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External Momentum Expansion in NJL Model*

Huang Mei[1] Zhuang Pengfei[2,3] Zhao Weiqin[1,3]

1 (*Institute of High Energy Physics, The Chinese Academy of Sciences, Beijing* 100039)

2 (*Physics Department, Tsinghua University, Beijing* 100084)

3 (*CCAST, Beijing* 100080)

Abstract In the large N_c expansion beyond mean-field approximation, we develop a general scheme of $SU(2)$ NJL model including current quark mass explicitly. In our scheme, the constituent quark's propagator is expanded in pion's external momentum k, and all the Feynman diagrams are naturally expanded to k^2 term in a unified way. Our numerical results show that in the mean field approximation, the effect of current quark mass is invisible, however, the effect of current quark mass can be seen explicitly beyond mean-field approximation for reasonable choices of the parameters in NJL model.

Key words NJL model, large N_c expansion, external momentum expansion.

The NJL model in the leading $1/N_c$ approximation, i.e., Hartree plus random-phase approximation (RPA), has been quite successful for describing low-energy meson physics in zero and finite temperature and density[1-3]. At this level there is no back contribution of meson modes to the quark propagator. If one tries to apply NJL model to a real physical process including pion at low energy, one must consider massive pion's contribution, which can not be obtained at the mean-field approximation level.

Among many efforts considering meson corrections, only [4] and [5] gave us chirally symmetric self-consist approximation schemes, in which, all the chiral theorems, i.e., Goldstone's theorem, the Goldberger-Treiman relation, and the conservation of the axial quark current, are obeyed in the chiral limit. In this paper, we extend the method of[5], and develop a general scheme for explicit chiral symmetry breaking of $SU(2)$ NJL model[6], which is necessary for our future work to analyze processes related to pion at low momentum.

The two-flavor NJL model is defined through the Lagrangian density,

$$\mathscr{L}=\bar{\psi}(\mathrm{i}\gamma^{\mu}\partial_{\mu}-m_0)\psi+G[(\bar{\psi}\psi)^2+(\bar{\psi}\mathrm{i}\gamma_5\tau\psi)^2], \tag{1}$$

where G is the effective coupling constant with dimension GeV^{-2}, and m_0 is the current quark mass, assuming isospin degeneracy of the u and d quarks, and ψ, $\bar{\psi}$ are quark fields with flavor, colour and spinor indices suppresseed.

The complete description is represented by two Schwinger-Dyson (SD) integral

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equations, i.e., the constituent quark propagator and the composite meson propagator, see Fig. 1 a and Fig. 1 b, and the two SD equations must couple to each other self-consistently and keep chiral symmetric relations. In Fig. 1, the one-vertex grey bubble *kernel a* represents the quark self-energy, and the two-vertex grey bubble *kernel b* indicates the meson polarization function.

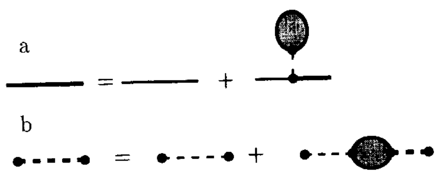

Fig. 1. The quark propagator a and the meon propagator b. The light dashed line in a and b represents a four-fermion vertex $2iG$ of the NJL type.

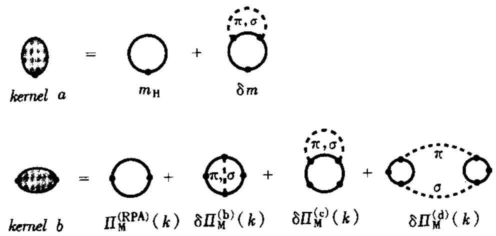

Fig. 2. *kernel a* and *kernel b*. m_H and δm indicate the leading and subleading order of quark self-energy, and $\Pi_M^{(RPA)}$ and $\delta\Pi_M^{(b,c,d)}$ represent the leading and subleading order of meson polarization function. The heavy dashed lines indicate internal meson propagator.

It is difficult to give the full expressions of the two kernels. Usually an approximation scheme called large N_c expansion is adopted in NJL model. V. Dmitrašinović et al. in their paper[5] proved that the *kernel a* and *kernel b* shown in Fig. 2 are self-consistent leading and subleading order in $1/N_c$ expansion and can keep all the chiral symmetric relations in the chiral limit. The heavy solid line indicates the constituent quark propagator with total mass m, and the heavy dashed line is the internal meson propagator $D_M^{(RPA)}(q)$, i. e., a string of single quark loops. The leading $O(1)$ and the subleading $O(1/N_c)$ order of *kernel a* are named m_H and δm, and the leading $O(N_c)$ and subleading $O(1)$ order of *kernel b* are expressed as $\Pi_M^{(RPA)}(k)$ and $\delta\Pi_M(k)$, respectively, where M represents pseudoscalar meson π and scalar meson σ.

Including current quark mass m_0, the gap equation of Fig. 1 a can be expressed as

$$m = m_0 + m_H + \delta m \ , \tag{2}$$

The meson propagator $D_M(k)$ of Fig. 1 b has the form

$$-iD_M(k) = \frac{2iG}{1 - 2G\Pi_M(k)} \ . \tag{3}$$

Meson mass m_M satisfies meson propagator's pole condition

$$1 - 2G\Pi_M(k^2 = m_M^2) = 0, \tag{4}$$

while the coupling constant g_{Mqq} is determined by the residue at the pole

$$g_{Mqq}^{-2} = (\partial\Pi_M(k) / \partial k^2)^{-1}|_{k^2 = m_M^2} \ . \tag{5}$$

Another important quantity in NJL model is the pion decay constant f_π, which generally satisfies

$$\frac{m_\pi^2 f_\pi}{g_{\pi qq}} = \frac{m_0}{2G} \ . \tag{6}$$

In the chiral limit, f_π satisfies Goldberger-Treiman relation $f_\pi(k)g_{\pi qq}(k) = m$.

To calculate the two-loop Feynman diagrams of *kernel b*, we adopt the small external momentum expansion. In the NJL model, the constituent quark is the fundamental element, and mesons are bound states of constituent quark and anti-quarks. Differently from [5], we only expand constituent quark propagator in small external momentum k

$$S(p \mp k) = \frac{1}{\not p \mp \not k - m} = S(p) \pm S(p)\not k S(p) + S(p)\not k S(p)\not k S(p) + \cdots. \tag{7}$$

With this expansion form, all the two-loop Feynman diagrams can be expanded naturally in k in a unified way. The pion propagator pole condition expanded to k^2 term is

$$m_\pi^2 = \frac{m_0}{Gm(-8N_c \mathrm{i}I(m_\pi) + 2\delta\Pi_\pi^{(2)}(0))}. \tag{8}$$

Correspondingly, the approximate form of $g_{\pi qq}$ is

$$g_{\pi qq}^{-2} = (\partial \Pi_\pi^{(\mathrm{RPA})}(k) / \partial k^2)^{-1}|_{k^2 = m_\pi^2} + \delta\Pi_\pi^{(2)}(0), \tag{9}$$

$\delta\Pi_\pi^{(2)}(0)$ can be calculated from the first term of axial-vector matrix element in the external momentum expansion,

$$\delta\Pi_\pi^{(2)}(0) = M(0) = M^{(b)}(0) + M^{(c)}(0) + M^{(d)}(0). \tag{10}$$

$M(0)$ are calculated and expressed as following:

$$M^{(b)}(0) = \mathrm{i}N_c\left\{\int\frac{\mathrm{d}^4q}{(2\pi)^4}[-\mathrm{i}D_\pi^{(\mathrm{RPA})}(q)](-3q^2L(q)) + \int\frac{\mathrm{d}^4q}{(2\pi)^4}[-\mathrm{i}D_\sigma^{(\mathrm{RPA})}(q)](4K(q) + 3(4m^2 - q^2)L(q))\right\}, \tag{11}$$

$$M^{(c)}(0) = \mathrm{i}N_c\left\{6\int\frac{\mathrm{d}^4q}{(2\pi)^4}[-\mathrm{i}D_\pi^{(\mathrm{RPA})}(q)](K(q) + 3K(0) - 3q^2M(q)) + 2\int\frac{\mathrm{d}^4q}{(2\pi)^4}[-\mathrm{i}D_\sigma^{(\mathrm{RPA})}(q)](5K(q) + 3K(0) - 3(q^2 - 4m^2)M(q))\right\}, \tag{12}$$

$$M^{(d)}(0) = -32\mathrm{i}N_c\int\frac{\mathrm{d}^4q}{(2\pi)^4}N_c[-\mathrm{i}D_\pi^{(\mathrm{RPA})}(q)][-\mathrm{i}D_\sigma^{(\mathrm{RPA})}(q)]\{-(I(q) + 2m^2K(0))(I(q) - I(0)) + (I(q) + I(0) - (q^2 + 2m^2)K(q))I(q) - q^2I(q)(I(q) + 2m^2K(0))[-\mathrm{i}D_\sigma^{(1)}(q)]\}, \tag{13}$$

$$[-\mathrm{i}D_\sigma^{(1)}(q)] = 8N_c(I(q) + ((4m^2 - q^2)/2q^2)(I(q) - I(0) + q^2K(q))[-\mathrm{i}D_\sigma^{(\mathrm{RPA})}(q)]), \tag{14}$$

where, $I(q)$, $K(q)$, $L(q)$, and $M(q)$ are quark-loop integrals

$$I(q) = \int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{1}{(p^2 - m^2)((p+q)^2 - m^2)}, \quad K(q) = \int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{1}{(p^2 - m^2)^2((p+q)^2 - m^2)},$$

$$L(q) = \int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{1}{(p^2 - m^2)^2((p+q)^2 - m^2)^2}, \quad M(q) = \int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{1}{(p^2 - m^2)^3((p+q)^2 - m^2)},$$

and the $O(1/N_c)$ internal meson propagators are $[-\mathrm{i}D_\pi^{(\mathrm{RPA})}(q)] = 1/[4N_c(-m_\pi^2 I(m_\pi) + q^2 I(q))]$ for π, and $[-\mathrm{i}D_\sigma^{(\mathrm{RPA})}(q)] = 1/[4N_c(-m_\pi^2 I(m_\pi) + (q^2 - 4m^2)I(q))]$ for σ.

Now we turn to the numerical evaluation. We have introduced the quark momentum cut-off Λ_f in Pauli-Villars regularization and the meson momentum cut-off Λ_b in covariant regularization.

By comparing with two observables $m_\pi = 139\text{MeV}$, $f_\pi = 92.4\text{MeV}$ and one reasonable experimental range of $-300\text{MeV} < \langle\bar{q}q\rangle^{1/3} < -200\text{MeV}$, we can not give fixed values of the four parameters, the current quark mass m_0, coupling constant G, quark momentum cut-off Λ_f and meson momentum cut-off Λ_b. Here we regard the ratio $z = \Lambda_b / \Lambda_f$ as one free parameter. For each z, we can get a series of solutions from the above conditions. The meson cloud effects are now characterized by z. The larger z means more meson contributions. Specially, when $z = 0$, i.e., $\Lambda_b = 0$, which returns to the mean-field approximation. For each z, there is a region where the quark condensate is almost a constant when other quantities change, and we define this "plateau region" by $(-\langle\bar{q}q\rangle^{1/3}) < (-\langle\bar{q}q\rangle^{1/3}_{\min} + 0.0015)\text{GeV}$.

Our numerical results are shown in Table 1, where we list the corresponding region of constituent quark mass m, current quark mass m_0, quark momentum cut-off Λ_f within the quark condensate "plateau". This table shows that:

Table 1. Quantities in the region of defined plateaus

		$-\langle\bar{q}q\rangle^{1/3}$/ GeV	m/GeV	m_0/MeV	Λ_f/GeV
$z=0$	$m_0=0$ $m_0\neq0$	0.2096+0.0015	0.47∓0.10	8.90∓0.30	0.615±0.002
$z=1$	$m_0=0$	0.2397+0.0015	0.39∓0.05		0.725±0.010
	$m_0\neq0$	0.2360+0.0015	0.40∓0.05	7.78∓0.20	0.710±0.010
$z=1.5$	$m_0=0$	0.2599+0.0015	0.38∓0.04		0.801±0.008
	$m_0\neq0$	0.2566+0.0015	0.39∓0.04	7.29∓0.20	0.785±0.008

1) In the mean-field approximation $z=0$, the values of constituent quark mass m and the current mass m_0 is a little higher than the empirical values $m\simeq1/3$ proton mass and $m_0 \simeq 5\sim7\text{MeV}$, and the quark condensate within the plateau is much lower than 0.25 GeV; however, these quantities in the plateaus at $z = 1, 1.5$ are more reasonable comparing with the empirical values.

2) In the mean-field approximation, the values of $\langle\bar{q}q\rangle$ in the case of $m_0=0$ and $m_0 \neq 0$ are the same, and so are the values of m and Λ_f. The values become different when considering meson corrections for $z\neq0$, i.e., beyond mean-field approximation. This shows that meson modes have feedback to the quark self-energy beyond mean-field approximation, and it is reflected by the quantities calculated from quark self-energy.

3) Comparing the values of $-\langle\bar{q}q\rangle^{1/3}$ and Λ_f at $z = 0$, they are all corrected by the order of 30% at $z = 1.5$.

Our conclusions are, only beyond mean-field approximation, can we see the effects of current quark mass explicitly, and the parameters become more reasonable in the quark condensate plateau comparing to the empirical values.

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NJL 模型中的外动量展开*

黄　梅[1]　庄鹏飞[2,3]　赵维勤[1,3]

1（中国科学院高能物理研究所　北京　100039）
2（清华大学物理系　北京　100084）
3（中国高等科学技术中心　北京　100080）

摘要　建立了包括流夸克质量的 $SU(2)$ NJL 模型超出平均场近似的一般框架．在这个框架内，组份夸克传播子按外动量展开，所有的费曼图可统一地得到计算．数值结果表明：在平均场近似下，流夸克质量的效应几乎看不见，只有超出平均场近似，流夸克质量的效应才能体现出来，并且 NJL 模型的参数更为合理．

关键词　NJL 模型　大 N_c展开　外动量展开

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