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Description of Staggering Phenomenon of the Superdeformed Bands in $SU_q(2)$ Symmetry *

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Abstract The staggering phenomenon of the superdeformed bands ^{149}Gd(1), ^{148}Gd(6) and ^{148}Eu (1) is analyzed with a $SU_q(2)$ rotational spectroscopy expression. When the spin of the head of band is chosen properly, the calculating bifurcation of $\Delta I = 4$ is agreement with that extracted from experiment.

Key words superdeformed bands, staggering phenomenon, $SU_q(2)$ symmetry

1 Introduction

Since the phenomenon known as $\Delta I = 4$ bifurcation, or staggering is observed in the yrast superdeformed (SD) band of ^{149}Gd[1], the study of staggering phenomenon has brought many physicist's attention. A large quantity of experimental and theoretical effort has been expanded in the study of $\Delta I = 4$ bifurcation. On the experimental side[2], very few examples of staggering have been identified. Theoretical endeavors have yielded a number of proposed explanations. An intuitive explanation of this specific feature is the presence of a new C_4 symmetry in SD Hamiltonian[3], or in other words, the presence of Y_{44} deformation in SD nucleus. On the other hand, I. N. Mikhailov et al.[4] attributed the $\Delta I = 4$ bifurcation to a typical collective model including two quantized quantities, the angular momentum and the Kelvin circulation associated with an intrinsic uniform vertical motion. Y. SUN et al.[5] proposed a possibility of $\Delta I = 4$ bifurcation from a single two-band mixture near the yrast line through the usual two-body shell model interaction. Moreover, the $\Delta I = 4$ bifurcation may also appear in the $U(5)$ limit of the sdg interacting boson model[6]. In this paper, the $SU_q(2)$ symmetrical theory is used to analyze the staggering phenomenon which can give out a excellent description.

2 Analysis of Staggering Phenomenon in $SU_q(2)$ Symmetry

It is well known that the Hamiltonian of the rotator is of the following type

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$$H = \frac{C_2}{2\mathscr{J}}, \tag{1}$$

where C_2 is the Casimir operator of $SU(2)$, $C_2 = \boldsymbol{L}^2 = L_- L_+ + L_0(L_0+1)$, $\boldsymbol{L}$ is the operator of angular momentum and $L_\pm = L_x \pm \mathrm{i}L_y$, $L_0 = L_z$ are the components of $\boldsymbol{L}$. These operators satisfy the standard commutation relations for the generators of $SU(2)$

$$[L_0, L_\pm] = \pm L_\pm, [L_+ L_-] = 2L_0. \tag{2}$$

The spectrum of the Hamiltonian (1) is given by the eigenvalues $l(l+1)$ of the Casimir operator, where $l = 0, 1, 2, \cdots$. Three generators L_+, L_-, L_0 can be mapped to the 2×2 fundamental realization with a pair of independent boson operators:

$$L_- = a_1^+ a_2, L_- = a_2^+ a_1, L_0 = \frac{1}{2}(a_1^+ a_1 - a_2^+ a_2) = \frac{1}{2}(N_1 - N_2), \tag{3}$$

where $[a_i, a_j^+] = \delta_{ij}$ with all other brackets vanishing.

While the Hamiltonian of a q - rotator is of the following type[7]

$$H = \frac{C_2^q}{2\mathscr{J}_q}, \tag{4}$$

where C_2^q is the Casimir operator of $SU_q(2)$, $C_2^q = \boldsymbol{J}^2 = J_- J_+ + J_0(J_0+1)$. Three generators J_+, J_-, J_0 satisfy the commutation relations of $SU_q(2)$

$$[J_0, J_\pm] = \pm J_\pm, [J_+, J_-] = [2J_0]. \tag{5}$$

The q - numbers

$$[x] = \frac{q^x - q^{-x}}{q - q^{-1}} \tag{6}$$

are given in square brackets. The spectrum of the Hamiltonian (4) is given by the eigenvalues $[j][j+1]$ of the Casimir operator of $SU_q(2)$, where $j = 0, 1, 2, \cdots$. Three generators J_+, J_-, J_0 can also be mapped to the 2×2 fundamental realization with a pair of independent q - deformed boson operators:

$$J_+ = b_1^+ b_2, J_- = b_2^+ b_1, \tag{7}$$

where $b_i b_i^+ = [N_i + 1]$, $b_i^+ b_i = [N_i]$ or $b_i b_i^+ - q^{-1} b_i^+ b_i = q^{N_i}$.

To a system with the symmetry of $SU(2)$ and the break of $SU(2)$ [i.e. $SU_q(2)$], the Hamiltonian can be written

$$H = \frac{C_2}{2\mathscr{J}} + \frac{C_2^q}{2\mathscr{J}_q}, \tag{8}$$

with the definition

$$b_i^+ = \sqrt{\frac{[N_i]}{N_i}} a_i^+, \quad b_i = \sqrt{\frac{[N_i+1]}{N_i+1}} a_i, \qquad (i = 1, 2). \tag{9}$$

It is easy to verify

$$[C_2, C_2^q] = 0. \tag{10}$$

In the space of basic vectors $|IM\rangle$, we have obtained the eigenvalues of Hamiltonian (8)

$$E(I) = \frac{I(I+1)}{2\mathscr{J}} + \frac{[I][I+1]}{2\mathscr{J}_q}. \tag{11}$$

With $q = e^{i\tau}$

$$E(I) = aI(I+1) + bI^2(I+1)^2 + c\,\frac{\sin[I\tau]\sin[(I+1)\tau]}{\sin^2\tau}, \tag{12}$$

the second term is a oscillating modification. When $\tau = 0$, Formula (12) is changed two-parameter expression of Bohr-Mottelson: $E(I) = aI(I+1) + bI^2(I+1)^2$, which is successful in describing normal deformed bands. However, it can not explain the staggering phenomenon emerged in SD bands. Formula (12) can be used to analyze the staggering property of SD bands. In order to see it, we calculate the difference of four-order for ^{149}Gd(1), ^{148}Gd(6) and ^{148}Eu(1) by the formula, which is proposed by B. Cederwall[8]

$$\delta^4 E_\gamma(I) = \frac{1}{16}[E_\gamma(I-4) - 4E_\gamma(I-2) + 6E_\gamma(I) - 4E_\gamma(I+2) + E_\gamma(I+4)], \tag{13}$$

here $\delta^4 E_\gamma(I)$ is only caused by the third term of Formula (12).

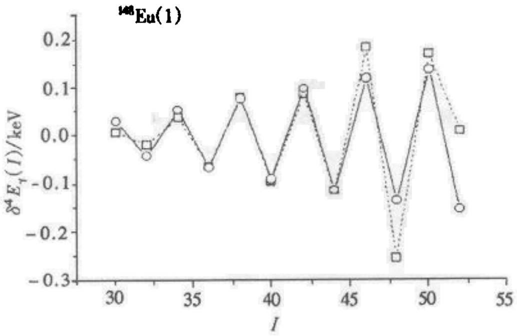

Fig. 1.

…□…Exp; —○—Cal.

$I_0 = 26$, $c = 0.111$, $\tau = 0.80$.

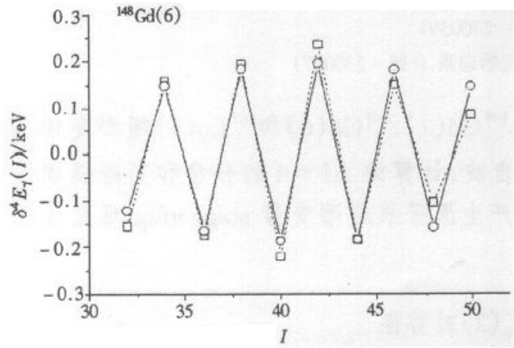

Fig. 2.

…□…Exp; —○—Cal.

$I_0 = 28$, $c = 0.089$, $\tau = 0.74$.

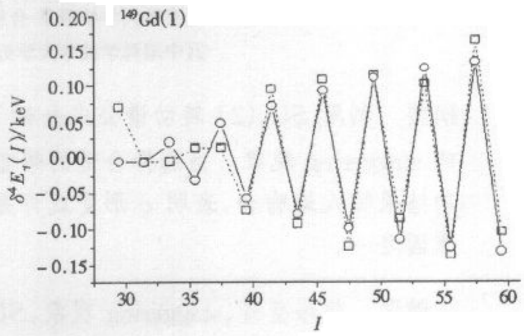

Fig. 3.

…□…Exp; —○—Cal.

$I_0 = 25$, $c = 0.071$, $\tau = 0.81$.

The experimental values of $\delta^4 E_\gamma(I)$ are also extracted from the observed SD transition energies for ^{149}Gd(1), ^{148}Gd(6) and ^{148}Eu(1). The results are given in Fig.1.—3.

The calculating bifurcation of $\Delta I = 4$ is agreement in excellent with that of extracted from experiment. When a correct I_0 is chosen, deviation is very little. To the ^{149}Gd(1), I_0 is assigned as 25.5. For ^{148}Eu(1), I_0 is assigned as 26 and for ^{148}Gd(6), I_0 is assigned as 28. All of them give the least deviations between theoretical calculation and experimental energy spectroscopy. This shows that it can also be used to determine the spin of head of SD bands.

3 Conclusion

By using rotational spectroscope expression with the $SU_q(2)$ symmetry, the staggering phenomenon of superdeformed bands can be reproduced with a correct I_0. It shows that the break of $SU(2)$ may be one of the causes which produces $\Delta I = 4$ bifurcation.

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超形变带 Staggering 现象的 $SU_q(2)$ 描述

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摘要 利用 $SU_q(2)$ 转动谱公式分析了^{149}Gd(1),^{148}Gd(6)和^{148}Eu(1)超形变带的 staggering 现象。当选择合适的带首自旋,计算的 $\Delta I = 4$ 的分岔和实验提取的结果惊人地吻合,表明 q 形变或许是产生原子核超形变带 staggering 现象的原因之一。

关键词 超形变带,staggering 现象,$SU_q(2)$对称性

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