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Twist-3 Distribution Amplitudes in the B→π Transition Form Factor*

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Abstract We derive and express for the B→π transition form factor only depending on the twist-3 distribution amplitudes by choosing an adequate chiral current correlator in the light-cone QCD sum rules. Our result shows that the contribution from the twist-3 distribution amplitudes to the $f^+_{B\pi}(q^2)$ gives a constraint on the twist-3 light-cone distribution amplitude.

Key words transition form factor, distribution amplitudes, light-cone QCD sum rules

Heavy-to-light exclusive decays are important for understanding and testing the Standard Model, and it is of crucial interest to make a reliable prediction for these exclusive processes. Theoretically, the precise calculations of heavy-to-light form factors are of great importance. Especially, it will be helpful for a clear understanding of $B\to\pi l\bar{\nu}_l\,(l=e,\mu)$ which provides us with a good chance to extract the CKM matrix element $|V_{ub}|$ from the available data. Recent progress on QCD factorization formula[1], which was proposed for B→ππ, πK and πD, shows that the amplitudes for these nonleptonic decays can be expressed in terms of the semileptonic form factors, hadronic light-cone distribution amplitudes and hard-scattering functions that are calcuated in perturbative QCD (pQCD). For the semileptonic form factors, one can take them as imputs from experimental data directly.

Many papers have tried to confront calculations of the semileptonic form factors. For example, these form factors can be calculated by pQCD[2] and by applying the light-cone QCD sum rules[3,4]. In fact, a considerable long-distance contribution may dominate the heavy-to-light form factors. The pQCD approach adopts the modified hard-scattering amplitude to them by a resummation of Sudakov logarithms, which can suppress the soft contribution beyond naive power counting. In the light-cone QCD sum rules, the contribution of nonperturbative dynamics is attributed to the distribution amplitudes which are classified by their twists.

The B→π transition form factor was calculated in the light-cone QCD sum rules[3]. Remarkably, the main uncertainties in these calculations arise from the light-cone distribution amplitudes which include not only the twist-2 distribution amplitude but also the twist-3 and the twist-4 distribution amplitudes. The latter two distribution amplitudes are understood poorly. It was shown that the contribution of the twist-3 distribution amplitudes is about 30% ~ 50% and the contribution of the twist-4 distribution amplitudes is about 5% to B→π transition form factor. Thus the great uncertainty, if possible, would be due to the uncertainty in the twist-3 distribution amplitudes in the framework of the light-cone QCD sum rules. In order to reduce the uncertainty Ref.[4] takes an adequate chiral current correlator to make the contribution of the twist-3 distribution amplitudes vanish in the B→π transition form factor. Consequently, the possible pollution by them can be avoided in the B→π transition form factors.

It is very interesting to ask a similar question if one can derive an expression for the B→π transition form factor only depending on the twist-3 distribution amplitudes by choosing the chiral current correlator. The answer is positive. We will discuss the question in this paper.

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Let us start with the following definition of B→π transition form factor $f^{+}_{B\pi}(q^2)$,

$$\langle\pi(p)\,|\,\bar{u}\gamma_{\mu}b\,|\,B(p+q)\rangle = 2f^{+}_{B\pi}(q^2)p_{\mu} + f^{-}_{B\pi}(q^2)q_{\mu} \tag{1}$$

with q being the momentum transfer. In order to calculate the form factor we need to construct a correlator. The different correlator gives the different expression (see Ref.[3] and Ref.[4]). For example, Ref.[4] proposed an improved approach by choosing the chiral current and they got the transition form factor,

$$\begin{aligned} f^{+}_{B\pi}(q^2) = {} & \frac{m_0^2 f_{\pi}}{m_B^2 f_B}e^{m_B^2/M^2}\left\{\int_{\Delta}^{1}\frac{du}{u}\exp\left[-\frac{m_b^2 - q^2(1-u)}{uM^2}\right]\cdot\right. \\ & \left[\phi_{\pi}(u) - \frac{4m_b^2}{u^2M^4}g_1(u) + \frac{2}{uM^2}\cdot\right. \\ & \left.\int_0^u dv g_2(v)\left(1 + \frac{m_b^2 + q^2}{uM^2}\right)\right] + \\ & \int_0^1 dv\int D\alpha_i\frac{\theta(\alpha_1 + v\alpha_3 - \Delta)}{(\alpha_1 + v\alpha_3)^2M^2}\cdot \\ & \exp\left[-\frac{m_b^2 - (1-\alpha_1 - v\alpha_3)q^2}{M^2(\alpha_1 + v\alpha_3)}\right]\cdot \\ & [2\phi_{\perp}(\alpha_i) + 2\tilde{\phi}_{\perp}(\alpha_i) - \phi_{\parallel}(\alpha_i) - \\ & \tilde{\phi}_{\parallel}(\alpha_i)] - 4m_b^2e^{-s_0/M^2}\cdot \\ & \left[\frac{1}{(m_b^2 - q^2)^2}\left(1 + \frac{s_0 - q^2}{M^2}\right)g_1(\Delta) - \right. \\ & \left.\frac{1}{(s_0 - q^2)(m_b^2 - q^2)}\frac{dg_1(\Delta)}{du}\right] - \\ & 2e^{-s_0/M^2}\left[\frac{m_b^2 + q^2}{(s_0 - q^2)(m_b^2 - q^2)}g_2(\Delta) - \right. \\ & \frac{1}{m_b^2 - q^2}\left(1 + \frac{m_b^2 + q^2}{m_b^2 - q^2}\right)\left(1 + \frac{s_0 - q^2}{M^2}\right)\cdot \\ & \left.\left.\int_0^{\Delta}dv g_2(v)\right]\right\} \end{aligned} \tag{2}$$

with $D\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3\delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$, $m_0 = \frac{m_{\pi}^2}{m_u + m_d}$ and $\Delta = (m_b^2 - q^2)/(s_0 - q^2)$. Here ϕ_{π} is π meson twist-2 distribution amplitude, $g_1(u)$, $g_2(u)$, $\phi_{\perp}(\alpha_i)$, $\tilde{\phi}_{\perp}(\alpha_i)$, $\phi_{\parallel}(\alpha_i)$, $\tilde{\phi}_{\parallel}(\alpha_i)$ are π meson twist-4 distribution amplitudes, s_0 is the threshold parameter which should be set to the value near the squared mass of the lowest scalar B^{*} meson, and M is the Borel parameter.

Now we propose to chose another chiral current to construct the correlator,

$$\begin{aligned} \Pi_{\mu}(p,q) = {} & i\int d^4x e^{iq\cdot x}\langle\pi(p)\,|\,T\{\bar{u}(x)\gamma_{\mu}(1+\gamma_5)b(x); \\ & \bar{b}(0)im_b(1-\gamma_5)d(0)\}\,|0\rangle = \\ & \Pi(q^2,(p+q)^2)p_{\mu} + \widetilde{\Pi}(q^2,(p+q)^2)q_{\mu}, \end{aligned} \tag{3}$$

which is different from that in Ref.[4]. Here the chiral limit $p^2 = m_{\pi}^2 = 0$ is made.

This correlator can be calculated in two ways. First, we discuss the hadronic representation for the correlator by inserting a complete set of intermediate states with the same quantum number as that or the current operator $\bar{b}\ i(1-\gamma_5)d$ in it, and then by isolating the pole term of the lowest pseudoscalar B meson, we get,

$$\begin{aligned} \Pi^{H}_{\mu}(p,q) = {} & \Pi^{H}(q^2,(p+q)^2)p^{\mu} + \widetilde{\Pi}^{H}(q^2,(p+q)^2)q_{\mu} = \\ & -\frac{m_b\langle\pi\,|\,\bar{u}\gamma_{\mu}b\,|\,B\rangle\langle B\,|\,\bar{b}\gamma_5 d\,|0\rangle}{m_B^2 - (p+q)^2} + \sum\nolimits_{H}\cdot \\ & \frac{m_b\langle\pi\,|\,\bar{u}\gamma_{\mu}(1+\gamma_5)b\,|\,B_H\rangle\langle B_H\,|\bar{b}(1-\gamma_5)d\mid 0\rangle}{m_{B_H}^2 - (p+q)^2}. \end{aligned} \tag{4}$$

Here the intermediate states B_H contain not only the pseudoscalar resonances of the masses greater than m_B, but also the scalar resonance with $J^P = 0^{+}$, corresponding to the operator $\bar{b}d$. Substituting Eq.(1) and the definition $m_b\langle B\,|\,\bar{b}i\gamma_5\,d\,|0\rangle = m_B^2 f_B$ into Eq.(4), the invariant amplitudes Π^H and $\widetilde{\Pi}^H$ become

$$\begin{aligned} \Pi^{H}[q^2,(q+p)^2] = {} & \frac{-2f^{+}_{B\pi}m_B^2 f_B}{m_B^2 - (p+q)^2} + \\ & \int_{s_0}^{\infty}\frac{\rho^{H}(s)}{s - (p+q)^2}ds + \text{subtraction} \end{aligned} \tag{5}$$

and

$$\begin{aligned} \widetilde{\Pi}^{H}[q^2,(q+p)^2] = {} & \frac{-f^{-}_{B\pi}m_B^2 f_B}{m_B^2 - (p+q)^2} + \\ & \int_{s_0}^{\infty}\frac{\tilde{\rho}^{H}(s)}{s - (p+q)^2}ds + \text{subtraction}\ . \end{aligned} \tag{6}$$

The terms in the integration are the contribution from higher resonances and continuum states above threshold s_0. Due to the quark-hadron duality ansatz, the spectral densities $\rho^H(s)$ and $\tilde{\rho}^H(s)$ can be approximated by the following expression,

$$\rho^{H}(s) = \rho^{QCD}(s)\theta(s - s_0)\ \text{and}\ \tilde{\rho}^{H}(s) = \tilde{\rho}^{QCD}(s)\theta(s - s_0)\ . \tag{7}$$

On the other hand, the correlator can be calculated in QCD theory, to obtain the desired sum rule for $f^{+}_{B\pi}(q^2)$, we work in the large space-like momentum regions $(p+q)^2 - m_b^2 \ll 0$ for the $b\bar{d}$ channel, and $q^2 \ll m_b^2 - 0(1\text{GeV}^2)$ for the momentum transfer, which correspond to the light-one region $x^2 \simeq 0$ and are required by the validity of the operator product expansion

(OPE). First we can write down a full b-quark propagator,

$$\begin{aligned}\langle 0|Tb(x)\bar{b}(0)|0\rangle = {} & \mathrm{i}\int\frac{\mathrm{d}^4k^4}{2\pi}\mathrm{e}^{\mathrm{i}k\cdot x}\frac{\hat{k}+m_\mathrm{b}}{k^2-m_\mathrm{b}^2} - \\ & \mathrm{i}g_\mathrm{s}\int\frac{\mathrm{d}^4k^4}{2\pi}\mathrm{e}^{\mathrm{i}k\cdot x}\int_0^1\mathrm{d}v\cdot \\ & \left[\frac{1}{2}\frac{\hat{k}+m_\mathrm{b}}{(m_\mathrm{b}^2-k^2)^2}G^{\mu\nu}(vx)\sigma_{\mu\nu} + \right. \\ & \left.\frac{1}{m_\mathrm{b}^2-k^2}vx_\mu G^{\mu\nu}(vx)\gamma_\nu\right], \end{aligned}\tag{8}$$

where $G_{\mu\nu}$ is the gluonic field strength and g_s donotes the strong-coupling constant. Carrying out the OPE for the correction and making use of Eq. (8), we require several formulas in Ref. [5]

$$\begin{aligned}&\langle\pi(q)\mid\bar{u}(x)\mathrm{i}\gamma_5 d(0)\mid 0\rangle = \frac{f_\pi m_\pi^2}{m_\mathrm{u}+m_\mathrm{d}}\int_0^1 du\,\mathrm{e}^{\mathrm{i}uq\cdot x}\phi_\mathrm{p}(u)\\ &\langle\pi(q)\mid\bar{u}(x)\sigma_{\mu\nu}\gamma_5 d(0)\mid 0\rangle = \mathrm{i}(q_\mu x_\nu - q_\nu x_\mu)\cdot\\ &\qquad\frac{f_\pi m_\pi^2}{6(m_\mu+m_\mathrm{d})}\cdot\\ &\qquad\int_0^1 du\,\mathrm{e}^{\mathrm{i}uq\cdot x}\phi_\sigma(u)\\ &\langle\pi(q)\mid\bar{u}(x)g_\mathrm{s}G_{\mu\nu}(vx)\sigma_{\alpha\beta}\gamma_5 d(0)\mid 0\rangle = \\ &\quad \mathrm{i}f_{3\pi}[(q_\mu q_\alpha g\gamma_{\nu\beta} - q_\nu q_\alpha g_{\mu\beta}) - (\alpha\leftrightarrow\beta)]\cdot\\ &\quad\int D\alpha_\mathrm{i}\phi_{3\pi}(\alpha_\mathrm{i})\mathrm{e}^{\mathrm{i}q\cdot x(\alpha_1+v\alpha_3)}.\end{aligned}\tag{9}$$

Here $\phi_\mathrm{p}(u)$ and $\phi_\sigma(u)$ are twist-3 distribution amplitudes of π meson, $\phi_{3\pi}(\alpha_i)$ is twist-3 three-particle distribution amplitude of π meson. Substituting the above b-quark full propagator and the corresponding π meson distribution amplitudes into the correlator and completing the integrations over x and k variables, finally we obtain an expression,

$$\begin{aligned}\Pi^{\mathrm{QCD}}[q^2,(p+q)^2] = {} & \frac{-2f_\pi m_\pi^2}{m_\mathrm{u}+m_\mathrm{d}}\int_0^1\mathrm{d}u\frac{1}{m_\mathrm{b}^2-(p+uq)^2}\cdot\\ & \left[u\phi_\mathrm{p}(u)+\frac{1}{6}\left(2+\right.\right.\\ & \left.\left.\frac{p^2+m_\mathrm{b}^2}{m_\mathrm{b}^2-(p+uq)^2}\right)\phi_\sigma(u)\right]+\\ & \int_0^1\mathrm{d}v\int D\alpha_i\frac{-8f_{3\pi}\phi_{3\pi}(\alpha_i)vq\cdot p}{(m_\mathrm{b}^2-(p+(\alpha_1+v\alpha_3)q)^2)^2}.\end{aligned}\tag{10}$$

After substituting Eq. (10) into Eq. (5) and performing the Borel transformation with respect to $(p+q)^2$, a sum rule for the B→π transition form factor can be obtained

$$\begin{aligned}f_{\mathrm{B}\pi}^{+}(q^2) = {} & \frac{m_\mathrm{b}f_\pi}{m_\mathrm{B}^2 f_\mathrm{B}}\frac{m_\pi^2}{m_\mathrm{u}+m_\mathrm{d}}\exp\left[\frac{m_\mathrm{B}^2}{M^2}\right]\cdot\\ & \left\{\int_\Delta^1\frac{\mathrm{d}u}{u}\exp\left[-\frac{m_\mathrm{b}^2-q^2(1-u)}{uM^2}\right]\cdot\right.\\ & \left[u\phi_\mathrm{p}(u)+\frac{1}{6}\left(2+\frac{m_\mathrm{b}^2+q^2}{uM^2}\right)\phi_\sigma(u)\right]-\\ & \frac{2f_{3\pi}}{f_\pi}\frac{m_\mathrm{u}+m_\mathrm{d}}{m_\pi^2}\int_0^1 v\mathrm{d}v\int D\alpha_i\frac{\theta(\alpha_1+v\alpha_3-\Delta)}{(\alpha_1+v\alpha_3)^2}\cdot\\ & \exp\left[-\frac{m_\mathrm{b}^2-q^2(1-\alpha_1-v\alpha_3)}{(\alpha_1+v\alpha_3)M^2}\right]\cdot\\ & \left.\left[1-\frac{m_\mathrm{b}^2-q^2}{(\alpha_1+v\alpha_3)M^2}\right]\phi_{3\pi}(\alpha_\mathrm{i})\right\},\end{aligned}\tag{11}$$

where M is the Borel parameter. Eq. (11) shows that $f_{\mathrm{B}\pi}^{+}(q^2)$ only depends on the twist-3 distribution amplitudes. It means that the contribution from the twist-3 distribution amplitudes to the $f_{\mathrm{B}\pi}^{+}(q^2)$ has the same order of magnitude as that from the leading twist distribution amplitude.

Now we need to make a choice of input parameters entering the sum rule Eq. (11) for $f_{\mathrm{B}\pi}^{+}(q^2)$. Let us specify the twist-3 model of the pion distribution amplitudes, $\phi_p(u)$, $\phi_\sigma(u)$ and $\phi_{3\pi}(\alpha_i)$ (Ref. [3,5]),

$$\begin{aligned}\phi_p(u) = {} & 1+B_2\frac{1}{2}(3(2u-1)^2-1)+\\ & B_4\frac{1}{8}(35(2u-1)^4-30(2u-1)^2+3),\end{aligned}$$

$$\begin{aligned}\phi_\sigma(u) = {} & 6u(1-u)\left[1+C_2\frac{3}{2}(5(2u-1)^2-1)+\right.\\ & \left.C_4\frac{15}{8}(21(2u-1)^4-14(2u-1)^2+1)\right]\end{aligned}$$

and

$$\begin{aligned}\phi_{3\pi}(\alpha_i) = {} & 360\alpha_1\alpha_2\alpha_3^2\left[1+\bar{w}_{1,0}\frac{1}{2}(7\alpha_3-3)+\right.\\ & \bar{w}_{2,0}(2-4\alpha_1\alpha_2-8\alpha_3+8\alpha_3^2)+\\ & \left.\bar{w}_{1,1}(3\alpha_1\alpha_2-2\alpha_3+3\alpha_3^2)\right],\end{aligned}$$

where $B_2 = 30R$, $B_4 = \frac{3}{2}(4\bar{w}_{2,0}-\bar{w}_{1,1}-2\bar{w}_{1,0})$, $C_2 = R(5-\frac{1}{2}\bar{w}_{1,0})$, $C_4 = \frac{1}{10}R(4\bar{w}_{2,0}-\bar{w}_{1,1})$ with $R = \frac{f_{3\pi}}{m_0 f_\pi}$, $f_\pi = 133\mathrm{MeV}$, $f_{3\pi} = 0.0026\mathrm{GeV}^2$ and $\bar{w}_{1,0} = -2.18$, $\bar{w}_{2,0} = 8.12$, $\bar{w}_{1,1} = -2.59$. Other input parameters are taken in the following: $s_0 = 33\mathrm{GeV}^2$, $M^2 = 16\mathrm{GeV}^2$, $m_\mathrm{b} = 4.7\mathrm{GeV}$, $m_\mathrm{B} = 5.28\mathrm{GeV}$ and $f_\mathrm{B} = 165\mathrm{MeV}$. With these inputs, we can carry out the numerical analysis. The form factor Eq. (11) in this paper is depicted by the solid curve in Fig. 1. The dashed and dotted curves in Fig. 1 are taken from Ref. [3] and Ref. [4] respectively. It shows that three curves are consistent in the region $q^2 < 16\mathrm{GeV}^2$. In fact, the applicability of the light-cone

QCD sum rules is questionable as $q^2 \geqslant 18\text{GeV}^2$[4], and a comparison between the different approaches in the regions is meaningless. Also one can see from Fig. 1 that the form factor goes up very quickly beyond the region $q^2 = 15\text{GeV}^2$ as long as the twist-3 distribution amplitudes make a contribution to the sum rules.

In summary, we show that the different expressions for the B→π transition form factor by choosing the different adequate current correlator in the light-cone QCD sum rules. Especially, we derive the expression for $f^+_{B\pi}(q^2)$ only depending on the twist-3 distribution amplitudes. It is consistent with other expressions by employing the present model for the pion distribution amplitudes. Conversely, they provide constraints on the pion distribution amplitudes.

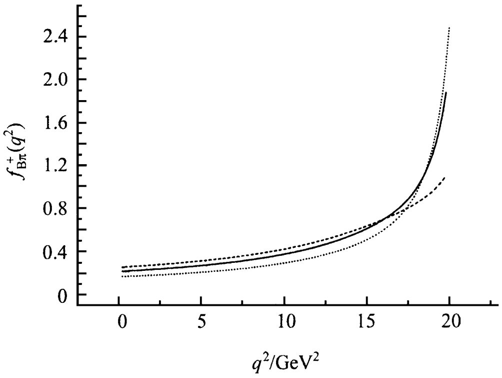

Fig. 1. The transition form factor $f^+_{B\pi}(q^2)$ in the light-cone QCD sum rules at $M^2 = 16\text{GeV}^2$ with $s_0 = 33\text{GeV}^2$, $m_b = 4.7\text{GeV}$, $m_B = 5.28\text{GeV}$, $f_B = 165\text{MeV}$, $f_\pi = 132\text{MeV}$.

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B→π跃迁形状因子中扭度为3的分布振幅*

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摘要　我们选择了恰当的手征流关联函数，用光锥QCD求和规则去计算B到π的跃迁形状因子，得到的结果仅仅依赖于π介子的3扭度光锥分布振幅．这样从$f^+_{B\pi}$的研究中，我们就可以对π介子的3扭度光锥分布振幅给出一个约束条件．

关键词　跃迁形状因子　光锥分布振幅　光锥QCD求和规则

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