

# Calculation of Absolute Intensities of $\gamma$ -Rays from $\alpha$ Decay

ZHOU Chun-Mei<sup>1)</sup> WU Zhen-Dong

(China Nuclear Data Center, Department of Nuclear Physics, China Institute of Atomic Energy, Beijing 102413, China)

**Abstract** A brief introduction is given how to calculate the absolute intensities of  $\gamma$ -rays arising from  $\alpha$  decay. Examples are given to illustrate the applications of the calculation methods. The intensity balance and some discussion are also given in text.

**Key words**  $\alpha$  decay,  $\gamma$ -ray, intensity calculation, intensity balance

## 1 Introduction

The  $\alpha$  decay data and schemes are some of the basic data for nuclear physics research, nuclear technology application, especially radioactive isotope application. In the  $\alpha$  decay process,  $\alpha$ -particles with different energies are emitted from the parent nuclide to different excited states and the ground state of the daughter nuclide, then the  $\gamma$ -rays and their internal conversion electrons are emitted in the de-excited process from the higher excited states to the lower excited states and to the ground state. In common uses, the users are interested in the  $\gamma$ -ray emission probability (the  $\gamma$ -ray absolute intensity per 100  $\alpha$  decays of parent). In most radioactive decay measurements, the  $\gamma$ -ray relative intensities are measured, because the measurements of the  $\gamma$ -ray absolute intensities are very difficult and the measurement accuracy is quite high. Therefore, it is required to convert the  $\gamma$ -ray relative intensity to  $\gamma$ -ray absolute intensity (the emission probability). This conversion factor is called  $\gamma$ -ray normalization factor of decay scheme. The calculation methods of absolute intensities of  $\gamma$ -rays from  $\alpha$  decay and the intensity balance checking are introduced briefly, examples are also given to illustrate their applications and some discussions given in the paper.

## 2 Calculation methods

### 2.1 Calculation from decay intensity $I_{\alpha,0}$ to ground state of daughter nuclide

In  $\alpha$  decay, the decay intensity  $I_{\alpha,0}$  to the ground state

of the daughter nuclide has been measured. The  $\gamma$ -ray relative decay intensity from the  $J$  excited state to the ground state is  $I_{\gamma^{J,0}}$ , its internal conversion coefficient is  $\alpha_{J,0}$ , where  $J \geq 1$ . From the  $\gamma$ -ray intensity balance, Eq. (1) is obtained, where  $N_\gamma$  is the  $\gamma$ -ray intensity normalization factor of the decay scheme. From Eq. (1), the  $\gamma$ -ray intensity normalization factor can be calculated by Eq. (2). Therefore, the  $\gamma$ -ray absolute intensities (the emission probabilities) can be calculated very easily by Eq. (3), where  $P_\gamma$  is the  $\gamma$ -ray absolute intensity, and  $I_\gamma$  is the  $\gamma$ -ray relative intensity.

$$(100 - I_{\alpha,0}) = N_\gamma \sum_j I_{\gamma^{j,0}}(1 + \alpha_{j,0}), \quad (1)$$

$$N_\gamma = \frac{(100 - I_{\alpha,0})}{\sum_j I_{\gamma^{j,0}}(1 + \alpha_{j,0})}, \quad (2)$$

$$P_\gamma = N_\gamma I_\gamma. \quad (3)$$

For example, the decay intensity  $I_{\alpha,0}$  to the ground state of the daughter nuclide for  $^{226}\text{Ra}$   $\alpha$  decay<sup>[1]</sup> is known,  $I_{\alpha,0} = (94.45 \pm 0.05)\%$ . In Table 1, the gamma-ray intensities and the internal conversion coefficients for  $^{226}\text{Ra}$   $\alpha$  decay are listed. The  $\gamma$ -ray normalization factor,  $N_\gamma = (3.59 \pm 0.06)\%$ , can be obtained by using Eq. (2). The  $\gamma$ -ray absolute intensities,  $P_\gamma$  (the emission probabilities), as shown in Table 1, are calculated by using Eq. (3). In Fig. 1, scheme for  $^{226}\text{Ra}$   $\alpha$  decay is given, where the  $\alpha$  decay intensities to different states of the daughter nuclide,  $\gamma$ -energies,  $\gamma$ -multipolarities,  $\gamma$ -absolute intensities and absolute intensities of their internal conversion electron, and places for  $\gamma$ -rays are also shown (for example,  $\% \alpha = 100$  means 100  $\alpha$  decays for the

parent nuclide, 186.211 E2  $\gamma$  3.59  $\epsilon$  2.49 means  $\gamma$ -energy is 186.211 keV, its multipolarity is E2, its intensity is 3.59, and its intensity of internal conversion electron is 2.49).

**Table 1.  $\gamma$ -ray intensities and internal conversion coefficients for  $^{226}\text{Ra}$   $\alpha$  decay.**

$E_\gamma/\text{keV}$	$I_\gamma^*$	$\alpha$	$P_\gamma^*/\%$
34.8 <sup>§</sup>	16		
186.211	13	100	0.693
187.10	20		
262.27	5	0.139	14
414.60	5	0.0836	0.212
449.37	10	0.00592	0.0164
600.66	5	0.0136	0.0030
			0.00766
			0.00049
			2

\* relative intensity; + absolute intensity; § uncertainty (error): the uncertainty in any number is given space after the number itself: for example, 34.8 16 means  $34.6 \pm 1.6$  (all the same below.)

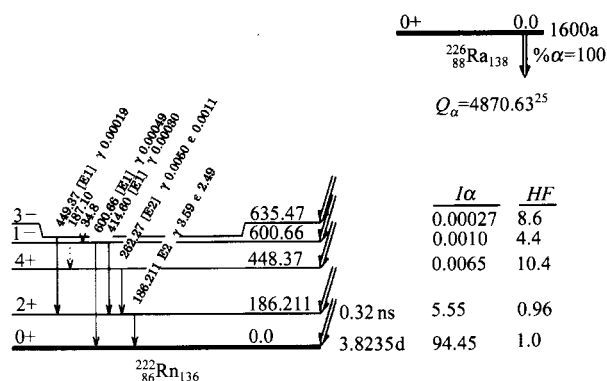


Fig. 1. Scheme for  $^{226}\text{Ra}$   $\alpha$  decay.

### 2.2 Calculation from decay intensity $I_{\alpha,J}$ to $J$ excited state of daughter nuclide

In  $\alpha$  decay, the decay intensity  $I_{\alpha,J}$  to the  $J$  excited state (level) of the daughter nuclide has been measured. The relative total transition intensity feeding  $J$ -state(level) is as follows:

$$TI_J(\text{in}) = \sum_i I_{\gamma^{i,J}}(1 + \alpha_{i,J}). \quad (4)$$

The relative total transition intensity leaving  $J$ -state(level) is as follows:

$$TI_J(\text{out}) = \sum_o I_{\gamma^{J,o}}(1 + \alpha_{J,o}). \quad (5)$$

On the basis of the intensity balance, the Eq. (6) can be obtained,

$$I_{\alpha,J} = N_\gamma \left[ \sum_o I_{\gamma^{J,o}}(1 + \alpha_{J,o}) - \sum_i I_{\gamma^{i,J}}(1 + \alpha_{i,J}) \right], \quad (6)$$

or

$$N_\gamma = \frac{I_{\alpha,J}}{\left[ \sum_o I_{\gamma^{J,o}}(1 + \alpha_{J,o}) - \sum_i I_{\gamma^{i,J}}(1 + \alpha_{i,J}) \right]}. \quad (7)$$

In Eqs. (6) and (7),  $N_\gamma$  is the  $\gamma$ -ray normalization factor of the decay scheme;  $I_{\gamma^{J,o}}$  and  $I_{\gamma^{i,J}}$  are the  $o$ -th and  $i$ -th  $\gamma$ -ray relative intensity of leaving and feeding  $J$ -level;  $\alpha_{J,o}$  and  $\alpha_{i,J}$  are their internal conversion coefficient, respectively. Therefore, the  $\gamma$ -ray absolute intensities (the emission probabilities) can also be calculated very easily by using Eq. (3).

For example, the decay intensity  $I_{\alpha,74.664}$  to 74.664 keV state of the daughter nuclide for  $^{243}\text{Am}$   $\alpha$  decay<sup>[2]</sup> is known,  $I_{\alpha,74.664} = (87.1 \pm 0.03)\%$ . On the basis of the scheme of  $^{243}\text{Am}$   $\alpha$  decay (Fig. 2), and their  $\gamma$ -ray intensities and their internal conversion coefficients, the 74.664 keV  $\gamma$ -ray absolute intensity (the emission probability) and the normalization factor  $N_\gamma = 0.684 \pm 0.013$  can be calculated by using Eq. (7). The radiation data for  $^{243}\text{Am}$   $\alpha$  decay are listed in Table 2, where data calculations<sup>[3,4]</sup> of the internal conversion electron (eCe), Auger electron (eAu), and X-ray are also done. The scheme for  $^{243}\text{Am}$   $\alpha$  decay is given in Fig. 2, where the  $\alpha$  decay energies and the intensities to different states of the daughter nuclide,  $\gamma$ -energies,  $\gamma$ -multipolarities,  $\gamma$ -absolute intensities and the absolute intensities of their internal conversion electrons, and places for  $\gamma$ -rays are also shown.

### 3 Intensity balance checking

In general, intensity balance for radioactive nuclides is one of the most important physical consistent checking. In principle, Eq. (6) satisfies each excited state (level) of the daughter nuclide. The difference of absolute transition intensity of leaving and feeding  $J$ -state (level) of the daughter nuclide equates to  $I_{\alpha,J}$  ( $\alpha$  decay intensity to the excited  $J$ -state (level) of the daughter nuclide from the parent nuclide). Eq. (6) satisfies the ground state of the daughter nuclide. For the highest excited  $L$ -state (level) of the daughter nuclide, Eq. (8) satisfies,

$$I_{\alpha,L} = N_\gamma \left[ \sum_o I_{\gamma^{L,o}}(1 + \alpha_{L,o}) \right]. \quad (8)$$

In Eq. (8),  $N_\gamma$  is the  $\gamma$ -ray normalization factor,  $I_{\gamma^{L,o}}$  is the

Table 2. Radiation data for  $^{243}\text{Am}$   $\alpha$  decay.

radiation type	energy/keV		absolute intensity(%)		radiation type	energy/keV		absolute intensity(%)	
$\alpha_1$	4695	3	0.0017	5	e Ce <sub>4L</sub>	28.17		0.0036	6
$\alpha_2$	4919	3	$8.5 \times 10^{-5}$		$\gamma_5$	55.40		0.0168	11
$\alpha_3$	4930	3	$1.8 \times 10^{-4}$		$\gamma_6$	68.10		0.0045 <sup>#</sup>	6
$\alpha_4$	4946	3	$3.4 \times 10^{-4}$		$\gamma_7$	71.2		0.0014 <sup>#</sup>	
$\alpha_5$	5008	3			$\gamma_8$	74.660	20	68.4	13
$\alpha_6$	5029				e Ce <sub>8L</sub>	52.233	20	14.4	6
$\alpha_7$	5035				e Ce <sub>8M</sub>	68.937	20	3.54	13
$\alpha_8$	5088	3	0.0055	6	e Ce <sub>8N+</sub>	73.159	20	1.25	5
$\alpha_9$	5113.0	10	0.0101		$\gamma_9$	86.710	20	0.344	9
$\alpha_{10}$	5181.0	10	1.1		e Ce <sub>9L</sub>	64.283	20	0.0488	20
$\alpha_{11}$	5233.3	10	11.2	2	e Ce <sub>9M</sub>	80.987	20	0.0120	5
$\alpha_{12}$	5275.3	10	87.1	3	e Ce <sub>9N+</sub>	85.209	20	0.00413	17
$\alpha_{13}$	5321.0	10	0.21	3	$\gamma_{10}$	98.50		0.0151	21
$\alpha_{14}$	5349.4	23	0.25	2	e Ce <sub>10L</sub>	76.07		0.174	25
e Au <sub>L</sub> <sup>*</sup>	10.10		14.2	21	e Ce <sub>10M</sub>	92.78		0.048	7
XL <sup>+</sup>	13.90		19.6	21	e Ce <sub>10N+</sub>	97.00		0.018	3
$\gamma_1$	31.14	3	0.0477	13	$\gamma_{11}$	117.60	15	0.57	5
e Ce <sub>1L</sub> <sup>‡</sup>	8.71	3	6.2	3	e Ce <sub>11L</sub>	95.17	15	0.037	4
e Ce <sub>1M</sub> <sup>‡</sup>	25.42	3	1.52	7	e Ce <sub>11M</sub>	111.88	15	0.0089	9
e Ce <sub>1N+</sub> <sup>‡</sup>	29.64	3	0.506	21	e Ce <sub>11N+</sub>	116.10	15	0.0032	3
$\gamma_2$	43.10		0.067		$\gamma_{12}$	141.89	3	0.0107	3
e Ce <sub>2L</sub>	20.67		7.89	24	e Ce <sub>12K</sub>	23.21	3	0.00186	8
e Ce <sub>2M</sub>	37.38		2.09	7	$\gamma_{13}$	169.0		0.0014	
e Ce <sub>2N+</sub>	41.60		0.697	21	$\gamma_{14}$	195		0.0010	
$\gamma_3$	43.530	20	5.72	17	$\gamma_{15}$	*220			
e Ce <sub>3L</sub>	21.103	20	4.98	21	$\gamma_{16}$	544.58			
e Ce <sub>3M</sub>	37.807	20	1.25	6	$\gamma_{17}$	587.77			
e Ce <sub>3N+</sub>	42.029	20	0.412	18	$\gamma_{18}$	631.09		0.0003	
$\gamma_4$	50.60		0.0062	10	$\gamma_{19}$	662.2		0.0012	

\* auger electron of L shell; + X-ray of L shell; ‡ internal conversion electrons of L, M, N + shell produced from  $\gamma_1$ -ray, respectively; x unplaced in decay scheme; # total intensity including  $\gamma$ -ray and internal conversion electron.

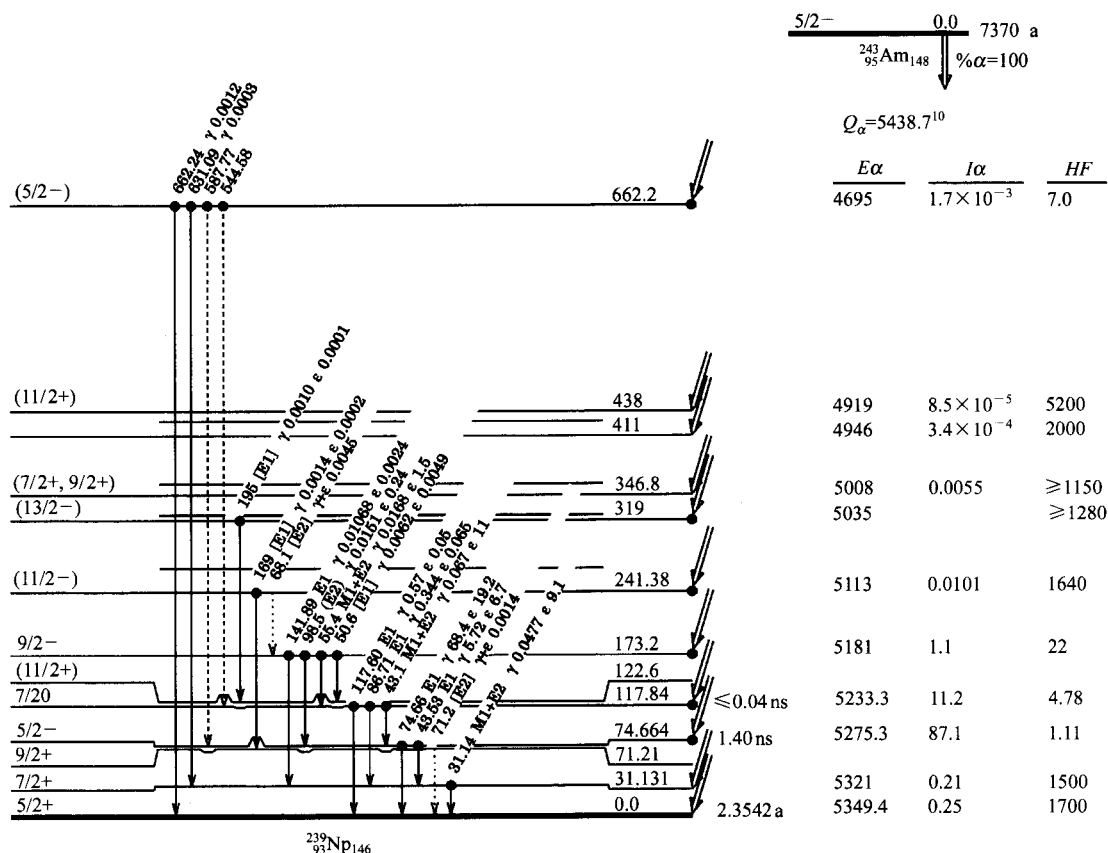


Fig. 2. Scheme for  $^{243}\text{Am}$   $\alpha$  decay.

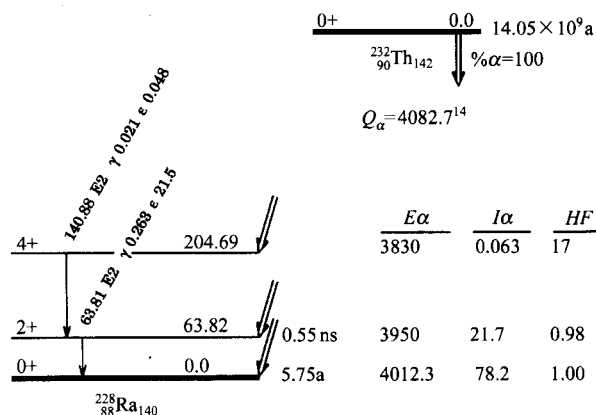


Fig. 3. Scheme of  $^{232}\text{Th}$   $\alpha$  decay.

$o$ -th  $\gamma$ -ray relative intensity of leaving  $L$ -state (level),  $\alpha_{L_o}$  is its internal conversion coefficient,  $I_{\alpha L}$  is the  $\alpha$  decay intensity to the highest excited  $L$ -state (level) of the daughter nuclide.

In Fig. 3, the scheme of  $^{232}\text{Th}$   $\alpha$  decay<sup>[5]</sup> is given, where the  $\alpha$  decay intensities to different states of the daughter nuclide,  $\gamma$ -energies,  $\gamma$ -multipolarities,  $\gamma$ -absolute intensities and absolute intensities of their internal conversion electrons, and places for  $\gamma$ -rays are also shown. In Table 3, the intensity balances of each level of the daughter nuclide for  $^{232}\text{Th}$   $\alpha$  decay is listed. From Table 3, we can see that the  $\gamma$ -ray transition intensity balances for all levels are good within their uncertainties.

Table 3. Intensity balance of each levels of the daughter nuclide for  $^{232}\text{Th}$   $\alpha$  decay.

level/keV	$RI^{\text{a}}$				$TI^{\text{b}}$				net feeding <sup>c</sup>						
	out	in	$(RI_{\text{out}} - RI_{\text{in}})$		out	in	$(TI_{\text{out}} - TI_{\text{in}})$		calculation	$I_o(\%)$					
0.0	0.000	0.263	13	-0.263	13	0.000	21.8	13	-21.8	13	78.2	13	78.2	13	
63.82	1	0.263	13	0.021	4	0.242	14	21.8	13	0.069	14	21.8	13	21.8	13
204.69	2	0.021	4	0.000		0.021	4	0.069	14	0.000		0.069	14	0.069	13

<sup>a</sup> relative  $\gamma$  intensity; <sup>b</sup> total relative intensity including  $\gamma$ -ray and internal conversion electron; <sup>c</sup> absolute  $\alpha$  intensity.

## 4 Discussion

### 4.1 Calculation from decay intensity $I_{\alpha,o} = 0$ to the ground state of the daughter nuclide

The  $\alpha$  decay intensity to the ground state of the daughter nuclide is zero for some radionuclides of  $\alpha$  decay,  $I_{\alpha,o} = 0$ . In this case, Eq. (9) can be gotten from Eq. (2).

$$N_{\gamma} = \frac{100}{\sum_j I_{\gamma^{j,o}}(1 + \alpha_{j,o})}. \quad (9)$$

Therefore, the  $\gamma$ -ray absolute intensities can also be easily calculated by using Eq. (3), when the  $\gamma$ -ray normalization

factor  $N_{\gamma}$  has been calculated.

### 4.2 Decay scheme

In general, when the relative intensity measurements of  $\gamma$ -rays are done, their absolute intensities can be obtained from the calculation of  $\gamma$ -ray normalization factor of the decay scheme. In this case, the accuracy and completeness of the decay scheme are very important. Besides, the  $\alpha$  decay intensity measurements are very difficult because of its short range and high stopping power. Therefore, the  $\gamma$ -ray intensity uncertainties from the calculation of decay scheme are higher. When a high accuracy of the  $\gamma$ -ray intensities for  $\alpha$  decay is required, the measurement of their absolute intensities must be done if possible.

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## $\alpha$ 衰变的 $\gamma$ 射线强度计算

周春梅<sup>1)</sup> 吴振东

(中国原子能科学研究院核物理研究所, 中国核数据中心 北京 102413)

**摘要** 简要介绍了  $\alpha$  衰变的  $\gamma$  射线强度的计算方法, 并以实例进行说明, 还给出了有关强度平衡的物理自洽检验方法及其讨论.

**关键词**  $\alpha$  衰变  $\gamma$  射线 强度计算 强度平衡