第24卷第6期 高能物理与核物理 Vol.24, No.6
2000年6月 HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS June, 2000

Half-Life and the Nuclear Matrix Element of the 2νββ Decay in ^{76}Ge *

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Abstract We apply the shell-model wave functions of ^{76}Ge to calculate the nuclear matrix elements and the half-life of the 2νββ decay of ^{76}Ge. Our result is comparable to the recent observed 2νββ decay half-life of ^{76}Ge. Furthermore it provides the upper limit of the effective neutrino mass is about 0.4eV with the recent experimental data of the 0νββ decay half-life.

Key words double β decay, half-life, shell-model

1 Introduction

The two-neutrino double beta (2νββ) decay is allowed in the standard theory of the electroweak interaction and has been observed in the laboratory[1]. A number of experiments have reported the direct observation of the 2νββ modes. For instance, Elliott and collaborators first observed the 2νββ decay of ^{82}Se[2] in 1986. Ejili et al and Alston-Garnjost et al have observed the 2νββ decay of ^{100}Mo[3]. And the 2νββ decay of ^{76}Ge have been observed by three groups: Russian[4], American[5] and Heidelberg-Moscow[6] groups. The future of double beta experiments will be dominated by the use of enriched detectors. Among them, ^{76}Ge plays a particular favorable role. Theoretically, one can predict the decay half-life with the calculated nuclear matrix elements, since it is parameter-free from the particle physics side. The experiment for the 2νββ decay will test the theoretical prediction of the nuclear matrix elements.

However the situation is different for the neutrinoless double beta (0νββ) decay, where no neutrino is emitted and lepton number is violated. These processes can only occur, as there is an exchange of Majorana neutrino. Consequently the 0νββ decay half-life depends not only on the nuclear matrix elements, but also on unknown parameters of the decay mechanism.

Furthermore, both modes are related to each other by the nuclear matrix elements. Thus one can use the same wave functions of nuclei to calculate the nuclear matrix elements for the 2νββ − and 0νββ − decays. The available experimental data of the 2νββ decay will test 2νββ nuclear matrix elements and tell us how reliable the wave functions are. With a more accurate wave functions one can extract the neutrino mass information from the experimental half − life limits of the 0νββ decay. The information about the neutrino mass and neutrino mixing attracts the most attention due to the recent experimental progress[7].

In this paper, following the shell-model wave functions of ^{76}Ge and ^{76}Se which were discussed in the previous publications[8,9], we evaluate the half-life and the nuclear matrix elements of the 2νββ

Received 25 November 1999

* Supported by National Natural Science Foundation of China and Grant of The Chinese Academy of Sciences

decay of ^{76}Ge. The latest Heidelberg-Moscow measurement for the $2\nu\beta\beta$ decay of ^{76}Ge provides a test for these wave functions. This comparison confirms our nuclear wave functions that apply to the $0\nu\beta\beta$ decay of ^{76}Ge. Our result shows that the theoretical prediction from our wave functions is comparable with the experimental data of the $2\nu\beta\beta$ decay of ^{76}Ge and the upper limit of the neutrino mass is less than 0.4eV with the measurement of $0\nu\beta\beta$ decay of ^{76}Ge.

2 Calculated Matrix Element of $2\nu\beta\beta$ Decay of ^{76}Ge

The $2\nu\beta\beta$ mode has two electrons and two antineutrinos in the final states and is expected to appear in the standard model in the second order process of weak interaction, such as

$$^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se} + 2e^- + 2\bar{\nu}_e \tag{1}$$

The decay amplitude associated with this process takes the form[10]

$$\mathcal{T}_{2\nu} = \sum_m \left[\frac{\langle e_1 e_2 \nu_1 \nu_2 \psi_f \mid H_\beta \mid e_1 \nu_1 \psi_m \rangle \langle e_1 \nu_1 \psi_m \mid H_\beta \mid \psi_i \rangle}{E_i - E_{e_1} - E_{\nu_1} - E_m} + \frac{\langle e_1 e_2 \nu_1 \nu_2 \psi_f \mid H_\beta \mid e_2 \nu_2 \psi_m \rangle \langle e_2 \nu_2 \psi_m \mid H_\beta \mid \psi_i \rangle}{E_i - E_{e_2} - E_{\nu_2} - E_m} - (e_1 \longleftrightarrow e_2) \right], \tag{2}$$

where H_β is the Hamiltonian of the weak interaction,

$$H_\beta = \frac{G}{\sqrt{2}} \int J_\mu(x) L_\mu(x) \mathrm{d}^3 x . \tag{3}$$

The ψ_i, ψ_m and ψ_f in Eq. (2) are the initial, intermediate and final state wave functions, respectively; E_i and E_m are the initial and intermediate state energies. G, L_μ and J_μ in Eq. (3) are Fermi coupling constant, the leptonic and hadronic currents respectively. L_μ and J_μ can be expressed as

$$L_\mu = e\gamma_\mu (1 - \gamma_5) \nu \tag{4}$$

and

$$J_\mu = \bar{N} \gamma_\mu (F_V - F_A \gamma_5) \tau_+ N , \tag{5}$$

with $F_V = 1.0$ and $F_A = 1.25$.

Employing the usual closure approximation to the intermediate states, the half-life $T_{1/2}^{2\nu}$ is given by

$$T_{1/2}^{2\nu} = \frac{\ln 2}{f_{GT} \mid M_{GT} \mid^2} , \tag{6}$$

where

$$M_{GT} = \left\langle \psi_f \middle| \sum_{i<j} \tau_+(i) \tau_+(j) \sigma(i) \sigma(j) \middle| \psi_i \right\rangle \tag{7}$$

and

$$f_{GT} = \zeta_{2\nu} \frac{2 m_e^{11} G^4}{7! \ \pi^7} \left[\frac{F^{PR}(Z)}{E_i - \bar{E}_m - T_0/2 - m_e} \right]^2 \times \left[1 + \frac{\tilde{T}_0}{2} + \frac{\tilde{T}_0^2}{9} + \frac{\tilde{T}_0^3}{90} + \frac{\tilde{T}_0^4}{1980} \right] \times \tilde{T}_0^7 . \tag{8}$$

Here $\tilde{T}_0 = T_0 / m_e$ with T_0 being the total kinetic energy of the outgoing electrons and m_e being the electron mass. For ^{76}Ge, $T_0 = 2.045$MeV. $\bar{E}_m$ is the average energy of the intermediate nuclear states. According to the statistics study of the β decay, the average intermediate energy is chosen reasonably and $E_m - E_i = 7.88$ MeV in the case of ^{76}Ge[10]. $F_{PR}(Z)$ is the nonrelativistic Coulomb correction factor[10] and its use permits analytic evaluation of the phase space integrals appearing in Eq. (8). $\zeta_{2\nu}$ in Eq(8) represents the difference between this approximation and an exact integrals of the phase space. For the $2\nu\beta\beta$ decay of ^{76}Ge, $\zeta_{2\nu} = 1.65$[10].

We have calculated the wave functions of ^{76}Ge and ^{76}Se by using a simple shell-model structure[8,9]. Ni^{56} is adopted as an inert core for ^{76}Ge and ^{76}Se. The shell-model space for these nuclei includes four single particle orbits $\{1P_{3/2}, 0f_{5/2}, 1P_{1/2}, 0g_{9/2}\}$. We employed the modified surface delta interaction (MSDI)[11] as the two body residual interaction. The single－particle energies and first set of parameters of MSDI in the Table 1 were taken from Faessler et al[12]. The second set of parameters of MSDI was used to the $2\nu\beta\beta$ decay of ^{82}Se in our previous paper[13].

Table 1. The nuclear matrix elements and the half－lifes of the $2\nu\beta\beta$ decay in ^{76}Ge Corresponding to the different parameters of MSDI.

	MSDI				ε_i/MeV				M_{GT}	$T_{1/2}^{2\nu}$ $\times 10^{21}$/years
	A_0	A_1	B	C	$1p_{3/2}$	$0f_{5/2}$	$1p_{1/2}$	$0g_{9/2}$		
1	0.43	0.35	0.33	0.00	0.00	1.75	2.20	3.39	0.353	5.17
2	0.31	0.30	0.30	0.00	0.00	1.75	2.20	3.39	0.479	2.82

3 Comparison between the Calculated Result and the Experimental Data

The recent experimental data for $2\nu\beta\beta$－and $0\nu\beta\beta$－decay modes of ^{76}Ge are given,

$$T_{1/2}^{2\nu}(\exp) = (1.77^{+0.01+0.13}_{-0.01-0.11}) \times 10^{21} \text{ years}^{[6]} \tag{9}$$

and

$$T_{1/2}^{0\nu}(\exp) > 5.7 \times 10^{25} \text{ years}^{[7]}. \tag{10}$$

The calculated result (see Table 1) is comparable to the observed $2\nu\beta\beta$ decay half－life of ^{76}Ge and the half－life with second set of parameters (last row) is close to the observed value in Eq.(9). Therefore, one can infer that our wave functions of ^{76}Ge and ^{76}Se may have more satisfied construction with the reasonable parameter choices. With the same wave functions one can obtain that the effective neutrino mass is less than 0.4eV by using the latest experimental lower limit (Eq.(10)) for the $0\nu\beta\beta$ decay half-life of ^{76}Ge.

As we know, the result of neutrino oscillation experiments on the effective neutrino mass[14] gives the upper limit may be around 10^{-2}eV. In order to approach this limit from the nuclear $0\nu\beta\beta$ decay we suggest to improve the existing experimental limit of the $0\nu\beta\beta$－decay half－life by using the enriched ^{76}Ge. More efforts to improve the theoretical calculation on the nuclear matrix element of ^{76}Ge are also necessary.

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^{76}Ge的2νββ衰变寿命和矩阵元*

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摘要 利用^{76}Ge的壳模型波函数计算^{76}Ge的2νββ衰变矩阵元和寿命.结果可以与最近观察到的^{76}Ge的2νββ衰变寿命相比拟.进而从最近的0νββ衰变的实验寿命可以算出有效中微子质量上限是0.4eV.

关键词 双β衰变 寿命 壳模型

1999－11－25收稿
* 国家自然科学基金和中国科学院理论物理特殊支持费资助