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Determining the Factorization Parameter and Strong Phase Differences in $B \to D^{(*)}\pi$ Decays

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Abstract The first observation of the color-suppressed decay mode $\bar{B}_d^0 \to D^{(*)0}\pi^0$ by the Belle and CLEO Collaborations makes a quantitative analysis of the isospin relations for the amplitudes of $B \to D^{(*)}\pi$ possible. The strong (isospin) phase difference in $B \to D\pi$ transitions is found to be about 29° by use of the Belle data or 26° by use of the CLEO data, implying that final-state interactions might not be negligible. Applying the factorization approximation to $I = 3/2$ and $I = 1/2$ isospin amplitudes of $B \to D\pi$ decays, we obtain the ratio of the effective Wilson coefficients $a_1^{\rm eff}$ and $a_2^{\rm eff}$: $a_2^{\rm eff}/a_1^{\rm eff} \approx 0.27$. A similar analysis shows that the magnitude of final-state interactions in $B \to D^*\pi$ might be comparable with that in $B \to D\pi$, and the factorization hypothesis works consistently in both of them.

Key words factorization, B-meson decay, isospin, strong phase

Two-body nonleptonic decays of the type $B \to D\pi$ have been of great interest in B physics for a stringent test of the factorization hypothesis and a quantitative analysis of final-state interactions[1]. The color-suppressed decay mode $\bar{B}_d^0 \to D^0\pi^0$ has for the first time been observed by the Belle[2] and CLEO[3] Collaborations. Its branching ratio is found to be $\mathcal{B}_{00} = (2.9^{+0.4}_{-0.3} \pm 0.6) \times 10^{-4}$ (Belle) or $(2.6 \pm 0.3 \pm 0.6) \times 10^{-4}$ (CLEO). In comparison, the branching ratios of the color-favored decay modes $\bar{B}_d^0 \to D^+\pi^-$ and $B_u^- \to D^0\pi^-$ are $\mathcal{B}_{+-} = (3.0 \pm 0.4) \times 10^{-3}$ and $\mathcal{B}_{0-} = (5.3 \pm 0.5) \times 10^{-3}$[3,4]. One can see that the naively expected color suppression does appear for $\bar{B}_d^0 \to D^0\pi^0$; i.e., $\mathcal{B}_{+-} \approx \mathcal{B}_{0-} \approx 3^2 \cdot \mathcal{B}_{00}$. With the help of the new experimental data, one is now able to analyze the isospin relations for the amplitudes of $B \to D\pi$ decays in a more complete way than before (see, e.g., Refs. [5] and [6]). Then it becomes possible to check whether final-state interactions are significant in such exclusive $|\Delta B| = |\Delta C| = 1$ transitions, and whether the factorization approximation works well.

The main purpose of this paper is to determine the strong (isospin) phase difference and the factorization parameter $a_2^{\rm eff}/a_1^{\rm eff}$ in $B \to D\pi$ decays. The former is found to be about 29° by use of the Belle data or 26° by use of the CLEO data, implying that final-state interactions might not be negligible. We obtain $a_2^{\rm eff}/a_1^{\rm eff} \approx 0.27$, a value in good agreement with the theoretical expectation. At the end of this paper, we present a similar isospin analysis for the decay modes $B \to D^*\pi$. Our result shows that the magnitude of final-state interactions in $B \to D^*\pi$ transitions might be comparable with that in $B \to D\pi$ transitions, and the factorization approximation works consistently in both of them.

The effective weak Hamiltonian responsible for $\bar{B}_d^0 \to D^+\pi^-$, $\bar{B}_d^0 \to D^0\pi^0$ and $B_u^- \to D^0\pi^-$ transitions[7] has the isospin configuration $|1, -1\rangle$. Therefore their amplitudes, defined respectively as A_{+-}, A_{00} and A_{0-}, can be decomposed as follows[6]:

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$$A_{+-} = A_{3/2} + \sqrt{2}A_{1/2}, \quad A_{00} = \sqrt{2}A_{3/2} - A_{1/2}, \quad A_{0-} = 3A_{3/2}, \tag{1}$$

where $A_{3/2}$ and $A_{1/2}$ stand respectively for $I = 3/2$ and $I = 1/2$ isospin amplitudes. In obtaining Eq. (1), we have assumed that there is no mixture of B→Dπ with other channels. It is obvious that three transition amplitudes form an isospin triangle in the complex plane: $A_{+-} + \sqrt{2}A_{00} = A_{0-}$. Of course, the sizes of A_{+-}, A_{00} and A_{0-} can straightforwardly be determined from the branching ratios given above. Then we are able to extract the ratio $A_{3/2}/A_{1/2}$, both its size and its phase, by use of Eq. (1). We find

$$r \equiv \left|\frac{A_{3/2}}{A_{1/2}}\right| = \sqrt{\frac{\mathscr{B}_{0-}}{3\kappa(\mathscr{B}_{+-} + \mathscr{B}_{00}) - \mathscr{B}_{0-}}},$$

$$\delta \equiv \arg\left(\frac{A_{3/2}}{A_{1/2}}\right) = \arccos\left\{\frac{3\kappa(\mathscr{B}_{+-} - 2\mathscr{B}_{00}) + \mathscr{B}_{0-}}{\sqrt{8\mathscr{B}_{0-}[3\kappa(\mathscr{B}_{+-} + \mathscr{B}_{00}) - \mathscr{B}_{0-}]}}\right\}, \tag{2}$$

where $\kappa \equiv \tau_{B_u^-}/\tau_{\bar{B}_d^0} = 1.073 \pm 0.027$[4] measures the difference between the life time of $\bar{B}_d^0$ and that of B_u^-. On the other hand, the tiny phase-space corrections induced by the mass differences $m_{D^0} - m_{D^-}$ and $m_{\pi^0} - m_{\pi^-}$ have been neglected in obtained Eq. (2).

Using the central values of $\mathscr{B}_{+-}$, $\mathscr{B}_{00}$, $\mathscr{B}_{0-}$ and κ, we obtain r and δ as follows:

$$r \approx \begin{cases} 1.0 & (\text{Belle}), \\ 1.0 & (\text{CLEO}); \end{cases} \quad \text{and} \quad \delta \approx \begin{cases} 29° & (\text{Belle}), \\ 26° & (\text{CLEO}). \end{cases} \tag{3}$$

If errors of the input parameters are taken into account, we find that r may change from 0.5 to 1.5 and δ can be as small as 0° in the extreme case. It is most likely, however, that δ takes the value given in Eq. (3), implying that final-state interactions in B→Dπ transitions might not be small. In addition, $r \approx 1$ means that the two isospin amplitudes have comparable contributions to the decay modes $\bar{B}_d^0 \to D^+\pi^-$ and $\bar{B}_d^0 \to D^0\pi^0$.

Let us proceed to calculate the isospin amplitudes $A_{3/2}$ and $A_{1/2}$ with the help of the factorization hypothesis. First of all, we assume that final-state interactions were absent (i.e., $\delta = 0$). In this assumption, the transition amplitudes of $\bar{B}_d^0 \to D^+\pi^-$, $\bar{B}_d^0 \to D^0\pi^0$ and $B_u^- \to D^0\pi^-$ can be expressed in terms of three topologically different quark-diagram amplitudes: X(color-favored topology), Y(color-suppressed topology) and Z(annihilation topology)[8]. Explicitly, we have

$$A_{+-}(\delta = 0) = X + Z, \quad A_{00}(\delta = 0) = \frac{Y}{\sqrt{2}} - \frac{Z}{\sqrt{2}}, \quad A_{0-}(\delta = 0) = X + Y, \tag{4}$$

where

$$X = \frac{G_F}{\sqrt{2}} a_1^{\rm eff}(V_{cb}V_{ud}^*)\langle \pi^- | (\bar{d}u)_{V-A} | 0\rangle \langle D^+ | (\bar{c}b)_{V-A} | \bar{B}_d^0\rangle,$$

$$Y = \frac{G_F}{\sqrt{2}} a_2^{\rm eff}(V_{cb}V_{ud}^*)\langle D^0 | (\bar{c}u)_{V-A} | 0\rangle \langle \pi^- | (\bar{d}b)_{V-A} | B_u^-\rangle,$$

$$Z = \frac{G_F}{\sqrt{2}} a_2^{\rm eff}(V_{cb}V_{ud}^*)\langle D^+\pi^- | (\bar{c}u)_{V-A} | 0\rangle \langle 0 | (\bar{d}b)_{V-A} | \bar{B}_d^0\rangle \tag{5}$$

in the QCD-improved factorization approximation. Here $a_1^{\rm eff}$ and $a_2^{\rm eff}$ are the effective Wilson coefficients, V_{cb} and V_{ud} are the relevant Cabibbo-Kobayashi-Maskawa matrix elements. Now we take final-state interactions into account (i.e., $\delta \neq 0$). The isospin amplitudes $A_{3/2}$ and $A_{1/2}$ can then be written as

$$A_{3/2} = \left(\frac{X}{3} + \frac{Y}{3}\right)e^{i\delta_{3/2}}, \quad A_{1/2} = \left(\frac{\sqrt{2}X}{3} - \frac{Y}{3\sqrt{2}} + \frac{Z}{\sqrt{2}}\right)e^{i\delta_{1/2}},$$

where $\delta_{3/2}$ and $\delta_{1/2}$ represent the strong phases of $I = 3/2$ and $I = 1/2$ isospin configurations, respectively. Note that $\delta_{3/2} - \delta_{1/2} = \delta$ holds by definition. Substituting Eq. (6) into Eq. (1) and taking $\delta_{3/2} = \delta_{1/2}$, we are able to reproduce Eq. (4).

In comparison with X and Y, the annihilation topology Z is expected to have significant form-factor suppression[10]. Therefore we neglect Z and obtain

$$r = \sqrt{2}\frac{X + Y}{2X - Y} = \sqrt{2}\frac{a_1^{\rm eff} + \zeta a_2^{\rm eff}}{2a_1^{\rm eff} - \zeta a_2^{\rm eff}} \tag{7}$$

from Eqs. (5) and (6), where

$$\zeta \equiv \frac{\langle D^0 | (\bar{c}u)_{V-A} | 0\rangle\langle \pi^- | (\bar{d}b)_{V-A} | B_u^-\rangle}{\langle \pi^- | (\bar{d}u)_{V-A} | 0\rangle\langle D^+ | (\bar{c}b)_{V-A} | \bar{B}_d^0\rangle} = \frac{(m_B^2 - m_\pi^2) f_D\, F_0^{B\to\pi}(m_D^2)}{(m_B^2 - m_D^2) f_\pi\, F_0^{B\to D}(m_\pi^2)}. \tag{8}$$

Using $r \approx 1.0$ obtained in Eq. (3) and $\zeta \approx 0.9$ given in Ref. [9], we can determine the ratio of the effective Wilson coefficients $a_1^{\rm eff}$ and $a_2^{\rm eff}$ with the help of Eq. (7):

$$\frac{a_2^{\rm eff}}{a_1^{\rm eff}} = \frac{\sqrt{2}}{\zeta}\cdot\frac{\sqrt{2}r - 1}{r + \sqrt{2}} \approx 0.27.$$

This result is in good agreement with the theoretical expectation[1].

Next we turn to the decay modes $\bar{B}_d^0 \to D^{*+}\pi^-$, $\bar{B}_d^0 \to D^{*0}\pi^0$ and $B_u^- \to D^{*0}\pi^-$. The Belle and CLEO Collaborations have recently reported the evidence for the color-suppressed transition $\bar{B}_d^0 \to D^{*0}\pi^0$, from which a preliminary value of the branching ratio can be obtained[2,3]: $\tilde{\mathscr{B}}_{00} = (1.5^{+0.6+0.3}_{-0.5-0.4}) \times 10^{-4}$ (Belle) or $(2.0 \pm 0.5 \pm 0.7) \times 10^{-4}$ (CLEO). In contrast, the color-favored decay modes $\bar{B}_d^0 \to D^{*+}\pi^-$ and $B_u^- \to D^{*0}\pi^-$ have the following branching ratios[4]: $\tilde{\mathscr{B}}_{+-} = (2.76 \pm 0.21) \times 10^{-3}$ and $\tilde{\mathscr{B}}_{0-} = (4.6 \pm 0.4) \times 10^{-3}$. As $B \to D^*\pi$ decays have the same isospin configurations as $B \to D\pi$, one may carry out an analogous analysis to determine the ratio of $I = 3/2$ and $I = 1/2$ isospin amplitudes (both its magnitude $\tilde{r}$ and its phase $\tilde{\delta}$) for the former. We obtain

$$\tilde{r} \approx \begin{cases} 0.98 & (\text{Belle}), \\ 0.97 & (\text{CLEO}); \end{cases} \quad \text{and } \tilde{\delta} \approx \begin{cases} 19^\circ & (\text{Belle}), \\ 25^\circ & (\text{CLEO}), \end{cases} \tag{10}$$

using the central values of $\tilde{\mathscr{B}}_{+-}$, $\tilde{\mathscr{B}}_{00}$, $\tilde{\mathscr{B}}_{0-}$ and κ. We see that the magnitude of final-state interactions in $B \to D^*\pi$ decays might be comparable with that in $B \to D\pi$ decays. A more precise measurement of $\bar{B}_d^0 \to D^{*0}\pi^0$ will narrow the error bar associated with its branching ratio $\tilde{\mathscr{B}}_{00}$ and allow us to extract the value of $\tilde{\delta}$ reliably. The result $\tilde{r} \approx 1$, similar to $r \approx 1$ for $B \to D\pi$, indicating that the two isospin amplitudes have comparable contributions to the decay modes $\bar{B}_d^0 \to D^{*+}\pi^-$ and $\bar{B}_d^0 \to D^{*0}\pi^0$.

Applying the factorization approximation to $B \to D^*\pi$ decays, one may analogously calculate the ratio of the effective Wilson coefficients $\tilde{a}_1^{\rm eff}$ and $\tilde{a}_2^{\rm eff}$. The result is

$$\frac{\tilde{a}_2^{\rm eff}}{\tilde{a}_1^{\rm eff}} = \frac{\sqrt{2}}{\tilde{\zeta}}\cdot\frac{\sqrt{2}\tilde{r} - 1}{\tilde{r} + \sqrt{2}} \approx 0.25,$$

where

$$\tilde{\zeta} \equiv \frac{\langle D^{*0} | (\bar{c}u)_{V-A} | 0\rangle\langle \pi^- | (\bar{d}b)_{V-A} | B_u^-\rangle}{\langle \pi^- | (\bar{d}u)_{V-A} | 0\rangle\langle D^{*+} | (\bar{c}b)_{V-A} | \bar{B}_d^0\rangle} = \frac{f_{D^*}\, F_+^{B\to\pi}(m_{D^*}^2)}{f_\pi\, A_0^{B\to D^*}(m_\pi^2)} \approx 0.9 \tag{12}$$

has been used[9]. One can see that the values of $\tilde{a}_2^{\rm eff}/\tilde{a}_1^{\rm eff}$ and $a_2^{\rm eff}/a_1^{\rm eff}$ are consistent with each other.

In summary, we have analyzed the isospin relations for the amplitudes of $\bar{B}_d^0 \to D^{(*)+}\pi^-$, $\bar{B}_d^0 \to D^{(*)0}\pi^0$ and $B_u^- \to D^{(*)0}\pi^-$ transitions. The strong phase differences are found to be about 29° (or

26°) and 19° (or 25°), respectively, in B→Dπ and B→D*π. We have also applied the factorization hypothesis to the decay modes under discussion. We find that the value of the factorization parameter $a_2^{\rm eff}/a_1^{\rm eff}$ extracted from B→Dπ is compatible with that extracted from B→D*π, and both of them are in good agreement with the theoretical expectation. We await more precise measurements of the color-suppressed decay modes $\overline{B}_d^0 \to D^0\pi^0$ and $\overline{B}_d^0 \to D^{*0}\pi^0$ at B-meson factories, in order to make a more stringent test of the factorization approximation and a more accurate analysis of final-state interactions.

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B→D$^{(*)}$π 衰变的因子化参数与强作用位相差

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摘要　利用 Belle 与 CLEO 实验组的最新实验数据，分析了 B→D$^{(*)}$π 衰变道的同位旋振幅与位相差，并确定了相关的因子化参数．结果表明这类衰变道中的强相互作用效应不可忽略，但是因子化近似对于单个同位旋振幅仍旧是合理与自洽的．

关键词　因子化　B 介子衰变　同位旋　强作用位相

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