第28卷 第7期
2004年7月

高 能 物 理 与 核 物 理

HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS

Vol. 28, No. 7
Jul., 2004

Theory of a Double-Undulator Free-Electron Laser *

DAI Zhi-Min[1)]
(Shanghai Institute of Applied Physics, CAS, Shanghai 201800, China)

Abstract In this paper, we present a general theory of the double-undulator free-electron laser, in which an additional undulator with period close to the beam electron betatron oscillation period in the main undulator is introduced. A set of self-consistent equations is developed to describe the evolution of the optical wave in this device. The basic nonlinear equations are analyzed in the low-gain regime, the high-gain regime, and the saturation regime, respectively. By properly selecting parameters of the additional undulator, we may enhance the gain or efficiency of the free-electron laser.

Key words free-electron laser, double-undulator, driven betatron

1 Introduction

In a free-electron laser (FEL), a relativistic electron beam (REB) passes through a periodic transverse magnetic field, the so-called undulator, to generate coherent radiation ranging from the infrared to the X-ray region[1,2]. The gain and efficiency are key parameters of FELs. Bazylev and Tulupov proposed a new construction of a free-electron laser (FEL), the double-undulator FFL, using driven betatron oscillations to enhance the gain or efficiency of FELs[3]. This new scheme is based on the fact that the magnetic field of a real undulator has a quadruple component which causes the beam electrons to perform betatron oscillation, and the betatron wavelength is dependent upon the beam energy. In this scheme, a second magnetic undulator with a period close to the beam electron betatron wavelength is introduced. Subjected to the second undulator field, the beam electron executes transverse driven betatron oscillation. Since the beam electron betatron wavelength depends parametrically on the beam energy, the amplitude of the beam electron driven betatron oscillation is sensitive to the beam energy. And then under proper conditions of the period and amplitude of the second undulator field, the driven betatron oscillations may determine beam electron grouping in the ponderomotive wave field, and it provides an additional mechanism of electron beam bunching.

In Ref. [3], the gain of the double-undulator FEL in the low-gain regime for the case of a cold, tenuous electron beam is obtained by using Madey's theorem. It is shown that, under proper conditions, the second undulator may enhance the FEL gain or efficiency greatly. In this paper, we present a general single-particle theory of the double-undulator FEL. Our main purpose is to show the effects of the second undulator on the FEL dynamics both in the low gain regime and the high-gain regime. In Sec. 2, a set of self-consistent equations of the double-undulator FEL is developed. In Sec. 3, the small-signal gain of the double-undulator FEL in the low-gain regime and the growth rate of FEL instability in the high-gain regime are derived from these basic equations, respectively. The efficiency of the double-undulator FEL in the regime close to saturation is also estimated. In Sec. 4, a conclusion is given.

Received 26 June 2003, Revised 9 December 2003

* Supported by the Major State Basic Research Development Programme of China (2002CB713600), the Shanghai Science and Technology Council (02QF14059) and the Exploration Project of Knowledge Innovation Program of CAS

1) E-mail: daizm@ssrc.ac.cn

2 Basic equations of the double-undulator FEL

The magnetic vector of the main undulator for the FEL may be expressed by[3]

$$\hat{A}_W = -\hat{e}_y A_u(1 + k_u^2 x^2/2)\cos(k_u z), \quad (1)$$

where A_u and λ_u are respectively the peak value and the period of the main undulator field with $\lambda_u = 2\pi/k_u$. Subjected to the magnetic field given in Eq. (1), the beam electrons execute fast transverse oscillations and slow betatron oscillations. The velocity of the beam electron fast oscillation is given by $\beta_y = (\sqrt{2}a_u/\gamma)\cos(k_u z)\hat{e}_y$, where the strength parameter for the main undulator $a_u = eA_u/\sqrt{2}m_e c^2$, m_e and $e = |e|$ are respectively the rest mass and charge of electron, and c the speed of light. The beam electron betatron wavelength is $\lambda_\beta = 2\pi/k_\beta$, where $k_\beta = a_u k_u/\gamma$, γ is the Lorentz factor of the beam electron. Following Bazylev and Tulupov, we introduce an additional magnetic undulator into this system, and it is

$$A_c = -\hat{e}_x A_c\cos(k_c z), \quad (2)$$

where A_c and λ_c are the peak value and the period of the second undulator field, with $\lambda_c = 2\pi/k_c \approx \lambda_\beta << \lambda_u$, and $A_c << A_u$. Clearly, the second undulator field causes the beam electron to perform driven betatron oscillations along the x-axis. These oscillations are determined by[3]

$$\frac{d^2x}{dz^2} + k_\beta^2 x = -\frac{a_c k_c}{\gamma}\sin(k_c z), \quad (3)$$

where $a_c = eA_c/\sqrt{2}\,m_e c^2$ denotes the second undulator strength. If $k_c \neq k_\beta$, one obtains the solution of Eq. (3)

$$x = \frac{\sqrt{2}a_c}{(k_c^2 - k_\beta^2)}[\sin(k_c z) + x_\beta \sin(k_\beta z + \phi_\beta)], \quad (4)$$

where x_β and ϕ_β are the initial values of the natural betatron motion, which is due to the initial beam emittance and determined by the initial condition. For our purpose, we neglect the beam electron natural betatron oscillations, and consider the effects of the beam electron driven betatron oscillations on the FEL dynamics. In fact, the natural oscillations can be made to vanish by an adiabatic tapering of entering path of the second undulator.

Since the amplitude of the beam electron driven betatron oscillation is sensitive to the beam energy, under proper the conditions of λ_c and a_c, the driven betatron oscillations may determine beam electron grouping in the ponderomotive wave field, and an additional mechanism of electron beam bunching is then provided. To show this effect, we firstly analyze the beam electron longitudinal velocity, which is given by

$$\beta_z = \bar{\beta}_z - \Delta\beta_z, \quad (5)$$

$$\bar{\beta}_z = 1 - \frac{1}{2\gamma_0^2}\left[1 + a_u^2 + \frac{a_c^2 k_c^4}{(k_c^2 - k_\beta^2)^2}\right], \quad (6)$$

$$\Delta\beta_z = -\frac{a_u^2}{2\gamma_0^2}\left[\cos(2k_u\bar{z}) + \frac{a_c^2 k_c^4}{a_u^2(k_c^2 - k_\beta^2)^2}\cos(2k_c\bar{z})\right], \quad (7)$$

where $\bar{z} = \bar{v}_z t$ is the average position of the beam electron with the velocity $\bar{v}_z = \bar{\beta}_z t$.

In the presence of a beam of relativeistic electrons, the main undulator may couple to a high frequency optical wave. The vector potential of the linearly polarized optical field may be taken as

$$\hat{A}_s = -\hat{E}_y(E_s/k_s)\sin(k_s z - \omega_s t + \varphi_s), \quad (8)$$

where E_s and φ_s are the slowly varied amplitude and phase of the optical value, respectively, and $\omega_s = k_s c$ the optical frequency. As it performs any additional oscillations on its longitudinal motion, the beam electron experiences any oscillations on its phase, which is

$$\Delta\psi = k_s\Delta z = -\sigma_1\sin(2k_u z) - \sigma_2\sin(2k_c z), \quad (9)$$

where the parameters σ_1 and σ_2 are respectively defined as $\sigma_1 = k_s a_u^2/4\gamma_0^2 k_u$ and $\sigma_2 = k_s k_c^3 a_c^2/4\gamma_0^2(k_c^2 - k_\beta^2)^2$. The fast osillations on beam electron phase given by the first term of Eq. (9) causes emission into all harmonics[4], e.g., $(2n_1 + 1)k_u c$, for $n_1 = 1, 2, 3, \cdots$. The slow oscillations on beam electron phase expressed by the second term of Eq. (9) causes each harmonic emission into many peaks, e.g., $(2n_1 + 1)k_u c \pm n_2 k_b c$, for $n_2 = 0, 1, 2, 3, \cdots$. In this paper, we assume that the parameter σ_1 is small, e.g., $\sigma_1 \leqslant 0.25$, and neglect the harmonic emission. Furthermore, to avoid radiation line splitting, the parameter σ_2 should be small, i.e., $\sigma_2 \leqslant 0.5$. In this case, the wave-number of the fundamental radiation may be expressed by $k_s \approx 2\gamma_0^2 k_u c(1 + a_u^2)$ and then the undulator parameters may be expressed by $\sigma_1 = a_u^2/2(1 + a_u^2)$, $\sigma_2 = a_c^2 k_c^3 k_u/2(k_c^2 - k_u^2)^2(1 + a_u^2)$.

Usually, theoretical treatment of the FEL based on the coupled Maxwell and Lorentz equations for optical field and electron motion[1,5,6], respectively. Following the classical methods, one obtains a reduced energy equation and wave equation for double-undulator FEL dynamics

$$\frac{d}{d\tau}\Gamma_j = a_s \exp(i\xi_j) + c.c., \tag{10}$$

$$\frac{d}{d\tau}a_s = i\delta a_s - [\exp(-i\xi_j)], \tag{11}$$

where the ponderomotive phase ξ_j is given by $\xi_s = (k_s + k_u - 2k_u\rho\delta)\bar{z}_j - \omega_s t$. Here we have introduced dimensionless variables[6]: $\tau = \bar{z}/L_G$, with $L_G = \lambda_u/4\pi\rho$, the gain length, $\rho = \gamma^{-1}(a_u\omega_b/4k_u c)^{2/3} f_B^{2/3}$, the FEL parameter, $f_B = [J_0(\sigma_1) - J_1(\sigma_1)]J_0(\sigma_2)$, $\omega_b = (4\pi n_b e^2/m_e)^{1/2}$ (n_b is the electron beam density), the plasma angle frequency of electron beam, $a_s = E_s \exp[i(\delta\tau + \varphi_s)]/(4\pi\gamma_0 m_e c^2 n_b \rho)^{1/2}$, the scaled amplitude of optical field, $\Gamma_j = \gamma_j/\rho\gamma_R$, the scaled beam energy, and $\delta = (<\gamma>_0^2 - \gamma_R^2)/2\rho\gamma_R^2$, the scaled initial detuning, with $\gamma_R = [k_s(1 + a_u^2)/2k_u]^{1/2}$, the resonant Lorentz factor.

The ponderomotive phase ξ_j is determined by

$$\frac{d\xi_j}{d\tau} = \frac{k_s}{2\rho}(1 - 1/\bar{\beta}_z) + \frac{1}{2\rho} - \delta. \tag{12}$$

Substituting Eq. (6) into Eq. (12), one may get

$$\frac{d}{d\tau}\xi_j = \frac{1}{2\rho}\left[1 - \frac{1}{\rho^2\Gamma_j^2}\right] - \frac{\Phi}{2\rho^3(\Gamma_c - \Gamma_j)^2}, \tag{13}$$

where the parameter Φ is defined as $\Phi = a_c^2/2(1 + a_u^2)$, $\Gamma_c = \gamma_c/\rho\gamma_R$, with $\gamma_c = a_u k_u/k_c$. Compared to the equations for one-undulator FEL[6], a new feature appears in these equations for the double-undulator FEL, which is the last term in Eq. (13) due to the electron driven betatron oscillations induced by the second undulator field. As this term is sensitive to the beam energy, it alters the nature of the linear regime and the saturation regime of the FEL for $\lambda_c \approx \lambda_\beta$. This effects may be seen clearly in the following. From Eqs. (10) and (13), one obtains

$$\frac{d^2}{d\tau^2}\xi_j = K a_s \exp(i\xi_j) + c.c., \tag{14}$$

$$K = 1 + \Phi\frac{\Gamma_j^3}{(\Gamma_j - \Gamma_c)^3}. \tag{15}$$

The Eqs. (11) and (14) clearly describe the evolution of the optical field in the double-undulator FEL, which are valid in weak or strong optical field, for high or low gain. It should be noted that the valid in weak or strong optical field, for the double-undulator FEL would require $\Delta\gamma/\gamma_0 << (\gamma_0 - \gamma_c)$, where $\Delta\gamma$ is the energy spread. The condition may be written as $\Delta\gamma/\gamma_0 << (k_c - k_\beta)/k_c$. In the conventional FEL, $\Phi = 0$, and then the coupling parameter $K = 1$. It can be seen from Eq. (15) that, by selecting the conditions for λ_c and a_c, the coupling parameter K for the double-undulator FEL may be much larger than unity. This implies that, under proper condition, the additional bunching mechanism may dominate the FEL dynamics, and great enhancement of the FEL gain may be achieved.

3 Gain and saturation of the double-undulator FEL

We now proceed to analyze the basic equation in the small signal low-gain regime, the small singal high-gain regime, and the regime close to saturation. In the small signal regime, the nonlinear Eqs. (11) and (14) may be linearized, and the reference to the individual electron phase can be explicitly removed.

3.1 Low-gain regime

Firstly, we analyze the gain of the optical field ($G = \Delta P/P/_{in}$) in the low-gain regime. In this case, the amplitude of the optical wave is almost constant. One can obtain the expression of the low gain by combining Eqs. (11) and (14) as

$$G = 2\pi g_0 K\frac{\partial}{\partial_{v_0}}\left(\frac{\sin^2(v_0/2)}{(v_0/2)^2}\right), \tag{16}$$

where $v_0 = (\delta - \delta_0)L_U/L_G$, L_U is the undulator length, $\delta_0 = \Phi\gamma_0^2/2\rho(\gamma_0 - \gamma_c)^2$ denotes the effective energy detuning induced by the second undulator, and g_0 is the so-called FEL gain parameter, which for low-gain ($g_0 < 1$) is given by

$$g_0 = 4\pi\frac{J}{I_A}a_u^2 N^3\frac{\lambda_u^2}{\gamma^3}f_B^2. \tag{17}$$

Here J denotes the electron current density, and $I_A = m_e c^3 \cong 17$kA is the Alfven current. This expression of the gain for the double-undulator FEL in the low-gain regime is identical to that obtain by Bazylev and Tulupov from the Madey's theorem[3]. The maximum gain is reached at $v_0 = 2.6$, and given by

$$G_{max} = 0.27\pi g_0 K. \tag{18}$$

Eq. (18) shows that the maximum gain of the double-undulator FEL in the low-gain regime is proportional to the coupling parameter K. And then the low gain is sensitively dependent of period and strength of the second undulator. From Eq. (15), it can be seen that by selecting the condition for λ_c and a_c, the value of the coupling parameter K may be much larger than unity. This indicates that under proper condition the second undulator may enhance the gain of the FEL greatly. In this case, the additional bunching mechanism, due to the driven oscillation induced by the second undulator, dominates the beam electron dynamics in the ponderomotive phase.

To fine the condition for gain enhancement, we rewrite the parameter K as $K=1+2\sigma_2(\lambda_u/\lambda_c)k_c/(k_c-k_\beta)$. For the undulator period $\lambda_u=2$cm, undulator strength $a_u=1$, and beam energy $E_b=5$MeV, the beam electron betatron wavelength $\lambda_\beta=22$cm. When $\sigma_2=0.5$, and $k_c/(k_c-k_\beta)=70$, the coupling factor $K\cong 8$, and the gain of the double-undulator FEL increases as seven times as that compared with an ordinary FEL. Note that as the value of $k_c/(k_c-k_\beta)$ increases, a considerable increasing of the growth rate is obtained, but less energy spread is required.

3.2 High-gain regime

In the high-gain regime, the optical field get enough energy to increase exponently. In the study of linear stability conditions and of the development of the system for small τ, we expand the Eqs. (11) and (14) around their equilibrium values $a_s(0)=0$, $\langle\exp(i\xi_0)\rangle=0$, and seek the solution of the form $\exp(i\lambda\tau)$. In the limit of $\rho\ll 1$, we obtains the eigenvalue equation

$$\lambda^3-(\delta-\delta_0)\lambda^2+K=0, \tag{19}$$

a cubic equation which is well known from the theory of traveling-wave tubes. The instability condition for Eq. (19) is given by $(\delta-\delta_0)^3\geqslant 27/4$. The maximum growth rate $\Gamma=2\rho k_u\,\mathrm{Im}(\lambda)$ occurs at $\delta=\delta_0$, and its value is

$$\Gamma_{\max}=\sqrt{3}\rho k_u K^{1/3}. \tag{20}$$

In the high-gain regime, evolution of the radiation field is dominated by the growth mode, and the gain of double-undulator FEL may be expressed by

$$G_{\max}=\frac{1}{9}\exp(\sqrt{3}\rho K^{1/3}k_u\bar{z}). \tag{21}$$

It shows that the growth rate of the FEL instability in the high gain is proportional to the FEL parameter ρ, the undulator wave-number, k_u and $K^{1/3}$. As the value of the coupling parameter K may be larger than unity under proper condition, growth rate enhancement of the double-undulator FEL may be achieved. For example, in the case whtn undulator period $\lambda_u=2$cm, the beam electron betatron wavelength $\lambda_\beta=22$cm, $\sigma_2=0.5$, and $k_c/(k_c-k_\beta)=70$, the coupling factor $K\cong 8$, and the growth rate of double-undulator FEL instability increases one time as compared with and one-undulator FEL. The higher growth rate, the larger gain with the same undulator length, the shorter undulator length required to reach saturation, and the less cost.

3.3 Saturation regime

In this section, we study the nonlinear saturation of the FEL instability. It is known that the nonlinear saturation of the FEL is due to the beam electrons trapped by the ponderomotive potential well[7]. And then the efficiency of the double-undulator FEL in the regime close to saturation may be estimated to be

$$\eta=\frac{1}{\gamma}\frac{d\gamma}{dv_z}2\Delta v_m, \tag{22}$$

where $\Delta v_m=v_{z0}-v_{ph}$ is the difference between the longitudinal velocity and the ponderomotive wave phase velocity in the case when the growth rate Γ is maximal. Δv_m can be expressed as a function of the parameter λ, i. e., $\Delta v_m=2\bar{v}_z\rho\mathrm{Re}(\lambda)/k_s$. In the limit of $\gamma_0\gg 1$, for $k_c\neq k_\beta$, the efficiency of this device may be expressed by

$$\eta=2\rho\frac{\mathrm{Re}(\lambda)}{K}, \tag{23}$$

In the case when the growth rate Γ is maximal, $\mathrm{Re}(\lambda)=K^{1/3}/2$, and then one obtains

$$\eta=2\rho K^{-2/3}, \tag{24}$$

which shows that the efficiency of the instability is proportional to the FEL parameter ρ, and inversely proportional to $K^{2/3}$. This indicates that for $K\geqslant 1$ (the condition of growth rate enhancement) the efficiency of the FEL is reduced. For the undulator period $\lambda_u=2$cm, the beam electron betatron wavelength $\lambda_\beta=22$cm, $\sigma_2=0.5$, and $k_c/(k_c-k_\beta)=70$, the coupling factor $K\cong 8$, and the efficiency of the double-undulator FEL instability decreases three times as compared with an ordinary FEL.

It can be also seen from Eq. (24) that by selecting the conditions, one can greatly enhance the efficiency of the FEL, but the growth rate is reduced.

For instance, when the undulator period $\lambda_u = 2$cm, the beam betatron wavelength $\lambda_\beta = 22$cm, $\sigma_2 = 0.5$, and $k_c/(k_c - k_\beta) = -10$, the coupling factor $K \cong 0.1$ and the efficiency of double-undulator FEL instability increases three times as compared with an ordinary FEL.

Note that the expressions of the growth rate and efficiency of the double-undulator FEL instability has been based on the cold beam limit and the assumption that $\rho\gamma_0 << (\gamma_0 - \gamma_c)$. The valid of cold beam approximation would require $v_s' < |v_{ph}'|$, where v_s' is the velocity spread with the superscript denoting the quantities in the beam frame. In the laboratory frame, for $\gamma_0 >> 1$, the condition becomes $\Delta\gamma/\gamma << \rho$. Therefore, for the grouping mechanism considered to be realized, it is necessary that $\Delta\gamma/\gamma << \rho << (k_c - k_\beta)/k_c$.

4 Conclusion

In this paper, we present a general theory of the double-undulator FEL. Subjected to the additional undulator field, the beam electron executes transverse driven betatron oscillation. Since the amplitude of the beam electron betatron oscillation is sensitive to the beam energy, it provides an additonal electron beam bunching. Under proper conditions, the driven betatron oscillations may determine beam electron grouping in the ponderomotive wave field, and then the additional bunching mechanism dominates the FEL dynamics. Here, a set of single-particle equations is developed to describe the double-undulator FEL dynamics. The basic nonlinear equations are analyzed in the low-gain regime, the high-gain regime, and the saturation regime, respectively. By properly selecting the parameters of the second undulator, we may enhance the gain or efficiency of the free-electron laser.

For the grouping mechanism induced by the driven betatron oscillations to be realized, it is necessary that a beam electron would be performed more than one driven betatron oscillation over the length of the interaction. As the beam electron betatron wave-number quickly decreases with the increasing of beam energy. For beam energy near 1GeV or more, which is necessary for VUV or soft X-ray production with available undulator, the beam electron makes less than one betatron oscillation in a realistic undulator length, and then the mechanism considered here is broken down, For VUV or soft X-ray FEL, one may use the ion-focusing channel together with a double undulator to enhance the gain[8]. In this case, the beam electron betatron wave-number for such an ion channel is $k_\beta = k_p/(2\gamma)^{1/2}$, where $k_p = \omega_p/c$, and $\omega_p = (4\pi e^2 n_p/m_e c^2)^{1/2}$ is the plasma frequency. As the betatron wave-number for the ion channel slightly depends on γ, and the plasma wave-number k_p can be much larger than $k_u a_u$, the driven betatron oscillations can easily be excited for high beam energy, e. g., 1GeV or more, in VUV or soft X-ray FEL.

References

1 Colson W B, Pellegrini C, Renieri A. Laser Handbook. North-Holland: Amsterdam, 1990. 6

2 Brau C A. Free-Electron Lasers. New York: Academic, Press, 1990

3 Bazyler V A, Tulupov A V. Nucl. Instrum. Methods, 1993, **A331**: 742

4 Colson W B. IEEE J. Quant. Electron, 1981, **17**:1417

5 Colson B W. Free-Electron Lasers. New York: Acadmic Press, 1990

6 Bonifacio R. In High Gain High Power Free Electron Laser: Physics and Aplications to Tev Particle Acceleration. edited by Bonifacio R, De Souza L, Pellegrini C. North-Holland: Amsterdam, 1989. 35—94

7 Sprangle P, Smith R A. Phys. Rev., 1980, **A21**: 293

8 Tulupov A V. Phys. Lett., 1993, **A178**: 189

双波荡器自由电子激光理论*

戴志敏[1)]
（中国科学院上海应用物理研究所　上海　201800）

摘要　给出了双波荡器自由电子激光器的通用理论. 在双波荡器自由电子激光器中，引入了一个周期长度非常接近主波荡器中电子束感应加速振荡周期的附加波荡器. 推导出了一套自治方程组描述双波荡器中自由电子激光场演变过程，并分别给出了低增益、高增益和饱和三种情况下的解析解. 研究表明，适当选择附加波荡器的参数，可以提高自由电子激光器的增益或转换效率.

关键词　自由电子激光　双波荡器　驱动感应加速器振荡

2003－06－26 收稿，2003－12－09 收修改稿
* 中国国家重点基础研究发展计划（2002CB713600），上海市科学技术委员会（02QF14059）和中国科学院知识创新工程资助

1）E-mail：daizm@ssrc.ac.cn