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Dependence of ρ Meson mass on Temperature and Chemical Potential *

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Abstract We studied the medium effects on ρ meson mass by taking account of the vacuum effects with the effective Lagrangian approach. We show that the ρ meson mass decreases with both temperature and chemical potential. Then we checked the validity of Brown-Rho scaling law in different conditions.

Key words meson mass, vacuum effects, ρNN interaction, scaling law

Recently, the problem of how the properties of meson change at high temperature and / or density has acquired a lot of attention[1—12,16]. It can help us to understand the properties of the chiral phase transition. Among these topics one of the most interesting is the vector meson masses in medium. So far it has been studied in many articles. However, the results obtained by different methods do not coincide. Some[3,4] find the vector meson masses decrease with temperature and / or density which is interpreted as evidence of the partial restoration of chiral symmetry[5,6]. For example, using effective chiral Lagrangians with a suitable incorporation of the scaling property of QCD, it has been shown[1] that the effective masses of σ, ρ, ω mesons and nucleons satisfy the approximate in-medium scaling law

$$\frac{m_\sigma^*}{m_\sigma} \approx \frac{m_\rho^*}{m_\rho} \approx \frac{m_\omega^*}{m_\omega} \approx \frac{m_N^*}{m_N}. \tag{1}$$

Whereas others[7,8] prove that the vector meson masses increase in hot and/or dense matter. Because the structure of the vacuum in hot and/or dense medium will be changed, however, the vacuum effect on the vector meson masses should be taken into account[1,9—12]. With this idea some authors[10—12] calculate the vector meson masses again by using the methods which give arising vector meson masses before and find now the vector meson masses reduced at either finite temperature of finite chemical potential. In this letter, based on the ρNN interaction and finite temperature field theory, we investigate the ρ meson mass in hot and dense matter by including the vacuum contribution. We will get the temperature and chemical potential dependence of ρ meson mass in an unified method, and find the ρ meson mass decrease with temperature and chemical potential. Then we will check the Brown-Rho scaling law Eq. (1) in different temperature and chemical potential.

At first we can use the Walecka model (often called QHD-I)[13] to get the change of the nucleon mass at finite temperature and density. The Lagrangian density in the Walecka model is given by

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$$\mathscr{L} = \bar{\psi}[\gamma_\mu(i\partial^\mu - g_v V^\mu) - (M - g_s\phi)]\psi + \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2) -$$

$$\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_v^2 V_\mu V^\mu + \delta\mathscr{L},$$

which contains fields for nucleon p, n(ψ), neutral scalar meson $\sigma(\phi)$ and vector meson $\omega(V_\mu)$. From the mean-field theory[14], the self-consistency condition can be derived as

$$M^* = M - g_s\phi = M - \frac{g_s^2}{m_s^2}\frac{2}{\pi^2}\int p^2 \mathrm{d}p \frac{M^*}{\omega_N^*}[n_p(T) + \bar{n}_p(T)], \tag{2}$$

where M^* is the nucleon effective mass and

$$n_p(T) = \frac{1}{e^\beta(\omega_N^* + \mu) + 1},\ \bar{n}_p(T) = \frac{1}{e^\beta(\omega_N^* - \mu) + 1},\ \omega_N^* = \sqrt{p^2 + M^{*2}},\ \beta = \frac{1}{T}.$$

We can see that the main contributions of the reduction of M^* come from the effect of scalar meson σ. Taking the parameters[11] $g_s^2 = 109.626$, $m_s = 520\mathrm{MeV}$, we can numerically get the nucleon effective mass M^* from Eq.(2).

Next we will think of the ρ meson effective mass in hot and dense matter. In Minkowski space the self-energy can be generally expressed as

$$\Pi^{\mu\nu} = FP_L^{\mu\nu} + GP_T^{\mu\nu}, \tag{3}$$

where $P_L^{\mu\nu}$ and $P_T^{\mu\nu}$ are standard longitudinal and transverse projection tension respectively, and are defined as

$$P_T^{00} = P_T^{0i} = P_T^{i0} = 0,\ P_T^{ij} = \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2},P_L^{\mu\nu} = \frac{k^\mu k^\nu}{\mathbf{k}^2} - g^{\mu\nu} - P_T^{\mu\nu}.$$

So the self-energy is related to the inverse of the full and bare propagators by

$$\Pi^{\mu\nu} = (\mathscr{D}^{-1})^{\mu\nu} - (\mathscr{D}_0^{-1})^{\mu\nu}. \tag{5}$$

As for ρ meson, we have

$$(\mathscr{D}_0^{-1})^{\mu\nu} = (k^2 - m_\rho^2)g^{\mu\nu} - k^\mu k^\nu$$

By using Eqs.(3)—(5), we can get the full propagator

$$\mathscr{D}^{\mu\nu} = -\frac{P_L^{\mu\nu}}{k^2 - m_\rho^2 - F} - \frac{P_T^{\mu\nu}}{k^2 - m_\rho^2 - G} - \frac{k^\mu k^\nu}{m_\rho^2 k^2} \tag{6}$$

For $P_T^{00} = 0$, we can know $P_L^{00} = \frac{k^{0^2}}{k^2} - g^{00} - P_T^{00} = \frac{\mathbf{k}^2}{k^2}$, and

$$\Pi^{00}(k) = F(k)P_L^{00} + G(k)P_T^{00} = F(k)\frac{\mathbf{k}^2}{k^2},$$

$$F(k) = \frac{k^2}{\mathbf{k}^2}\Pi^{00}(k).$$

In order to get the effective mass of ρ meson, we can take the limit $\mathbf{k} \longrightarrow 0$, so $F = G$. Then the effective mass of ρ meson in hot and dense medium will be obtained from the pole of the full propagator

$$\omega^2 - m_\rho^2 - \mathrm{Re}[\lim_{\mathbf{k}\to 0}F(k)] = 0.$$

Now we think of the nucleon-nucleon-meson interactions. The Lagrangian density for ρNN interaction is

$$\mathscr{L}_1 = g_{\rho NN}[\bar{\psi}\gamma_\mu\tau\psi\rho^\mu(x) + \frac{k_\rho}{2M}\bar{\psi}\sigma_{\mu\nu}\tau\psi\partial^\mu\rho^\nu(x)], \tag{9}$$

where ρ^μ and ψ are the ρ meson and nucleon fields, respectively. Using the imaginarytime formalism of the finite temperature field theory, we obtain the ρ meson selfenergy (Fig. 1) as

$$\prod_{\rho}^{\mu\nu} = 2g_{\rho NN}^2 T\sum_{n=-\infty}^{+\infty}\int\frac{d^3p}{2\pi^3}\mathrm{Tr}\Big[\Gamma_\mu(\kappa)\frac{1}{\gamma^s p_s - M^*}\Gamma_\nu(-k)\frac{1}{\gamma^s(p_s - k_s) - M^*}\Big], \qquad (10)$$

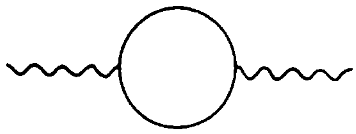

Fig. 1. One-loop diagram for ρ meson self-energy in the ρNN interaction

with $p_0 = (2n+1)\pi T\mathrm{i} + \mu$, $k_0 = 2l\pi T\mathrm{i}$, $\Gamma_\mu(k) = \gamma_\mu + \frac{\mathrm{i}\kappa_\rho}{2M}\sigma_{\mu\nu}k^\nu$ From the mathematical theorem

$$T\sum f(p_0 = (2n+1)\pi T\mathrm{i} + \mu) = \frac{T}{2\pi\mathrm{i}}\oint \mathrm{d}p_0 f(p_0)\frac{1}{2}\beta\tanh\Big[\frac{1}{2}\beta(p_0 - \mu)\Big],$$

we can separate the vacuum and the matter contributions as[15]

$$T\sum f(p_0 = (2n+1)\pi T\mathrm{i} + \mu) = -\frac{1}{2\pi\mathrm{i}}\int_{-\mathrm{i}\infty+\mu+\epsilon}^{+\mathrm{i}\infty+\mu+\epsilon}\mathrm{d}p_0 f(p_0)\frac{1}{\mathrm{e}^{\beta(p_0-\mu)}+1}$$
$$\frac{1}{2\pi\mathrm{i}}\int_{-\mathrm{i}\infty+\mu-\epsilon}^{+\mathrm{i}\infty+\mu-\epsilon}\mathrm{d}p_0 f(p_0)\frac{1}{\mathrm{e}^{-\beta(p_0-\mu)}+1} + \frac{1}{2\pi\mathrm{i}}\int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty}\mathrm{d}p_0 f(p_0 - \mu).$$

The first and second terms are matter contribution related with temperature and chemical potential, the last term is the vacuum contribution.

We calculate the last term firstly and find it infinite. Using dimensional regularization to get rid of the divergence and from Eq. (7) we can get

$$\mathrm{Re}[\lim_{k\to 0}F_{\rho,\mathrm{vac}}(\kappa)] = \frac{g_{\rho NN}^2}{\pi^2}\omega^2\Big[P_1 + M^*\Big(\frac{\kappa_\rho}{2M}\Big)P_2 + \frac{1}{2}\Big(\frac{\kappa_\rho}{2M}\Big)^2(\omega^2 P_1 + M^{*2}P_2)\Big], \qquad (11)$$

where

$$P_1(\omega) = \int_0^1 \mathrm{d}zz(1-z)\ln\Big[\frac{M^{*2} - \omega^2 z(1-z)}{M^2 - m_\rho^2 z(1-z)}\Big],$$
$$P_2(\omega) = \int_0^1 \mathrm{d}z\ln\Big[\frac{M^{*2} - \omega^2 z(1-z)}{M^2 - m_\rho^2 z(1-z)}\Big].$$

It is easily to be seen when $T = 0$ and $\mu = 0$ the matter contribution will vanish and return to the result of the zero temperature field.

Now we calculate the matter contribution. With the Residue Theorem and the periodicity condition of κ_0 in the finite temperature field theory[15], we can get

$$\mathrm{Re}[\lim_{k\to 0}F_{\rho,\mathrm{matt}}(k)] = -\frac{4g_{\rho NN}^2}{\pi^2}\int p^2\mathrm{d}p\cdot$$
$$\frac{1}{\omega_N^*(\omega^2 - 4\omega_N^{*2})}\Big[\frac{1}{\mathrm{e}^{\beta(\omega_N^*+\mu)}+1} + \frac{1}{\mathrm{e}^{\beta(\omega_N^*-\mu)}+1}\Big]\cdot$$
$$\Big[\Big(\frac{4}{3}p^2 + 2M^{*2}\Big) + 2M^*\Big(\frac{\kappa_\rho}{2M}\Big)\omega^2 + \Big(\frac{\kappa_\rho}{2M}\Big)^2\Big(\frac{2p^2}{3} + 2M^*\Big)\omega^2\Big]. \qquad (12)$$

One point should be noted that ρ NN interaction will affect M^*. However, as pointed out by some authors[16] that the contribution due to vector meson exchange for M^* is very small and then can be neglected. This coincides with our treatment of nucleon mass M^*, and then there will be no effect of vector mesons on M^* because M^* is determined by the scalar field only.

After taking the parameters[17] as $\frac{g_{\rho NN}^2}{4\pi} = 0.92$, $\kappa_\rho = 6.1$, we can solve Eq. (8) to deter-

mine m_ρ^* where M^* is given by Eq. (2). Therefore, we can get the modified ρ meson mass as a function of the temperature and chemical potential. In Fig. 2 we show the change of the ρ meson mass with temperature at $\mu = 0$ and $\mu = 200$MeV respectively and find the ρ meson mass decreases with temperature. In Fig. 3 we show the chemical potential dependence of ρ meson mass at $T = 100$MeV and find the ρ meson mass decreases also with increasing chemical potential μ which is consistent with the result derived with another method[18]. With our result we can check the Brown-Rho scaling law, Eq. (1). We find that the scaling law is good when T and μ is not too high. When the temperature and chemical potential is very high the scaling law will be broken(Fig. 4 and Fig. 5). It possibly means that the scaling law will be broken when the temperature and density are near the critical point of phase transition as the vacuum will change violently in the vicinity of the critical point. This is an interesting problem which needs further discussion.

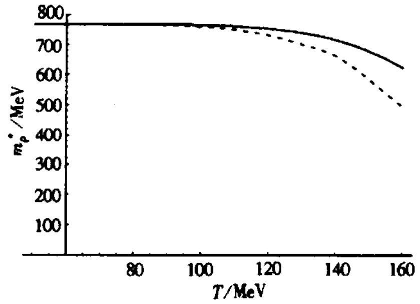

Fig. 2. The temperature dependence of ρ meson effective mass,
The solid line represents $m_\rho^*(T)$ at $\mu = 0$, the dashed line represents $m_\rho^*(T)$ at $\mu = 200$MeV.

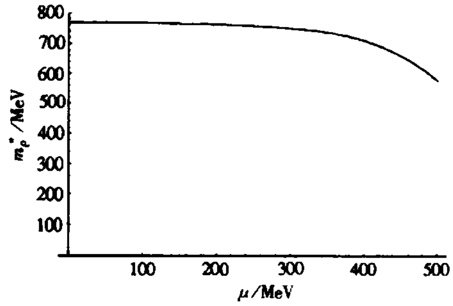

Fig. 3. The chemical potential dependence of ρ meson effective mass at $T = 100$MeV.

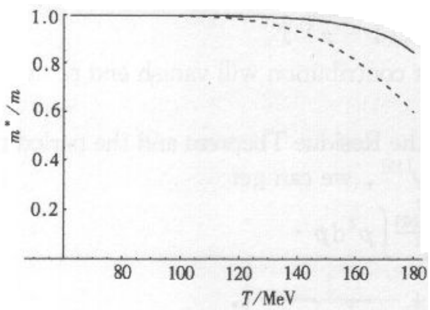

Fig. 4. The temperature dependence of $\frac{M_N^*}{M_N}$ and $\frac{m_\rho^*}{m_\rho}$ at $\mu = 0$,
The solid line represents $\frac{M_N^*}{M_N}$, the dashed line represents $\frac{m_\rho^*}{m_\rho}$.

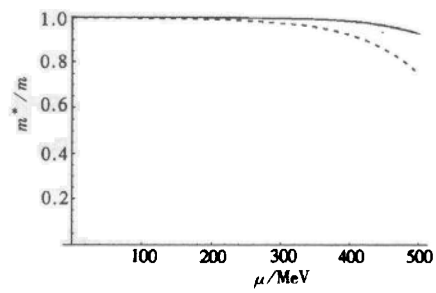

Fig. 5. The chemical potential dependence of $\frac{M_N^*}{M_N}$ and $\frac{m_\rho^*}{m_\rho}$ at $T = 100$MeV
where the solid line represents $\frac{M_N^*}{M_N}$, and the dashed line represents $\frac{m_\rho^*}{m_\rho}$.

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ρ 介子质量对温度和化学势的依赖性*

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摘要　考虑到真空效应,用有效拉氏量方法给出了介质对 ρ 介子质量的影响,发现 ρ 介子质量随温度和化学势的升高而降低. 然后用所得到的结果检验了 Brown-Rho 标度定理的适用范围.

关键词　介子质量　真空效应　ρNN 相互作用　Brown-Rho 标度定理

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