Faddeev-Senjanovic Quantization of the Supersymmetrical Electrodynamic System

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Abstract　According to the method of path integral quantization for the canonical constrained system in Faddeev-Senjanovic scheme, we quantize the supersymmetrical electrodynamic system in general situation, and obtain the generating functional of Green function. Another first class constraint is obtained by making the linear combination of several primary constraints, the generator of gauge transformation is constructed, gauge transformations of all the different fields are deduced. Utilizing the consistency equation of gauge fixing condition to deduce another gauge fixing condition, we find that the secondary constraint of the system is an Euler-Lagrange equation which is just the conversation law of electric charge. Thus, we do not need to calculate the other secondary constraints step by step, and get no new constraints naturally. So, the Faddeev-Senjanovic path integral quantization of the supersymmetrical electrodynamic system is simplified.

Key words　supersymmetry, quantum electrodynamics, Faddeev-Senjanovic quantization, Dirac-Bergmman algorithm

1　Introduction

The minimal supersymmetrical standard model (MSSM) is currently the most favored candidate for extension of the standard model. Researches on supersymmetrical quantum field theories have great meaning to the possible discovery of supersymmetry.

Kushreshtha and Müller-Kirsten quantized 1+1 dimensional superfields in Faddevv-Jackiw scheme[1]; Batalin presented a superfield formulation of the quantization program for the theories with the first and second class constraints, and set up a phase-space path integral expression entirely in terms of superfields, further made BRST transformations and canonical transformations enter on equal footing[2]. Rupp et al. obtained, in the level of supersymmetrical quantum field theory, Slazvnov-Taylor iden-

tity satisfying the invariance of the supersymmetrical transformations[3].

Supersymmetrical quantum electrodynamics (SQED) is a simple model of MSSM, and its action is a singular Lagrange system. Using Faddeev-Senjanovic path integral quantization scheme we quantize the system of SQED, construct the generator of gauge transformation, and further give gauge transformation and the generating functional of Green functions.

2　Hamilton system

Under Wess-Zumino gauge, the SQED action is obtained in terms of its component fields after the integral of Grassmann coordinates of the action, i.e., the Lagrangian density is[4]

$$L_{\mathrm{SQED}} = -F_{\mu\nu}(x)F^{\mu\nu}(x) - 4i\lambda(x)\sigma^\mu\partial_\mu\overline{\lambda}(x) +$$
$$i(D_\mu^*\overline{\psi_1}(x))\overline{\sigma}^\mu\psi_1(x) + i(D_\mu^*\overline{\psi_2})\overline{\sigma}^\mu\psi_2 +$$
$$(D_\mu B_1)(D_\mu^* B_1^*) + (D_\mu B_2(x))(D_\mu^* B_2^*(x)) -$$
$$\frac{e}{\sqrt{2}}\big[\overline{\lambda}(x)(\overline{\psi_1}(x)B_1(x) - \overline{\psi_2}(x)B_2(x)) +$$
$$\lambda(x)(\psi_1(x)B_1^*(x) - \psi_2(x)B_2^*(x))\big] +$$
$$m\big[\psi_1(x)\psi_2(x) + \overline{\psi_1}(x)\overline{\psi_2}(x)\big] -$$
$$m^2\big[B_1(x)B_1^*(x) + B_2(x)B_2^*(x)\big] -$$
$$\frac{e^2}{32}\big[B_1(x)B_1^*(x) - B_2(x)B_2^*(x)\big], \tag{1}$$

where

$$D_\mu = \partial_\mu - \frac{1}{2}ieA_\mu, D_\mu^* = \partial_\mu + \frac{1}{2}ieA_\mu, F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{2}$$

A_μ is gauge boson field, λ is the gaugino fermionic partner of A_μ, B_1 and B_2 are supersymmetrical partners of ψ_1 and ψ_2 which are two-component fermionic matter fields，$F_{\mu\nu}(\mu,\nu = 0,1,2,3)$ are gauge field strength tensors，e and m, respectively, represent electro-charge and mass parameters, All spinors are two-component Weyl spinors. We take metrics[5] to be $g_{\mu\nu} = (1,-1,-1,-1)$，and introduce the Pauli matrices $\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\overline{\sigma}^0 = \sigma^0$, $\overline{\sigma}^i = -\sigma^i$.

3 Analysis for constraints of the Hamilton system

Since the Lagrangian of the SQED system is singular, we discuss its constraints in phase space, the canonical momenta conjugate to the component fields are

$$\pi_\mu = 4F_{\mu 0}, \quad \pi_\lambda = 0, \quad \pi_{\overline{\lambda}} = -4i\lambda\sigma^0,$$
$$\pi_{B_1} = D_0^* B_1^* = \left(\partial_0 + \frac{1}{2}ieA_0\right)B_1^*,$$
$$\pi_{B_1}^* = D_0 B_1 = \left(\partial_0 - \frac{1}{2}ieA_0\right)B_1,$$
$$\pi_{B_2} = D_0^* B_2^* = \left(\partial_0 + \frac{1}{2}ieA_0\right)B_2^*, \tag{3}$$
$$\pi_{B_2}^* = D_0 B_2 = \left(\partial_0 - \frac{1}{2}ieA_0\right)B_2,$$
$$\pi_{\psi_1} = 0, \quad \pi_{\overline{\psi_1}} = -i\overline{\sigma}^0\psi_1,$$
$$\pi_{\psi_2} = 0, \quad \pi_{\overline{\psi_2}} = -i\overline{\sigma}^0\psi_2.$$

The canonical Hamiltonian density is given by

$$H_c = -A_0\partial_i\pi^i - \frac{1}{8}\pi_i\pi^i + \pi_{B_1}\pi_{B_1^*} +$$
$$\frac{1}{2}ieA_0(\pi_{B_1}B_1 - \pi_{B_1^*}B_1^*) + \pi_{B_2}\pi_{B_2^*} +$$
$$\frac{1}{2}ieA_0(\pi_{B_2}B_2 - \pi_{B_2^*}B_2^*) +$$
$$\frac{1}{2}eA_0(\overline{\psi_1}\,\overline{\sigma^0}\psi_1 + \overline{\psi_2}\,\overline{\sigma^0}\psi_2) +$$
$$F_{ij}F^{ij} - (D_iB_1)(D_i^*B_1^*) - (D_iB_2)(D_i^*B_2^*) -$$
$$i(D_i^*\overline{\psi_1})\overline{\sigma}^i\psi_1 - i(D_i^*\overline{\psi_2})\overline{\sigma}^i\psi_2 +$$
$$\frac{e}{\sqrt{2}}\big[\overline{\lambda}(\overline{\psi_1}B_1 - \overline{\psi_2}B_2) + \lambda(\psi_1 B_1^* - \psi_2 B_2^*)\big] +$$
$$4i\lambda\sigma^k\partial_k\overline{\lambda} - m(\psi_1\psi_2 + \overline{\psi_1}\overline{\psi_2}) -$$
$$m^2(B_1 B_1^* + B_2 B_2^*) + \frac{e^2}{32}(B_1 B_1^* - B_2 B_2^*). \tag{4}$$

According to Dirac constraint theory[6], it follows that there are 7 primary constraints

$$\phi_1^0 = \pi_0 \approx 0, \quad \phi_2^0 = \pi_\lambda \approx 0, \quad \phi_3^0 = \pi_{\overline{\lambda}} + 4i\lambda\sigma^0 \approx 0,$$
$$\phi_4^0 = \pi_{\psi_1} \approx 0, \quad \phi_5^0 = \pi_{\overline{\psi_1}} + i\overline{\sigma^0}\psi_1 \approx 0, \quad \phi_6^0 = \pi_{\psi_2} \approx 0,$$
$$\phi_7^0 = \pi_{\overline{\psi_2}} + i\overline{\sigma^0}\psi_2 \approx 0, \tag{5}$$

where the symbol "$\approx$" means weak equality in Dirac sense[7].

The total Hamiltonian is given by

$$H_T = \int d^4x\big(H_c + u_1\phi_1^0 + u_2\phi_2^0 + u_3\phi_3^0 + u_4\phi_4^0 +$$
$$u_5\phi_5^0 + u_6\phi_6^0 + u_7\phi_7^0\big). \tag{6}$$

The consistency equations of primary constraints are

$$\dot{\phi}_l^0 = \{\phi_l^0, H_T\}_P \approx 0, \quad (l = 1,2,3,4,5,6,7,). \tag{7}$$

Assume that F and G are functions of the Grassmann canonical variables $(\eta^\alpha, \pi_\alpha)$, the Possion Bracket is given by[8]

$$\{F,G\} = \frac{\partial_r F}{\partial\eta^\alpha}\frac{\partial_l G}{\partial\pi_\alpha} - (-1)^{n_F n_G}\frac{\partial_r G}{\partial\eta^\alpha}\frac{\partial_l F}{\partial\pi_\alpha}, \tag{8}$$

where n_F, n_G represent the Grassmann parities of functions F and G, respectively. The Lagrangian multipliers $u_2, u_3, u_4, u_5, u_6, u_7$ are solved out by consistency equations of primary constraints $\phi_l(l = 2,3,4,5,6,7)$. Consistency of ϕ_1 leads to a secondary

constraint

$$\phi_8^1 = \{\pi_0, H_T\}_P = \partial_i \pi^i + \frac{1}{2}ie(-B_1\pi_{B_1} - B_2\pi_{B_2} +$$

$$B_1^*\pi_{B_1^*} + B_2^*\pi_{B_2^*}) - \frac{1}{2}e(\overline{\psi_1}\,\overline{\sigma^0}\psi_1 + \overline{\psi_2}\,\overline{\sigma^0}\psi_2) \approx 0.$$

$$(9)$$

According to Dirac-Bergmman algorithm[8], three situations may occur from the consistency equations: (1) on the constraint's surface, we get an identity $0 = 0$; (2) the consistency equations are independent of Lagrangian multipliers, we get new constraints; (3) we get equations of Lagrangian multipliers. We get ϕ_8^1 as a secondary constraint, it expresses the charge conversation law of the SQED system in phase space, and is just an Euler-Lagrange equation when converted to configuration space, which cannot give new constraint. When substituting the solved Lagrangian multipliers into the consistency equation, we also get an identity $0 = 0$, and cannot obtain new constraint.

We obtain another first-class constraint by making the linear combination of $\phi_4^0, \phi_5^0, \phi_6^0, \phi_7^0, \phi_8^1$, and further renew to mark the constraints as follows

$$\Lambda_1 = \phi_1^0,$$

$$\Lambda_2 = (\phi_8^1)' = \phi_8^1 - \frac{1}{2}ie(\overline{\psi_2}\theta_6 - \overline{\psi_1}\theta_4 - \psi_1\theta_3 + \psi_2\theta_5) =$$

$$\partial_i\pi^i + \frac{1}{2}ie(-B_1\pi_{B_1} - B_2\pi_{B_2} + B_1^*\pi_{B_1^*} + B_2^*\pi_{B_2^*}) +$$

$$\frac{1}{2}ie(-\overline{\psi_2}\pi_{\psi_2} - \overline{\psi_1}\pi_{\psi_1} + \psi_1\pi_{\psi_1} + \psi_2\pi_{\psi_2}),$$

$$\theta_1 = \phi_2^0,\ \theta_2 = \phi_3^0,\ \theta_3 = \phi_4^0,\ \theta_4 = \phi_5^0,\ \theta_5 = \phi_6^0,\ \theta_6 = \phi_7^0.$$

$$(10)$$

In terms of the definitions of Dirac's first and second-classe constraints[9], we obtain that Λ_1, Λ_2 are the first class of constraints, and $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$ are the second-class constraints. According to Castellani's method to construct generator of gauge transformation[10], we get the generator of the system as follows

$$G = \int d^3x[\dot{\varepsilon}(x)\Lambda_1 - \varepsilon(x)\Lambda_2] = \int d^3x\Bigg\{\varepsilon(\dot{x})\pi_0 - \varepsilon(x)\times$$

$$\Bigg[\partial_i\pi^i + \frac{1}{2}ie(-B_1\pi_{B_1} - B_2\pi_{B_2} + B_1^*\pi_{B_1^*} + B_2^*\pi_{B_2^*}) +$$

$$\frac{1}{2}ie(\overline{\psi_2}\pi_{\psi_2} - \overline{\psi_1}\pi_{\psi_1} + \psi_1\pi_{\psi_1} - \psi_2\pi_{\psi_2})\Bigg]\Bigg\}. \qquad (11)$$

Therefore, the transformations of the component fields are

$$\delta A_\mu = \{A_\mu(x), G\}_P = \partial_\mu\varepsilon(x), \qquad\qquad \delta\lambda = \{\lambda(x), G\} = 0,$$

$$\delta\overline{\lambda} = \{\overline{\lambda}(x), G\} = 0, \qquad\qquad \delta B_1 = \{B_1(x), G\} = \frac{1}{2}ie\varepsilon(x)B_1,$$

$$\delta B_1^* = \{B_1^*(x), G\} = -\frac{1}{2}ie\varepsilon(x)B_1^*, \qquad\qquad \delta B_2 = \{B_2(x), G\} = \frac{1}{2}ie\varepsilon(x)B_2,$$

$$\delta B_2^* = \{B_2^*(x), G\} = -\frac{1}{2}ie\varepsilon(x)B_2^* \qquad\qquad \delta\overline{\psi_1} = \{\overline{\psi_1}(x), G\} = -\frac{1}{2}ie\varepsilon(x)\overline{\psi_1},$$

$$\delta\overline{\psi_2} = \{\overline{\psi_2}(x), G\} = -\frac{1}{2}ie\varepsilon(x)\overline{\psi_2}, \qquad\qquad \delta\psi_1 = \{\psi_1(x), G\} = \frac{1}{2}ie\varepsilon(x)\psi_1,$$

$$\delta\psi_2 = \{\psi_2(x), G\} = \frac{1}{2}ie\varepsilon(x)\psi_2, \qquad\qquad \delta\pi_\mu = \{\pi_\mu(x), G\} = 0, \qquad (12)$$

$$\delta\pi_\lambda = \{\pi_\lambda, G\} = 0, \qquad\qquad \delta\pi_{\overline{\lambda}} = \{\pi_{\overline{\lambda}}(x), G\} = 0,$$

$$\delta\pi_{B_1} = \{\pi_{B_1}(x), G\} = -\frac{1}{2}ie\varepsilon(x)\pi_{B_1}, \qquad\qquad \delta\pi_{B_2} = \{\pi_{B_2}(x), G\} = -\frac{1}{2}ie\varepsilon(x)\pi_{B_2},$$

$$\delta\pi_{B_1}^* = \{\pi_{B_1}^*(x), G\} = \frac{1}{2}ie\varepsilon(x)\pi_{B_1}^*, \qquad\qquad \delta\pi_{B_2}^* = \{\pi_{B_2}^*(x), G\} = \frac{1}{2}ie\varepsilon(x)\pi_{B_2}^*,$$

$$\delta\pi_{\overline{\psi_1}} = \{\pi_{\overline{\psi_1}}(x), G\} = \frac{1}{2}ie\varepsilon(x)\pi_{\overline{\psi_1}}, \qquad\qquad \delta\pi_{\overline{\psi_2}} = \{\pi_{\overline{\psi_2}}(x), G\} = -\frac{1}{2}ie\varepsilon(x)\pi_{\overline{\psi_2}},$$

$$\delta\pi_{\psi_1} = \{\pi_{\psi_1}(x), G\} = -\frac{1}{2}ie\varepsilon(x)\pi_{\psi_1}, \qquad\qquad \delta\pi_{\psi_2} = \{\pi_{\psi_2}(x), G\} = \frac{1}{2}ie\varepsilon(x)\pi_{\psi_2}.$$

These transformations are gauge transformations in phase space for the system.

4 Generating functional of Green function

The Lagrangian density is unchanged under the gauge transformations (12). According to path integral quantization in Faddeev-Senjanovic scheme[11], for each first-class constraint, we need to choose a gauge fixing condition. Consider the Coulomb gauge

$$\Omega_2 = \partial_i A^i \approx 0, \quad (i = 1, 2, 3). \tag{13}$$

Using the consistency of Ω_2, we obtain another gauge fixing condition as follows

$$\Omega_1 = \nabla^2 A_0 - \frac{1}{4} \partial_i \pi^i \approx 0. \tag{14}$$

We introduce exterior sources of fields and their conjugate momenta $(\varphi_\alpha, \pi^\alpha)$, the generating functional of Green function for this system is given by

$$Z[J,K] = \int D\varphi^\alpha D\pi_\alpha \prod_{i,k,l} \delta(\Lambda_i) \delta(\Omega_k) \delta(\theta_l) \times$$

$$\det|\{\Lambda_i, \Omega_k\}| \cdot (\det|\{\theta_{l_1}, \theta_{l_2}\}|)^{1/2} \times$$

$$\exp\{i \int d^4 x (\pi_\alpha \dot\varphi^\alpha - H_c +$$

$$J_\alpha \varphi^\alpha + K^\alpha \pi_\alpha)\}. \tag{15}$$

It is easy to check that $\det|\{\Lambda_i, \Omega_k\}|$, $\det|\{\theta_{l_1}, \theta_{l_2}\}|$ are independent of the fields, and thus we can omit them from the generating functional of Green function, then we have

$$Z[J,K] = \int D\varphi^\alpha D\pi_\alpha \prod_{i,k,l} \delta(\Lambda_i) \delta(\Omega_k) \delta(\theta_l) \times$$

$$\exp\{i \int d^4 x (L_{\mathrm{eff}} + J_\alpha \varphi^\alpha + K^\alpha \pi_\alpha)\}. \tag{16}$$

Using the property of δ function

$$\delta(\Lambda_\alpha) = \int \frac{du^\alpha}{2\pi} \exp(iu^\alpha \Lambda_\alpha). \tag{17}$$

The generating functional of Green function for this system is now deduced as follows

$$Z[J,K] = \int D\varphi^\alpha D\pi_\alpha Du_i Dv_j Dw_k \times$$

$$\exp\{i \int d^4 x (L_{\mathrm{eff}} + J_\alpha \varphi^\alpha + K^\alpha \pi_\alpha)\}, \tag{18}$$

where

$$L_{\mathrm{eff}} = L^p + L_m, \tag{19}$$

$$L^P = \pi_a \varphi^a - H_c, \tag{20}$$

$$L_m = u_i \Lambda_i + v_j \Omega_j + \omega_k \theta_k, \tag{21}$$

$$\varphi^\alpha = (A_\mu, \lambda, \overline{\lambda}, B_1, B_2, B_1^*, B_2^*, \psi_1, \psi_2, \overline{\psi_1}, \overline{\psi_2}, u_i, v_j, w_k), \tag{22}$$

$$\pi_\alpha = (\pi_\mu, \pi_\lambda, \pi_{\overline{\lambda}}, \pi_{B_1}, \pi_{B_1^*}, \pi_{B_2}, \pi_{B_2^*}, \pi_{\psi_1}, \pi_{\overline{\psi_1}}, \pi_{\psi_2}, \pi_{\overline{\psi_2}}), \tag{23}$$

$$J_\alpha = (J_\mu, J_\lambda, J_{\overline{\lambda}}, J_{B_1}, J_{B_2}, J_{B_1^*}, J_{B_2^*}, J_{\psi_1}, J_{\psi_2},$$

$$J_{\overline{\psi_1}}, J_{\overline{\psi_2}}, J_{u_i}, J_{v_i}, J_{w_i}), \tag{24}$$

$$K^\beta = (K_\mu, K_\lambda, K_{\overline{\lambda}}, K_{B_1}, K_{B_2}, K_{B_1^*}, K_{B_2^*}, K_{\psi_1}, K_{\psi_2},$$

$$K_{\overline{\psi_1}}, K_{\overline{\psi_2}}), \tag{25}$$

where u_i, v_j, ω_k are the multiplier fields, and exterior sources $J_{u_i}, J_{v_j}, J_{w_k}$ corresponding to the multiplier fields are induced.

5 Summary and conclusion

Based on the constrained Hamilton theory, we obtain the constraints in the singular SQED system in phase space, two first-class and six secondary constraints are obtained through combining the primary and secondary constraints. Using spinor electrodynamics, Ref. [12] rigorously proved that the secondary constraints act as independent generators of gauge transformations (Dirac conjecture) for the system possessing only the first-class constraints, spinor field ψ has the conjugate momentum $\pi_\psi = i\overline{\psi}\gamma^0$, but it is not considered as a constraint, and $\pi_{\overline{\psi}}$ is not introduced as the conjugate momentum of $\overline{\psi}$ in Ref. [12]. We find that the secondary constraint (9) is the electric charge conversation law of supersymmetry spinor electrodynamics. On the other hand, we may also use Faddeev-Jackiw quantization method[13] to quantize the supersymmetrical electrodynamic system.

According to Castellani's method to construct generator of gauge transformations[10], we get the generator of the gauge transformtions, and the gauge transformations of the component fields.

Using path integral quantization for canonical

constrained system in Faddeev-Senjanovic scheme, and considering Coulomb gauge and its consistent equation to fix gauge, we quantize the supersymmetrical electrodynamic system, and get the generating functional of Green function for this system. Furthermore, we can obtain the canonical Ward identities for the system with the generating functional of Green function.

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超对称电动力学系统的Faddeev-Senjanovic量子化

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摘要　用Faddeev-Senjanovic量子化方法对超对称电动力学系统在一般情况下进行了量子化, 得到了格林函数的生成泛函. 通过对一些约束作线性组合获得了另一个第一类约束, 构造出了该体系的规范生成元, 导出了该系统的规范不变的对称变换. 由一个规范条件的自恰性导出了另一个规范条件, 发现超对称电动力学系统的次级第一类约束对应物理电荷守恒律, 从而使过去要算很多次级约束才能截断的约束自然截断, 因而使超对称电动力学系统在一般情况下的Faddeev-Senjanovic量子化被简化.

关键词　超对称　量子电动力学　Faddeev-Senjanovic量子化　Dirac-Bergmman算法