarization of sea quarks, we obtain

$$x g_1^p(x) = \frac{2}{3}[F_2^p(x) - F_2^n(x)], \tag{66}$$

After integrating over x, we obtain

$$\Gamma_1^p = \frac{2}{3} S_G, \tag{67}$$

Equation (67) gives a crude relation between the Ellis-Jaffe sum Γ_1^p and the Gottfried sum S_G. Therefore, we conclude that the nontrivial Q^2-dependence of the Ellis-Jaffe sum rule is the same as that of the Gottfried sum. Thus we would expect a measurable Q^2-dependence in both the Gottfried and Ellis-Jaffe sum rules.

5. DISCUSSION AND SUMMARY

The nontrivial Q^2-dependence of the nucleon structure function results from the nonperturbative effect due to the quark condensate in the QCD vacuum. The magnitude of the Q^2-dependence of the nucleon structure function is of the order 0.06 GeV^2/Q^2 which is consistent with the Q^2-dependence as observed in experiments and as estimated from the perturbative QCD and higher twist effects in the other parton sums. For the whole investigation of the Q^2-dependence of the nucleon structure function, the nontrivial Q^2-dependence should not be neglected.

We investigate the modification of the QCD vacuum to the quark propagator and give the nonperturbative quark propagator which is independent of the gauge parameter ξ. By using the obtained nonperturbative quark propagator, we re-analyze the nucleon structure function. The results show that the nonperturbative effect results in a nontrivial Q^2-dependence in the nucleon structure function. We expect that this nontrivial Q^2-dependence can be measured by means of the Gottfried sum rule and the Ellis-Jaffe sum rule.

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