# Two-Body B Decays to Pseudoscalar and Vector Mesons in QCD Factorization Approach

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**Abstract** Motivated by recent CELO measurements and the progress of the theory of B decays, $B \to PV(P=\pi, K; V= K^*, \rho, \omega)$ decay modes are studied in the framework of QCD factorization. All the measured branching ratios are well accommodated in the reasonable parameter space and predictions for other decay modes are well below the experimental upper limits.

**Key words** factorization, weak decay, meson

B physics is one of the most important fields nowadays because it is of great help for testing the quark flavor mixing theory of the standard model and exploring the source of CP violation. Most of the theoretical studies of B decays to pseudocalar and vector final states are based on the popular Naive Factorization approach[1]. As it was ponited out years ago in Ref. [2], the dominant contribution in B decays comes from the so-called Feynman mechanism, where the energetic quark created in the weak decay picks up the soft spectator softly and carries nearly all of the final-state meson's momentum. It is also shown that Pion form factor in QCD at intermediate engery scale is dominated by Feynman mechanism[3−5]. From this point, we can understand why the naive factorization approach have worked well for B and D decays, and the many existing predictions for B decays based on naive factorization and spectator ansatz do have taken in the dominant physics effects although there are shortcommings. However, with the many new data available from CLEO and an abundance of data to arrive within few years from the B factories BaBar and Belle, it is demanded highly to go beyond the naive factorization approach.

Recently, Beneke et al., have formed an interesting QCD factorization formula for B exclusive nonleptonic decays[6,7]. The factorization formula incorporates elements of the naive factorization approach (as leading contribution) and the hard-scattering approach (as subleading corrections), which allows us to calculate systematically radiative(subleading nonfactorizable) corrections to naive factorization for B exclusive nonleptonic decays. An important product of the formula is that the strong final-state interaction phases are calculable, which arise from the hard-scattering kernel and hence process dependent. The strong phases are very important for studying CP violation in B decays.

The amplitude of B decays to two light mesons, say $M_1$ and $M_2$, is obtained through the hadronic matrix element $<M_1(p_1)\ M_2(p_2)\,|\,O_i\,|\,B(p)>$, here $M_1$ denotes the final meson that picks up the light spectator quark in the B meson, and $M_2$ is the another meson which is composed of the quarks produced from the weak decay point of b quark. Since the quark pair, forming $M_2$, is ejected from the decay point of b quark carrying the large energy of order of $m_b$, soft gluons with the momentum of order of $\Lambda_{QCD}$ decouple from it at leading order of $\Lambda_{QCD}/m_b$ in the heavy quark limit. As a consequence any interaction between the quarks of $M_2$ and the quarks out of $M_2$ is hard at leading power in the heavy quark expansion. On the other hand, the light spectator quark carries the momentum of the order of $\Lambda_{QCD}$, and is softly transferred into $M_1$ unless it undergoes a hard interaction.Any soft interaction between the spectator quark and other constituents in B and $M_1$ can be absorbed into the transition form factor of $B \to M_1$. The non-factorizable contribution to $B \to M_1 M_2$ can be calculated through the diagrams in Fig.1.

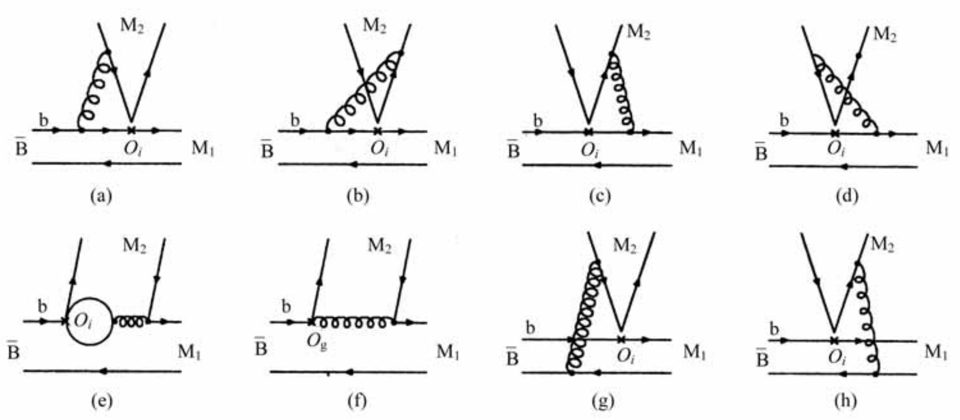


Fig. 1. Order $\alpha_s$ non-factorizable contributions in $B \to M_1 M_2$ decays.

The $O_i$'s incorporated in Fig.1 are the operators in the effective Hamiltonian for B decays[8],

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}\left[V_{ub}V_{uq}^*\left(\sum_{i=1}^{2} C_i O_i^u + \sum_{i=3}^{10} C_i O_i + C_g O_g\right) + V_{cb}V_{cq}^*\left(\sum_{i=1}^{2} C_i O_i^c + \sum_{i=3}^{10} C_i O_i + C_g O_g\right)\right], \tag{1}$$

Where

$$O_1^u = (\bar{q}_\alpha u_\alpha)_{V-A} \cdot (\bar{u}_\beta b_\beta)_{V-A}, \qquad O_2^u = (\bar{q}_\alpha u_\beta)_{V-A} \cdot (\bar{u}_\beta b_\alpha)_{V-A},$$

$$O_1^c = (\bar{q}_\alpha c_\alpha)_{V-A} \cdot (\bar{c}_\beta b_\beta)_{V-A}, \qquad O_2^c = (\bar{q}_\alpha c_\beta)_{V-A} \cdot (\bar{c}_\beta b_\alpha)_{V-A},$$

$$O_3 = (\bar{q}_\alpha b_\alpha)_{V-A} \cdot \sum\nolimits_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \qquad O_4 = (\bar{q}_\alpha b_\beta)_{V-A} \cdot \sum\nolimits_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A},$$

$$O_5 = (\bar{q}_\alpha b_\alpha)_{V-A} \cdot \sum\nolimits_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \qquad O_6 = (\bar{q}_\alpha b_\beta)_{V-A} \cdot \sum\nolimits_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A},$$

$$O_7 = \frac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A} \cdot \sum\nolimits_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \qquad O_8 = \frac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A} \cdot \sum\nolimits_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A},$$

$$O_9=\frac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A}\cdot\sum_{q'}e_{q'}(\bar{q}'_\beta q'_\beta)_{V-A},\quad O_{10}=\frac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A}\cdot\sum_{q'}e_{q'}(\bar{q}'_\beta q'_\alpha)_{V-A},$$

$$O_g=(g_s/8\pi^2)m_b\bar{d}_a\sigma^{\mu\nu}R(\lambda^A_{\alpha\beta}/2)b_\beta G^A_{\mu\nu}. \tag{2}$$

Here q=d, s and (q'ε {u, d, s, c, b}), $\alpha$ and $\beta$ are the $SU(3)$ color indices and $\lambda^A_{\alpha\beta}$, $A$=1,...,8 are the Gell-Mann matrices, and $G^A_{\mu\nu}$ denotes the gluonic field strength tensor. The Wilson coefficients evaluated at $\mu=m_b$ scale are[8]

$$C_1=1.082,\quad C_2=-0.185,\quad C_3=0.014,\quad C_4=-0.035,\quad C_5=0.009,\quad C_6=-0.041,$$
$$C_7=-0.002/137,\ C_8=0.054/137,\ C_9=-1.292/137,\ C_{10}=0.262/137,\ C_g=-0.143. \tag{3}$$

The non-factorizable contributions to $B\to M_1M_2$ can be calculated through the diagrams in Fig.1. The details of the calculations can be found in Ref. [9]. In the numerical calculations we use[10]

$$\tau(B^+)=1.65\times10^{-12}\text{s},\qquad \tau(B^0)=1.56\times10^{-12}\text{s},$$
$$M_B=5.2792\text{GeV},\qquad m_b=4.8\text{GeV},\qquad m_c=1.4\text{GeV},$$
$$f_B=0.180\text{GeV},\qquad f_\pi=0.133\text{GeV},\qquad f_K=0.158\text{GeV},$$
$$f_{K^*}=0.214\text{GeV},\qquad f_\rho=0.21\text{GeV},\qquad f_\omega=0.195\text{GeV}.$$

For the chiral enhancement factors for the pseudoscalar mesons, we take

$$R_{\pi^\pm}=R_{K^{\pm,0}}=-1.2,$$

which are consistent with the values used in [6, 11, 12]. We should take care for $R_{\pi^0}$. As pointed out in Ref. [7], $R_{\pi^0}$ for $\pi^0$ should be $-2M^2_\pi/(m_b(m_u+m_d))$ and equal to $R_{\pi^\pm}$ due to inclusion of isospin breaking effects correctly.

For the form factors, we take the results of light-cone sum rule[13,14]

$$F^{B\to\pi}(0)=0.3,\quad F^{B\to K}(0)=1.13F^{B\to\pi}(0),\quad A_0^{B\to\rho}=0.372,\quad A_0^{B\to K^*}=0.470,$$

and assume $A_0^{B\to\omega}(0)=1.2\,A_0^{B\to\rho}(0)$ since we find larger $A_0^{B\to\omega}(0)$ is preferred by experimental data.

We take the leading-twist distribution amplitude (DA) $\phi(x)$ and the twist-3 DA $\phi^0(x)$ of light pseudoscalar and vector mesons as the asymptotic form[15]

$$\phi_{P,V}(x)=6x(1-x),\quad \phi^0_P(x)=1. \tag{4}$$

For the B meson, the wave function is chosen as[16,17]

$$\phi_B(x)=N_Bx^2(1-x)^2\exp\left[-\frac{M_B^2x^2}{2\omega_B^2}\right], \tag{5}$$

with $\omega_B$=0.4GeV, and $N_B$ is the normalization constant to make $\int_0^1 dx\phi_B(x)=1$. $\phi_B(x)$ is strongly peaked around $x$=0.1, which is consistent with the observation of Heavy Quark Effective Theory that the wave function should be peaked around $\Lambda_{QCD}/M_B$.

We have used the unitarity of the CKM matrix $V^*_{uq}V_{ub}+V^*_{cq}V_{cb}+V^*_{tq}V_{tb}=0$ to decompose the amplitudes into terms containing , $V^*_{uq} V_{ub}$ and $V^*_{cq} V_{cb}$, and

$$|V_{us}| = \lambda = 0.2196,\ |V_{ub}/V_{cb}| = 0.085 \pm 0.02,\ |V_{cb}| = 0.0395 \pm 0.0017,\ |V_{ud}| = 1 - \lambda^2/2\ . \quad (6)$$

We leave the CKM angle γ as a free parameter.

The numerical results of the branching ratios B→PV are shown in Fig.2 as the function of CKM angle γ. We can see from Fig. 2(a), (b) and (c) that for the three detected channels the predicted branching ratios agree well with the CLEO experiment data[18]. Our predictions for other decay modes are well below their 90% C.L. upper limits.

There are several works available with detailed analysis of the CLEO new data of the decays of B to charmless PV states[11,12,19]. It is worth to note that the shortcomings in the "generalized factorization" are resolved in the framework of QCD Factorization. Nonfactorizable effects are calculated in a rigorous way here instead of being parameterized by effective color number. Since the hard scattering kernals are convoluted with the light cone DAs of the mesons, gluon virtuality $k^2 = \bar{x} m_b^2$ in the penguin diagram Fig. 1(e) has well defined meaning and leaves no ambiguity as to the value of $k^2$, which has usually been treated as a free phenomenological parameter in the estimations of the strong phase generated though the BSS mechanism[20]. So that CP asymmetries are predicted soundly in this approach. We present the numerical result of the branching ratios of B→PV decays in Table 1 with the relevant strong phases shown explicitly. It shows that the strong phases are generally mode dependent.

**Table 1. Strong phases in the branching ratios** (in units of $10^{-6}$) **for the charmless decays modes studied by CLEO.** ($\gamma = \mathrm{Arg} V^*_{ub}$)

| | |
|---|---|
| $B(B^- \to \pi^- \rho^0) = 6.65\,\lvert 0.11 e^{-i86.5°} + e^{-i\gamma}\rvert^2$ | $B(\overline{B}^0 \to \pi^+ \rho^-) = 19.79\,\lvert 0.11 e^{i9.02°} + e^{-i\gamma}\rvert^2$ |
| $B(\overline{B}^0 \to \pi^- \rho^+) = 13.43\,\lvert 0.03 e^{i172°} + e^{-i\gamma}\rvert^2$ | $B(B^- \to \pi^- \omega) = 10.59\,\lvert 0.065 e^{i26.01°} + e^{-i\gamma}\rvert^2$ |
| $B(\overline{B}^0 \to \pi^0 \rho^0) = 0.11\,\lvert 0.21 e^{2.90°} + e^{-i\gamma}\rvert^2$ | $B(B^- \to \pi^0 \rho^-) = 10.81\,\lvert 0.176 e^{i7.20°} + e^{-i\gamma}\rvert^2$ |
| $B(\overline{B}^0 \to \pi^- \omega) = 1.49 \times 10^{-3}\,\lvert 1.64 e^{i148°} + e^{-i\gamma}\rvert^2$ | $B(B^- \to K^- \rho^0) = 0.55\,\lvert 0.24 e^{-i162°} + e^{-i\gamma}\rvert^2$ |
| $B(B^- \to \pi^- \overline{K}^{*0}) = 0.0012\,\lvert 56.4 e^{-i15.7°} + e^{-i\gamma}\rvert^2$ | $B(B^- \to K^- K^{*0}) = 0.030\,\lvert 2.86 e^{i164°} + e^{-i\gamma}\rvert^2$ |
| $B(B^- \to \pi^0 K^{*-}) = 0.59\,\lvert 2.80 e^{-i169°} + e^{-i\gamma}\rvert^2$ | $B(B^- \to K^- \omega) = 0.80\,\lvert 0.48 e^{-i9.23°} + e^{-i\gamma}\rvert^2$ |
| $B(\overline{B}^0 \to K^0 \omega) = 0.72\,\lvert 0.81 e^{-i11.8°} + e^{-i\gamma}\rvert^2$ | $B(\overline{B}^0 \to K^- \rho^+) = 0.96\,\lvert 0.63 e^{-i7.20°} + e^{-i\gamma}\rvert^2$ |
| $B(\overline{B}^0 \to \pi^0 \overline{K}^{*0}) = 0.004\,\lvert 12.89 e^{i67.61°} + e^{-i\gamma}\rvert^2$ | |

Hou, Smith and Würthwein have performed a model dependent fit using the recent CLEO data and found $\gamma = 114^{+25}_{-21}$ degree. Using $SU(3)$ flavor symmetry, Gronau and Rosner have analyzed the decays of B to charmless PV final states extensively and found several processes are consistent with $\cos\gamma < 0$. In this paper we find $\cos\gamma < 0$ is favored by the $B^- \to \pi^- \rho^0$ and $\overline{B}^0 \to \pi^- \rho^+ + \pi^+ \rho^-$ if their experimental center values are taken seriously. To meet its center value with $\cos\gamma < 0$ , $B^- \to \pi$

$^{-}\omega$ would indicate larger form factor i.e. $A_0^{B\to\omega}(0) > A_0^{B\to\rho}(0)$. In our numerical calculation, we have taken $A_0^{B\to\omega}(0) = 0.446$ which is still consistent with the LCSR results $0.372 \pm 0.074$[13]. It is also interesting to note that $\bar{B}^0\to\pi^+\rho^-$ is suppressed by $\cos\gamma < 0$ while $\bar{B}^0\to\pi^-\rho^+$ is enchanced. The defference between $Br(\bar{B}^0\to\pi^+\rho^-)$ and $Br(\bar{B}^0\to\pi^-\rho^+)$ is much more sensitive to $\gamma$ than their sum.

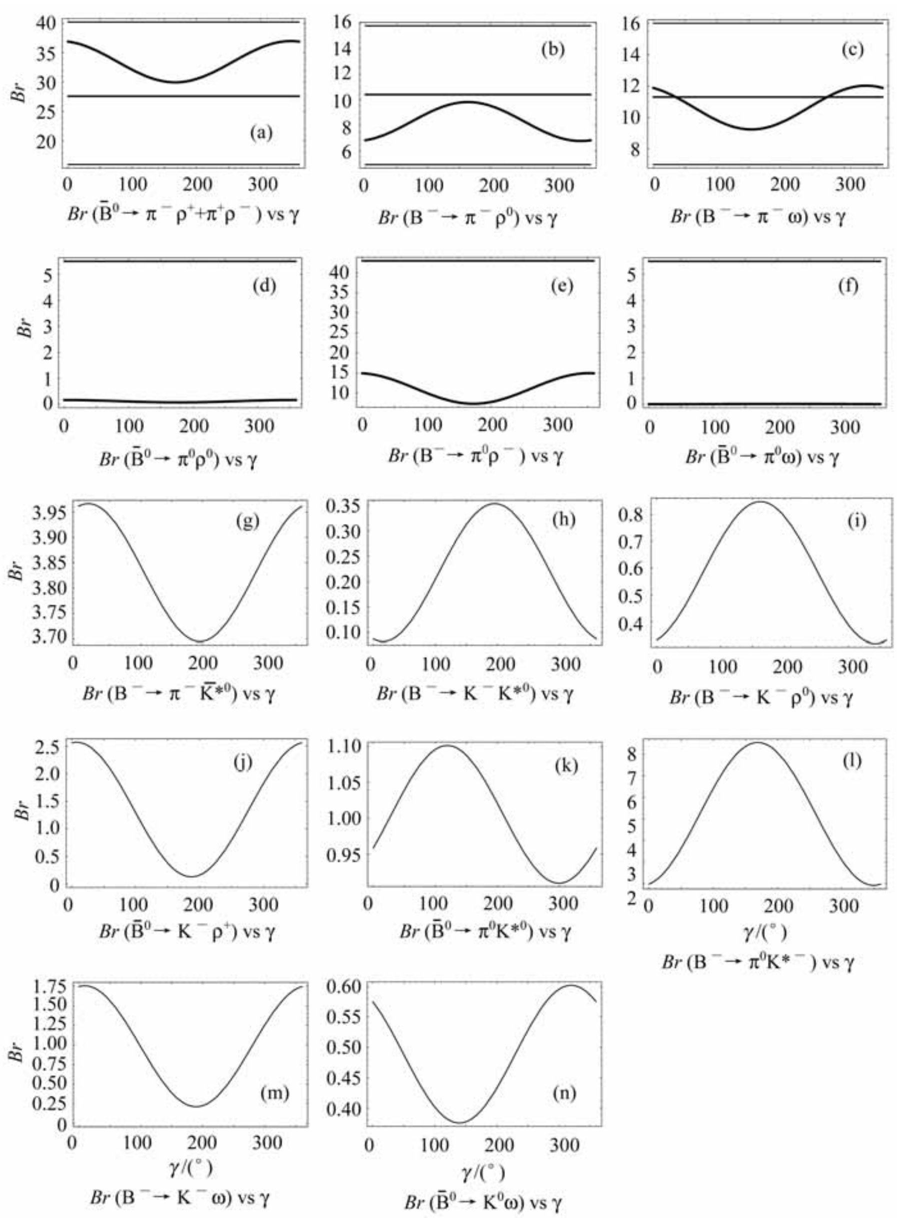


Fig.2. $Br(B\to PV)$ as a function of $\gamma$ are shown as curves in units of $10^{-6}$. The $Br$ measured by CLEO Collaboration are shown by horizontal solid lines. The thicker solid lines are its center values, thin lines are its error bars or the upper limit.

**Summary**

we have calculated the branching ratios and CP asymmetries of the charmless decays $B\to PV(P = (\pi, K), V= (\rho,\omega, K^*))$ in QCD factorization approach. We have used LCSR form factors $F^{B\to\pi,K}(0)$ and $A_0^{\rho,K^*}(0)$ as inputs. The results of $Br(B^-\to\pi^-\rho^0)$ and $Br(\bar{B}^0\to\pi^\pm\rho^\mp)$ agree with CLEO[18] very well and favor $\cos\gamma < 0$ if their experimental center values are taken seriously. To meet its experimental center value and $\cos\gamma < 0$, the decay $B^-\to\pi^-\omega$ will prefer larger form factor $A_0^{B\to\omega}(0)$. For the other decay modes, the branching ratios are predicted well below their 90% C.L. upper limits given in Ref. [18].

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# 用 QCD 因子化方法研究 B→PV 两体弱衰变过程

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**摘要**　基于最近 CLEO 实验和 B 介子物理中理论研究的进展, 在 QCD 因子化方案下研究了 B 介子到一个赝标π, K 和一个矢量介子ρ, ω的两体弱衰变过程.在合理的参数范围内, 理论计算与实验相符得很好.

**关键词**　因子化　弱衰变　介子