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Nonlinear Transport of the Intense Pulsed Gaussian Beams in Magnetic Quadrupoles

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Abstract The paper presents the nonlinear transport of the intense pulsed charged particle beams analyzed with the Lie algebraic method. The particles are supposed to be distributed in 3D dimensional ellipsoid in Gaussian manner. The analysis is performed for magnetic quadrupoles, and is similar for dipoles, sextupoles and other optical elements.

Key words magnetic quadrupoles, Gaussian beams, Lie map, nonlinearity

1 Introduction

When the particle energy is low and the beam current is high, the space charge forces of the beams can not be ignored. In the intense accelerators, such as medical proton linear accelerators, accelerator driven sub-critical system and so on, the nonlinear transport of intense beams should be taken into account, so that the high beam transmission can be obtained. In this situation, accuracy calculations for the particle trajectories are very complicated.

There are usually two ways to calculate the nonlinear transport for the intense beams:
numerical method (solving fields and calculating trajectories) and analytical method. The former one is often used for short beam transport systems, such as ion attracting systems of ion sources. Because of large memory needed in numerical calculations, analytical method is convenient for the very long beam line calculations.

Lie algebraic method[1] provides a good tool to study the nonlinear transport of intense beams. The key problem is how to express the self-excited potentials of the beams. Because different particle phase space distributions have different potentials, they will evolve with the particle motions. Different distribution functions of charged particle beams can be chosen as distributed in phase spaces in K-V, parabolic or Gaussian manner, etc. The particle distribution function in 3D dimensional ellipsoid can also be chosen as parabolic, Gaussian or uniform. Gaussian distribution function in 3D dimensional ellipsoid is used in our analysis.

2 Hamiltonian and its expansion[2]

In the Cartesian coordinates (x, y, z), the Hamiltonian of a particle with time t as an independent variable is

$$H_t = [m_0^2c^4 + c^2(p_x - qA_x)^2 + c^2(p_y - qA_y) + c^2(p_z - qA_z)^2]^{\frac{1}{2}} + q\varphi, \qquad (1)$$

where m_0 is the rest energy of a particle, q the charge, p_x, p_y and p_z the x, y and z components of the particle canonical momenta, respectively, A_x, A_y and A_z the x, y and z components of the magnetic vector potential, ϕ the electric potential, c the light velocity. Here, the canonical variables are $\eta = (x, y, z, p_x, p_y, p_z)$.

Introducing the variable $p_t = -H_t(x, y, z, p_x, p_y, p_z; t)$ and solving p_z from p_t, one obtains

$$K = -p_z = -[(p_t + q\phi)^2/c^2 - m_0^2c^2 - (p_x - qA_x)^2 - (p_y - qA_y)^2]^{\frac{1}{2}} - qA_z. \qquad (2)$$

Define the new canonical variables $\zeta = (x, y, \tau, x', y', p_\tau)$ as

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$$x = x, y = y, \tau = T - z/\beta_0,$$
$$x' = p_x/p_0, y' = p_y/p_0, p_\tau = p_T - p_T^0, \tag{3}$$

where $T = ct$, $\beta_0 = c/v_0$, v_0 is the velocity of the reference particle, p_0 the momentum of the reference particle, $p_T = p_t/(p_0 c)$, p_T^0 the value of p_T for the reference particle. Under the transformation expressed by Eq.(3), the new Hamiltonian is

$$H = -\left[\left(p_\tau + p_T^0 + \frac{q\phi}{p_0 c}\right)^2 - \frac{1}{\beta_0^2\gamma^2}\right.$$
$$\left.(x' - qA_x/p_0)^2 - (y' \quad qA_y/p_0)^2\right]^{\frac{1}{2}} -$$
$$qA_z/p_0 - (p_\tau + p_T^0)/\beta_0. \tag{4}$$

For the magnetic quadrupoles, $A = \frac{G}{2}(y^2 - x^2)e_z$ (G is the magnetic field gradient), and the selfexcited electric potential of the charged particle beams for the Gaussian distribution is

$$\phi = -\frac{IT_{rf}}{8\pi^{3/2}\varepsilon_0}$$
$$\int_0^\infty \frac{\exp\left[-\left(\frac{x^2}{2X^2+\xi} + \frac{y^2}{2Y^2+\xi} + \frac{z^2\gamma_0^2}{2Z^2\gamma_0^2+\xi}\right)\right]}{\sqrt{(2X^2+\xi)(2Y^2+\xi)(2Z^2\gamma_0^2+\xi)}}\mathrm{d}\xi, \tag{5}$$

where I is the average beam current of the beam bundles, T_{rf} the beam pulse repetition period, X, Y and Z the rms pulsed beam dimensions, and $\gamma_0 = 1/\sqrt{1-\beta_0^2}$. The Hamiltonian becomes

$$H = -\left[\left(p_\tau - \frac{1}{\beta_0} + \frac{q\phi}{p_0 c}\right)^2 - x'^2 - y'^2 - \frac{1}{\beta_0^2\gamma^2}\right]^{\frac{1}{2}}$$
$$\frac{qG}{2p_0}(y^2 - x^2) - (p_\tau + p_T^0)/\beta_0. \tag{6}$$

Expand the Hamiltonian in Eq.(6) about the equilibrium orbit, we have

$$H = \sum_{n=0}^{\infty} H_n,$$

where

$$H_0 = \frac{1}{\beta_0^2\gamma_0^2},$$
$$H_1 = -p_\tau\left(\frac{\alpha}{s} + \frac{1}{\beta_0}\right),$$
$$H_2 = \frac{x^2}{2}\left(\frac{qG}{p_0} + \frac{Q\mu_x\alpha}{p_0 s}\right) + \frac{y^2}{2}\left(-\frac{qG}{p_0} + \frac{Q\mu_y\alpha}{p_0 s}\right) +$$
$$\frac{1}{2s}(x'^2 + y'^2) + \frac{\tau^2}{2}\frac{2Q\beta_0^2\gamma_0^2\mu_z\alpha}{p_0 s} +$$
$$\frac{p_\tau^2}{2\beta_0^2\gamma_0^2 s^3},$$
$$H_3 = -x^2 p_\tau \frac{Q\mu_x}{\beta_0^2\gamma_0^2 p_0 s^3} - y^2 p_\tau \frac{Q\mu_y}{\beta_0^2\gamma_0^2 p_0 s^3} -$$
$$(x'^2 p_\tau + y'^2 p_\tau)\frac{\alpha}{2s^3} - \tau^2 p_\tau \frac{Q\mu_z}{p_0\ s^3} -$$
$$\frac{p_\tau^3\alpha}{2\beta_0^3\gamma_0^2 s^5},$$

……

and

$$\alpha = q\mu/p_0 - 1/\beta_0,$$
$$s = [(q\mu/p_0 - 1/\beta_0)^2 - 1/(\beta_0^2\gamma_0^2)]^{1/2}$$
$$Q = \frac{qIT_{rf}}{8\pi^{3/2}c\varepsilon_0},$$
$$\mu = \int_0^\infty \frac{\mathrm{d}\xi}{\sqrt{(2X^2+\xi)(2Y^2+\xi)(2Z^2\gamma^2+\xi)}},$$
$$\mu_x = \int_0^\infty \frac{\mathrm{d}\xi}{(2X^2+\xi)\sqrt{(2X^2+\xi)(2Y^2+\xi)(2Z^2\gamma^2+\xi)}},$$
$$\mu_y = \int_0^\infty \frac{\mathrm{d}\xi}{(2Y^2+\xi)\sqrt{(2X^2+\xi)(2Y^2+\xi)(2Z^2\gamma^2+\xi)}},$$
$$\mu_z = \int_0^\infty \frac{\mathrm{d}\xi}{(2Z^2\gamma^2+\xi)\sqrt{(2X^2+\xi)(2Y^2+\xi)(2Z^2\gamma^2+\xi)}}.$$

3 First order approximation

The linear map M_2 is expressed as

$$M_2 = \exp(-:f_2:), \tag{10}$$

where $:f_2:$ is the Lie operator. When acting on another function, it performs Poisson bracket operation, and

$$f_2 = -lH_2, (l \text{ is the length of quadrupoles}). \tag{11}$$

Let the subscript "1" express the first order terms of the map, and M_2 act on the components of the initial canonical variable ζ, one obtains the first order approximation solutions of the particle trajectories, expressed in matrix form as

$$\begin{bmatrix} x_1 \\ x'_1 \\ y_1 \\ y'_1 \\ \tau_1 \\ p \end{bmatrix} = \begin{bmatrix} \cos(k_x l) & \frac{1}{k_x s}\sin(k_x l) & 0 & 0 & 0 & 0 \\ k_x s\sin(k_x l) & \cos(k_x l) & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh(k_y l) & \frac{1}{k_y s}\sinh(k_y l) & 0 & 0 \\ 0 & 0 & k_y s\ \sinh(k_y l) & \cosh(k_y l) & 0 & 0 \\ 0 & 0 & 0 & 0 & \cosh(k_z l) & \frac{\sinh(k_z l)}{k_z \beta_0^2 \gamma_0^2 s^3} \\ 0 & 0 & 0 & 0 & k_z \beta_0^2 \gamma_0^2 s^3 \sinh(k_z l) & \cosh(k_z l) \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \\ y \\ y' \\ \tau \\ p_\tau \end{bmatrix},$$

where

$$k_x^2 = \frac{qG}{2s} + \frac{2Q\mu_x \alpha}{p_0 s}, k_y^2 = \frac{qG}{2s} - \frac{2Q\mu_y \alpha}{p_0 s}, k_z^2 = -\frac{2Q\mu_z \alpha}{\beta_0 \gamma_0 p_0 s^5}.$$

4 Second order approximation

The second order map M_3 can be expressed as $M_3 = :f_3:$, where

$$f_3 = -\int_0^l h_3^{\text{int}}(\zeta, z_1)\mathrm{d}z_1 = -\int_0^l M_2 H_3(\zeta, z_1)\mathrm{d}z_1. \quad (14)$$

Let the map M_3 act on the linear solution $\zeta_1 = (x_1, x'_1, y_1, y'_1, \tau_1, p_{\tau 1})$, one obtains the second order solutions ζ_2 (the subscript "2" expresses the second order) of the map. The results are listed as the following:

$$x_2 = \frac{x\tau}{4k_x^2 + k_z^2}\Big\{\frac{2Q\mu_x}{p_0 k_x s}[2k_x^2\sin(k_x l)(\cosh(k_z l) - 1) - k_z^2\sin(k_x l) + k_x k_z\cos(k_x l)\sinh(k_z l)] +$$
$$\beta_0\gamma_0^2 k_x s\alpha[2k_x^2\sin(k_x l)(\cosh(k_z l) - 1) + k_z^2\sin(k_x l)\cosh(k_z l) - k_x k_z\cos(k_x l)\sinh(k_z l)]\Big\} +$$
$$\frac{xp_\tau}{(4k_x^2 + k_z^2)k_z s^2}\Big\{\frac{2Q\mu_x}{\beta_0^2\gamma_0^2 p_0 s^2}[k_z\cos(k_x l)(\cosh(k_z l) - 1) + 2k_x\sin(k_x l)\sinh(k_z l)] +$$
$$k_x\alpha[(2k_x^2 + k_z^2)\sin(k_x l)\sinh(k_z l) - k_x k_z\cos(k_x l)(\cosh(k_z l) - 1)]\Big\} +$$
$$\frac{x'\tau}{4k_x^2 + k_z^2}\Big\{\frac{2Q\mu_x}{p_0 k_x s^2}[k_z\sin(k_x l)\sinh(k_z l) - 2k_x\cos(k_x l)(\cosh(k_z l) - 1)] -$$
$$\beta_0^2\gamma_0^2\alpha[k_x k_z\sin(k_x l)\sinh(k_z l) + (2k_x^2 + k_z^2)\cos(k_x l)(\cosh(k_z l) - 1)]\Big\} +$$
$$\frac{x'p_\tau}{(4k_x^2 + k_z^2)k_z s^3}\Big\{\frac{2Q\mu_x}{\beta_0^2\gamma_0^2 p_0 k_x s^2}[k_z\sin(k_x l)(1 + \cosh(k_z l)) - 2k_x\cos(k_x l)\sinh(k_z l)] -$$
$$\alpha[k_x k_z\sin(k_x l)(1 + \cosh(k_z l)) + (2k_x^2 + k_z^2)\cos(k_x l)\sinh(k_z l)]\Big\}$$

$$x'_2 = \frac{x\tau}{4k_x^2 + k_z^2}\Big\{\frac{2Q\mu_x}{p_0}[(2k_x^2 + k_z^2)\cos(k_x l)(\cosh(k_z l) - 1) + k_x k_z\sin(k_x l)\sinh(k_z l)] +$$
$$\beta_0^2\gamma_0^2 k_x^3 s^2\alpha[2k_x\cos(k_x l)(\cosh(k_z l) - 1) - k_z\sin(k_x l)\sinh(k_z l)]\Big\} +$$
$$\frac{xp_\tau}{(4k_x^2 + k_z^2)k_z s}\Big\{\frac{2Q\mu_x}{\beta_0^2\gamma_0^2 p_0 s^2}[2k_x k_z\sin(k_x l)(1 + \cosh(k_z l)) + (2k_x^2 + k_z^2)\cos(k_x l)\sinh(k_z l)] +$$
$$k_x^3\alpha[2k_x\cos(k_x l)\sinh(k_z l) - k_z\sin(k_x l)(1 + \cosh(k_z l))]\Big\} +$$
$$\frac{x'\tau}{4k_x^2 + k_z^2}\Big\{\frac{2Q\mu_x}{p_0 k_x s}[2k_x^2\sin(k_x l)(\cosh(k_z l) - 1) + k_z^2\sin(k_x l)\cosh(k_z l) - k_x k_z\cos(k_x l)\sinh(k_z l)] +$$

$$\beta_0^2\gamma_0^2 k_x s\alpha[2k_x^2\sin(k_x l)(\cosh(k_z l)-1)-k_z^2\sin(k_x l)+k_x k_z\cos(k_x l)\sinh(k_z l)]\Big\}+$$

$$\frac{x' p_\tau}{(4k_x^2+k_z^2)k_z s^2}\Big\{\frac{2Q\mu_x}{\beta_0^2\gamma_0^2 k_x p_0 s^2}[(2k_x^2+k_z^2)\sin(k_x l)\sinh(k_z l)-k_x k_z\cos(k_x l)(\cosh(k_z l)-1)]+$$

$$k_x^2\alpha[k_z\cos(k_x l)(\cosh(k_z l)-1)+2k_x\sin(k_x l)\sinh(k_z l)]\Big\},$$

$$y_2=\frac{y\tau}{4k_y^2-k_z^2}\Big\{\frac{2Q\mu_y}{p_0 k_y s}[-(2k_y^2-k_z^2)\sinh(k_y l)-k_y k_z\cosh(k_y l)\sinh(k_z l)+2k_y^2\sinh(k_y l)\sinh(k_z l)]+$$

$$\beta_0^2\gamma_0^2 k_y^2 s\alpha[2k_y^2\sinh(k_y l)(1-\cosh(k_z l))+k_z^2\sinh(k_y l)\cosh(k_z l)-k_y k_z\cosh(k_y l)\sinh(k_z l)]\Big\}+$$

$$\frac{y p_\tau}{(4k_y^2-k_z^2)k_z s^2}\Big\{\frac{2Q\mu_y}{\beta_0^2\gamma_0^2 p_0 s^2}[k_z\cosh(k_y l)(1-\cosh(k_z l))+$$

$$2k_y\sinh(k_y l)\sinh(k_z l)]+k_y\alpha[k_y k_z\cosh(k_y l)(1-\cosh(k_z l))-(2k_y^2-k_z^2)\sinh(k_y l)\sinh(k_z l)]\Big\}\cdot$$

$$\frac{y'\tau}{4k_y^2-k_z^2}\Big\{\frac{2Q\mu_y}{p_0 k_y s^2}[2k_y\cosh(k_y l)(\cosh(k_z l)-1)-k_z\sinh(k_y l)\sinh(k_z l)]-$$

$$\beta_0^2\gamma_0^2\alpha[(2k_y^2-k_z^2)\cosh(k_y l)(\cosh(k_z l)-1)+k_y k_z\sinh(k_y l)\sinh(k_z l)]\Big\}+$$

$$\frac{y' p_\tau}{(4k_y^2-k_z^2)k_z s^3}\Big\{\frac{2Q\mu_y}{\beta_0^2\gamma_0^2 k_y p_0 s^2}[k_y\cosh(k_y l)\sinh(k_z l)-k_z\sinh(k_y l)(1+\cosh(k_z l))]-$$

$$k_y\alpha[k_z\sinh(k_y l)(1+\cosh(k_z l))+2k_y\cosh(k_y l)\sinh(k_z l)]\Big\},$$

$$y'_2=\frac{y\tau}{4k_y^2-k_z^2}\Big\{\frac{2Q\mu_y}{p_0}[(2k_y^2-k_z^2)\cosh(k_y l)(\cosh(k_z l)-1)+k_y k_z\sinh(k_y l)\sinh(k_z l)]+$$

$$\beta_0^2\gamma_0^2 k_y^3 s^2\alpha[k_z\sin(k_y l)\sinh(k_z l)-2k_y\cosh(k_y l)(\cosh(k_z l)-1)]\Big\}+$$

$$\frac{y p_\tau}{(4k_y^2-k_z^2)k_z s}\Big\{\frac{2Q\mu_y}{\beta_0^2\gamma_0^2 p_0 s^2}[k_y k_z\sinh(k_y l)(1+\cosh(k_z l))+(2k_y^2-k_z^2)\cosh(k_y l)\sinh(k_z l)]+$$

$$k_y^3\alpha[k_z\sinh(k_y l)(1+\cosh(k_z l))-2k_y\cosh(k_y l)\sinh(k_z l)]\Big\}+$$

$$\frac{y'\tau}{(4k_y^2-k_z^2)k_y}\Big\{\frac{2Q\mu_y}{p_0 k_y s}[2k_y^2\sinh(k_y l)(\cosh(k_z l)-1)-k_z^2\sinh(k_y l)\cosh(k_z l)+k_y k_z\cosh(k_y l)\sinh(k_z l)]$$

$$\beta_0^2\gamma_0^2 k_y^2 s\alpha[k_z^2\sinh(k_y l)+2k_y^2\sinh(k_y l)(\cosh(k_z l)-1)-k_y k_z\cosh(k_y l)\sinh(k_z l)]\Big\}+$$

$$\frac{y' p_\tau}{(4k_y^2-k_z^2)k_z s^2}\Big\{\frac{2Q\mu_y}{\beta_0^2\gamma_0^2 p_0 k_z s^2}[k_y k_z\cosh(k_y l)(\cosh(k_z l)-1)+(2k_y^2-k_z^2)\sinh(k_y l)\sinh(k_z l)]+$$

$$k_y^2\alpha[k_z\cosh(k_y l)(\cosh(k_z l)-1)-2k_y\sinh(k_y l)\sinh(k_z l)]\Big\}.$$

$$\tau_2=\frac{x^2}{(4k_x^2+k_z^2)k_z s}\Big\{\frac{Q\mu_x}{\beta_0^2\gamma_0^2 p_0 s^2}[(2k_x^2+k_z^2)\sinh(k_z l)-2k_x k_z\sin(k_x l)\cos(k_x l)]+k_x^3\alpha[k_z\sin(k_x l)\cos(k_x l)-$$

$$k_x\sinh(k_z l)]\Big\}+\frac{xx'}{(4k_x^2+k_z^2)s^2}\left(\frac{2Q\mu_x}{\beta_0^2\gamma_0^2 p_0 s^2}-k_x^2\alpha\right)\cdot(\cos(2k_x l)-\cosh(k_z l)+$$

$$\frac{x'^2}{(4k_x^2+k_z^2)k_x k_z s^3}\Big\{\frac{Q\mu_x}{\beta_0^2\gamma_0^2 p_0 s^2}\cdot[k_z\sin(2k_x l)-k_x\sinh(k_z l)]-\frac{k_x\alpha}{2}[k_x k_z\sin(2k_x l)+(2k_x^2+k_z^2)\sinh(k_z l)]\Big\}$$

$$\frac{y^2}{(4k_y^2-k_z^2)k_z s}\Big\{-\frac{Q\mu_y}{\beta_0^2\gamma_0^2 p_0 s^2}[(2k_y^2-k_z^2)\sinh(k_z l)+k_y k_z\sinh(2k_y l)]+$$

$$k_y^3\alpha\Big[k_y\sinh(k_z l)-k_z\sinh(k_y l)\cosh(k_y l)\Big]\Big\}$$

$$\frac{yy'}{(4k_y^2-k_z^2)s^2}\left(\frac{2Q\mu_y}{\beta_0^2\gamma_0^2p_0s^2}+k_y^2\alpha\right)(\cosh(k_zl)-\cosh(2k_yl))+$$

$$\frac{y'^2}{(4k_y^2-k_z^2)k_yk_zs^3}\left\{\frac{2Q\mu_y}{\beta_0^2\gamma_0^2p_0s^2}[k_y\sinh(k_zl)-k_z\sinh(k_yl)\cosh(k_yl)]-\right.$$

$$\left.\frac{1}{2}k_y\alpha[(2k_y^2-k_z^2)\sinh(k_zl)+k_yk_z\sinh(2k_yl)]\right\}+$$

$$\frac{\tau^2}{k_zp_0s^3}[-Q\mu_z+\beta_0^2\gamma_0^2p_0k_0^2s^4\alpha(1-\cosh(k_zl))]\sinh(k_zl)+$$

$$\frac{\tau p_\tau\alpha}{s^2}[\cosh\{(k_zl)-\cosh(2k_zl)]-p_\tau^2\frac{1}{2\beta_0^3\gamma_0^2k_zs^5}(\sinh(k_zl)+\sinh(2k_zl)),$$

$$p_{\tau_2}:\frac{x^2}{4(4k_x^2+k_z^2)p_0}\left\{2Q\mu_x[(4k_x^2+k_z^2)+k_z^2\cos(2k_xl)-2(2k_x^2+k_z^2)\cosh(k_zl)]+\right.$$

$$\left.\beta_0^2\gamma_0^2p_0h_x^2s^2\alpha[(4k_x^2+k_z^2)-k_z^2\cos(2k_xl)-4k_x^2\cosh(k_zl)]\right\}+$$

$$\frac{xx'k_z}{(4k_x^2+k_z^2)k_xp_0s}(2Q\mu_x-\beta_0^2\gamma_0^2p_0k_x^2s^2\alpha)\cdot(k_z\sin(k_xl)\cos(k_xl)-k_x\sinh(k_zl))$$

$$\frac{x'^2}{4k_x^2(4k_x^2+k_z^2)p_0s^2}\left\{2Q\mu_x[(4k_x^2+k_z^2)-k_z^2\cosh(2k_xl)-4k_x^2\cos(k_zl)]+\right.$$

$$\left.\beta_0^2\gamma_0^2k_x^2\alpha[(4k_x^2+k_z^2)+k_z^2\cosh(2k_xl)-2(2k_x^2+k_z^2)\cos(k_zl)]\right\}+$$

$$\frac{y^2}{4(4k_y^2-k_z^2)p_0}\left\{2Q\mu_y[(4k_y^2-k_z^2)-k_z^2\cos(2k_yl)-2(2k_y^2-k_z^2)\cosh(k_zl)]+\right.$$

$$\left.\beta_0^2\gamma_0^2p_0k_y^2s^2\alpha[-(4k_y^2-k_z^2)-k_z^2\cos(2k_yl)+4k_y^2\cosh(k_zl)]\right\}+$$

$$\frac{yy'k_z}{(4k_y^2-k_z^2)p_0k_ys}(2Q\mu_y+\beta_0^2\gamma_0^2p_0k_y^2s^2\alpha)\cdot[k_y\sinh(k_zl)-k_z\sinh(k_yl)\cosh(k_yl)]+$$

$$\frac{y'^2}{4(4k_y^2-k_z^2)}\left\{\frac{2Q\mu_y}{p_0k_y^2s^2}[-(4k_y^2-k_z^2)-k_z^2\cos(2k_yl)+4k_y^2\cosh(k_zl)]+\right.$$

$$\left.\beta_0^2\gamma_0^2\alpha[(4k_y^2-k_z^2)-h_z^2\cosh(2k_zl)-2(2k_y^2-k_z^2)\cos(k_zl)]\right\}+$$

$$\tau^2\beta_0^2\gamma_0^2\left[\frac{Q\mu_z}{p_0}\cosh(k_zl)-\frac{1}{2}\beta_0^2\gamma_0^2k_z^2s^4\alpha(\cosh(k_zl)-1)\right](\cosh(k_zl)-1)$$

$$\tau p_\tau\left[\frac{2Q\mu_z}{p_0k_zs^3}\cosh(k_zl)-\beta_0^2\gamma_0^2k_zs\alpha(\cosh(k_zl)-1)\right]\sinh(k_zl)+$$

$$\frac{p_\tau^2}{2s^2}\left[\frac{2Q\mu_z}{\beta_0^2\gamma_0^2p_0k_z^2s^4}(1+\cosh(k_zl))-\alpha(2+\cosh(k_zl))\right](\cosh(k_zl)-1). \tag{15}$$

5 Discussions

It is a very complicated procedure to calculate the nonlinear transport of intense pulsed beams. Because the electrical potential of the beams depends on the beam dimensions, and the beam dimensions are related to the electric potential also, we should solve the problem by iterations. The procedure is: first, provide the initial beam dimensions, then calculate the self-excited electric potential. Next, calculate the particle trajectories, and last, go to the second step... After several iterations we can obtain self-consistent solutions of the particle trajectories.

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强流脉冲高斯分布束在磁四极透镜中的非线性传输

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摘要　用李代数方法分析了高斯分布下强流脉冲束在磁四极透镜中的非线性传输. 在高斯分布下,束流的空间电荷势可利用 Green 函数算出,进而可以得到包含束流自场的粒子运动的 Hamilton 函数. 再施加李变换,就可以得到粒子运动的各级近似解. 本文给出二级近似下的结果,根据需要,还可以扩展到更高级近似. 计算过程需要进行迭代,即根据每次算出的轨迹值,确定束团在三维实空间中的大小,然后再进行迭代,直到满足精度要求为止.

关键词　磁四极透镜　高斯分布束　Lie 映射　非线性

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