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Low-Lying Spectra and Electromagnetic Transition Rates in $^{140—162}$Gd in the Interacting Boson Model*

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Abstract Spectra and electromagnetic transition for the even-even $^{140—162}$Gd isotopes are studied in the framework of the interacting boson model. A schematic Hamiltonian can be used to describe their spectra and transition. The results show that $^{140—162}$Gd are in the transition from the vibrational limit to rotational limit.

Key words spectra, electromagnetic transition, positive parity collective state

1 Introduction

The interacting boson model (IBM)[1—3] is a very effective phenomenological model for describing low-lying collective properties of nuclei. Different from the geometrical picture, it has an inherent group structure. In the first version, nucleon pairs coupled out of an entire major shell are s boson and d boson whose angular momentums are $L=0$ and $L=2$ separately and no distinction is made between neutron boson and proton boson in this model. Scholars show strong interest in nucleons in the rare-earth region. Wen Wanxin and Gu Jingnan calculated $B(\mathrm{M1},\ 0_1^+ \to 1_\mathrm{M}^+)$ with the theory of the neutron-proton interacting boson model. The mixed symmetry states in nuclei of $^{150—154}$Gd were studied. They found that the deformation of isotope is a transition from $U(5)$ limit to $SU(3)$ limit or from vibration to rotation[4]. By using the generator coordinate method approach to the dynamic group representation (DGR-DCM), Liang Shidong and Fu Deji discussed the spherical nuclei and deformation nuclei at low excitation state[5]. The potential energy graphs of isotopes show that the potential surface of Gd isotope is a transition from spherical to axial symmetry deformation. The low-lying spectra and E2 transition rates in even-even $^{140—162}$Gd are studied in the framework of the interacting boson model IBM-1. A schematic Hamiltonian is used to calculate the low-lying spectra and E2 transition rates in even-even $^{140—162}$Gd, and the results show that $^{140—162}$Gd are in the transition from the vibration limit to rotational limit.

2 Schematic IBM Hamiltonian

The general IBM Hamilton contains seven terms. However, for our study, we take the following schematic Hamiltonian[6],

$$\hat{H}=\varepsilon_\mathrm{d}\hat{n}_\mathrm{d}+K\hat{Q}\cdot\hat{Q}+K_\mathrm{L}\hat{L}\cdot\hat{L}\ .$$

where

$$\hat{Q}_\mu=(\hat{s}^+\hat{\tilde{d}}+\hat{d}^+\hat{s})^2+\chi(\hat{d}^+\hat{\tilde{d}})^2_\mu\ ,$$

$$\hat{L}_\mathrm{q}=\sqrt{10}(\hat{d}^+\hat{\tilde{d}})^{(1)}_\mathrm{q},\quad \chi=-\sqrt{7}/2\ .$$

The meanings of the above terms are the same as those in other papers about IBM. The Hamiltonian has three terms. The first one is the single particle

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energy, the second one is the interaction of quadruple moments and the third one is the interaction of single polarity. $\varepsilon_{\rm d}$, K, and $K_{\rm L}$ denote the intensity of interaction. Because the term $K_{\rm L}(\hat{L}\cdot\hat{L})$ is diagonal, it contributes the same to the energy levels with identical spin, and is a term adjusting energy level L. If the boson numbers are given, the Hamiltonian is principally determined by the two parameters $\varepsilon_{\rm d}$ and K. However, this Hamiltonian is able to give a transition from $U(5)$ to $SU(3)$, if $\varepsilon_{\rm d}=0$, then the Hamilton reduces to a $SU(3)$ limit Hamiltonian. If $K=0$, the Hamiltonian becomes a $U(5)$ limit, describing the vibrational collective motion. $K_{\rm L}(\hat{L}\cdot\hat{L})$ term removes some of the degeneracy for different L values. Therefore the ratio of $k/\varepsilon_{\rm d}$ is a measure of the transition between $U(5)$ and $SU(3)$. The Hamiltonian is vibrational for $k/\varepsilon_{\rm d}=0$ and the Hamiltonian is rotational for $k/\varepsilon_{\rm d}=\infty$. The Hamiltonian is in the transition between $U(5)$ and $SU(3)$ for the other cases. The parameters in the Hamiltonian can be determined by fitting to the experimental spectra.

3　Results and discussion

In Table 1, we give the parameters the Hamiltonian and the E2 transition operator in each nucleus studied. From Table 1, the value of $\varepsilon_{\rm d}$ increases with the increase of neutron number in the lighter even Gd isotopes. The value of $\varepsilon_{\rm d}$ decreases with the increase of neutron number in heavier even Gd isotopes. $\varepsilon_{\rm d}$ reaches its maximum and K changes from positive to negative near ^{146}Gd. These results show the property of energy change at excitation state to Gd isotopes and its shape. They also show the change tendency of Gd isotope from oblate to prolate. ^{154}Gd is a special nuclei because of ten times the difference between $\varepsilon_{\rm d}$, K, $K_{\rm L}$ of ^{154}Gd and those of ^{152}Gd. ^{154}Gd with neutron number $N=90$ may be $X(5)$ dynamical symmetry. We cannot calculate energy spectra and B(E2) of ^{146}Gd. It is probable that ^{146}Gd is a double-magic nucleus with strong shell effect. There is a little change of e_2. By using these parameters we calculate energy level and B(E2).

Table 1.　Parameters of energy level and B (E2) operator for Gd isotopes.

nucleus	$\varepsilon_{\rm d}$/MeV	K/MeV	$K_{\rm L}$/MeV	e_2/(eb)
^{140}Gd	0.482	−0.0055	0.0045	
^{142}Gd	0.612	−0.0005	0.0005	
^{144}Gd	0.698	0.0055	0.0055	
^{148}Gd	0.685	0.0045	0.0000	
^{150}Gd	0.540	0.0050	0.0025	
^{152}Gd	0.449	−0.0045	0.0025	0.1689
^{154}Gd	0.049	−0.0115	0.0138	0.1185
^{156}Gd	0.048	−0.0150	0.0075	0.1201
^{158}Gd	0.048	−0.0150	0.0075	0.1155
^{160}Gd	0.044	−0.0125	0.0075	0.1094
^{162}Gd	0.040	−0.0095	0.0075	

3.1　Energy spectra

The comparisons between the calculated and experimental values[7] of energy levels and B(E2) for $^{140-162}$Gd are shown in Figs. 1—5 and Table 3. The theoretical energy levels accord with those of experiments, especially for the ground-state band levels in spite of high spin. γ and the first β band are the same. Because we do not consider multi-band coupling,

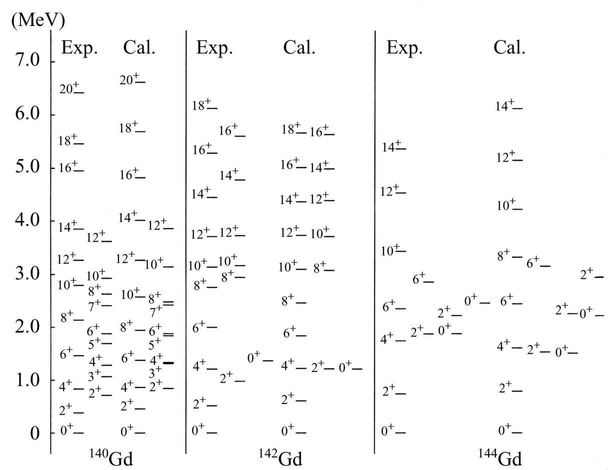

Fig. 1.　Spectra for ^{140}Gd, ^{142}Gd and ^{144}Gd.

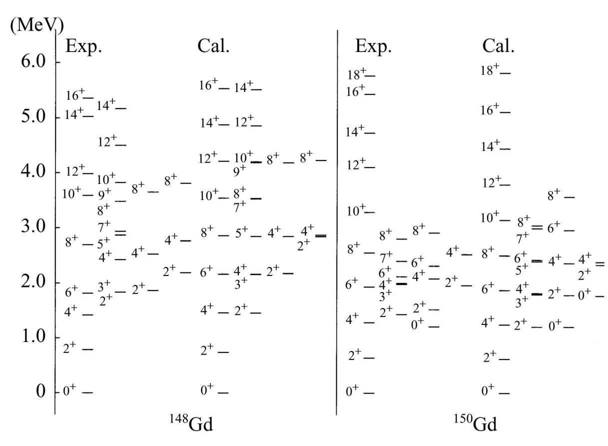

Fig. 2.　Spectra for ^{148}Gd and ^{150}Gd.

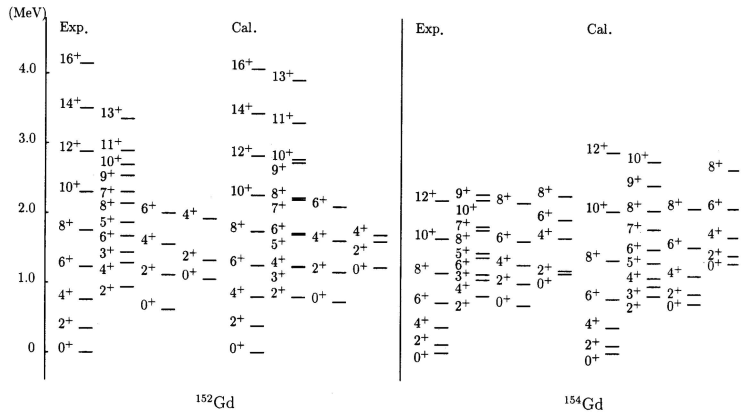

Fig. 3. Spectra for ^{152}Gd and ^{154}Gd.

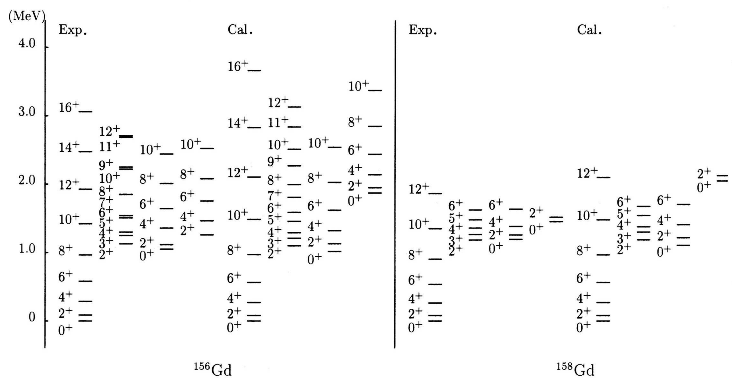

Fig. 4. Spectra for ^{156}Gd and ^{158}Gd.

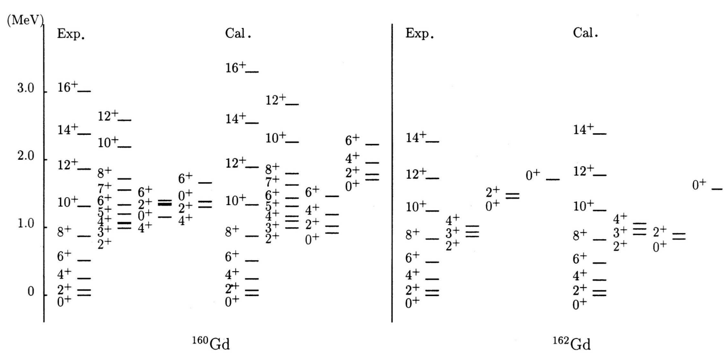

Fig. 5. Spectra for ^{160}Gd and ^{162}Gd.

there are differences between the theoretical and experimental values. The agreement is good for $^{140—144,148,150}$Gd, but there is a backbend at the ground-state band for each of them. This result can be explained with collective backbend theory put forward by Long Guilu[8]. Gd isotopes exhibit staggering in the excited states of $^{140—150}$Gd which exist in the γ band generally. The experimental values show obvious staggering, but they are more homogeneous than those of calculation. 3^+_γ and 4^+_γ, 5^+_γ and 6^+_γ, 7^+_γ and 8^+_γ exhibit strong degeneracy. The spectra are the vibration limit with larger $\varepsilon_{\rm d}$ and smaller K, so there is the staggering. The three-body interaction[9] between the d-boson is introduced to treat the staggering phenomenon in IBM-1. The staggering phenomenon was improved because of adding cubic term to Hamiltonian by Sevrim A and adding quadruple interaction terms of bosons to Hamilton by Liu Yuxin and Long Guilu in IBM2[10]. For $^{144—150}$Gd the two-phonon states are slightly split in energy, about 100keV, which may be understood by means of a small anharmonic term in the collective vibration. However the two-phonon states of ^{142,148}Gd and the three-phonon states of ^{150}Gd have a much larger energy splitting. It is difficult to envisage that such a large energy splitting can be caused by anharmonicties alone, which may be related to the excitation of single particle. The theoretical energy of 8_1^+ state of ^{144}Gd is 3.327MeV and that of 0_4^+ of ^{150}Gd is 1.766MeV.

For ^{152}Gd, the results obtained in the present work are in good agreement with those of experiments. ^{152}Gd exhibits strong property of vibration, which can be understood from energy spectra. We can see the differences between the theoretical and experimental data by comparing the experimental and theoretical energy spectra because there exists strong degeneracy between 3_γ^+ and 4_γ^+, 5_γ^+ and 6_γ^+, 7_γ^+ and 8_γ^+, $9_{\gamma 1}^+$ and 10_γ^+ in the calculated energy spectra. It can be improved by correcting Hamiltonian by adding cubic[11] or quadruple interaction boson[12]. The two-phonon states and the three-phonon sates of ^{152}Gd are slightly split in energy.

For $^{154—156}$Gd, the calculated values are in good agreement with the experimental ones, especially the ground-band, the gamma-band and the first beta-band. Because multi-band coupling is not considered, it is understood that there exist differences between the calculated and the experimental values. Most calculated values of beta-band are larger. The special states, 0^+ and 2^+, may be mixed state or intruder state.

For the ground-band, gamma-band and first beta-band of $^{158—160}$Gd, the calculated values are in good agreement with the experimental ones. The calculated values of the second beta-band are larger than that of the ground-band of ^{162}Gd. For the ground-band, gamma-band and first beta-band of ^{162}Gd the calculated values are in good agreement with the experimental ones. The calculated values of the second beta-band are smaller. There is no obvious staggering for the excited states of $^{158—162}$Gd.

For explaining the changes of $^{140—162}$Gd, the ratios $R=E(4_1^+)/E(2_1^+)$ are shown in Table 2. From Table 2, we can also see a transition from $U(5)$ to $SU(3)$. $^{140—152}$Gd isotopes exhibit obvious properties of vibration. The properties of vibration are becoming strong with the increase of nuclei number and $^{158—162}$Gd are nearly perfect rotors.

Table 2. The ratios of $E(4_1^+)/E(2_1^+)$.

A	R_{Exp}	R_{Cal}
140	2.19	2.13
142	2.35	2.01
144	2.35	2.03
148	1.81	1.98
150	2.02	2.01
152	2.19	2.08
154	3.02	3.32
156	3.24	3.33
158	3.30	3.32
160	3.31	3.32
162	3.34	3.32

3.2 E2 transition

After the determination of the spectra, the wave function is determined. The electric and magnetic transition properties can then be obtained accordingly. For example, the E2 transition operator is

$$\hat{T}(\mathrm{E2})^2_\mu = e_2\left[(\hat{s}^+\tilde{\hat{d}}+\hat{d}^+\hat{s})^2_\mu+\chi\left(\hat{d}^+\tilde{\hat{d}}\right)^2_\mu\right].$$

The meanings of the above terms are the same as those in other papers about IBM. B(E2) of each nuclei are calculated by using e_2 shown in Table 1. The results are shown in Table 3.

From Table 3, we can see that the results calculated are in good agreement with those of experiment.

Δn denotes phonon number of transition between different levels. Most B(E2) values shown in Table 3 are larger for $\Delta n=1$. That means these nuclei are $U(5)$ limit. However there exist some nuclei belonging to $SU(3)$ limit because of smaller B(E2) values for $\Delta n=1$. It is determined by the selection rule of $\Delta\lambda=0$ and $\Delta\mu=0$. Because the beta-band and gamma-band belong to the same $SU(3)$ expression ($\lambda=2N-4,\ \mu=0$) and g-band the other $SU(3)$ expression ($\lambda=2N,\ \mu=0$), the transition between gamma-band and g-band main caused by breaking terms of $SU(3)$ is weak. The fact that the transition probability of $B(\mathrm{E2},\,2_2^+\to 0_1^+)$ is not zero hints that there exist symmetry breaking terms.

Table 3. Comparison of B (E2) values in Gd nuclei[7].

nucleus	J_i	J_f	Exp./($e^2\cdot$fm^4)	Cal./($e^2\cdot$fm^4)
^{152}Gd	2_1^+	0_1^+	3517	3516
	2_2^+	0_1^+	15	20
	2_2^+	0_2^+	2024	1206
	2_2^+	4_1^+	1735	830
	0_2^+	2_1^+	8672	6751
	4_1^+	2_1^+	6456	6544
	6_1^+	4_1^+	9636	8858
	2_2^+	2_1^+	814	5019
^{154}Gd	2_1^+	0_1^+	7698	7698
	2_2^+	0_2^+	4752	5853
	2_3^+	0_2^+	63	67
	2_2^+	0_1^+	42	1
	2_3^+	0_1^+	279	1
	2_2^+	4_1^+	961	6
	2_3^+	4_1^+	84	0
	2_3^+	2_1^+	642	2
	2_2^+	2_1^+	328	2
	0_2^+	2_1^+	2550	9
	4_1^+	2_1^+	12012	10798
	6_1^+	4_1^+	13974	11498
	8_1^+	6_1^+	15297	11438
	10_1^+	8_1^+	17651	10945
^{156}Gd	2_1^+	0_1^+	9328	9328
	2_2^+	0_1^+	31	1
	2_3^+	0_1^+	234	1
	2_4^+	0_1^+	15	0
	2_2^+	2_1^+	170	1
	2_4^+	2_1^+	30	0
	2_3^+	2_1^+	364	1
	2_2^+	4_1^+	205	3
	2_4^+	4_1^+	214	39
	2_3^+	4_1^+	39	0
	4_1^+	2_1^+	13118	13121
	6_1^+	4_1^+	14715	14044
	8_1^+	6_1^+	15962	14086
	10_1^+	8_1^+	15662	13641
	12_1^+	10_1^+	14964	12843
^{158}Gd	2_1^+	0_1^+	10047	10046
	2_2^+	0_1^+	178	1
	2_3^+	0_1^+	16	0
	2_4^+	0_1^+	19	0
	2_2^+	2_1^+	299	1
	2_4^+	2_1^+	21	0
	2_4^+	4_1^+	19	0
	2_4^+	4_2^+	1218	5
	2_2^+	4_1^+	14	2
	2_3^+	4_1^+	71	0
	4_1^+	2_1^+	15664	14161
	8_1^+	6_1^+	16237	15363
	10_1^+	8_1^+	16744	15014
	12_1^+	10_1^+	15730	14317
^{160}Gd	2_1^+	0_1^+	10380	10379
	2_2^+	0_1^+	196	1
	2_2^+	2_1^+	366	1
	2_2^+	4_1^+	37	3

For decay from 2_3^+ to 4_1^+, the transition between states of $\Delta n=1$ can be described by vibration and the transition from beta-band to g-band the rotation. The B(E2) value of ^{154}Gd, ^{156}Gd, ^{158}Gd from 2_3^+ to 4_1^+ is nearly zero. That means these nuclei exhibit strong rotation property.

For further explaining the shape changes of $^{152-158}$Gd, we analyze g-band, B (E2:$L+2\rightarrow L$), values. Results are shown in Fig. 6. From Fig. 6, we can see that the g-band value increases with the increase of angular momentum and it begins to decrease at 1/4 maximum angular momentum. It is similar to $SU(3)$ property[13]. That means $^{156-158}$Gd exhibit transition from $U(5)$ to $SU(3)$.

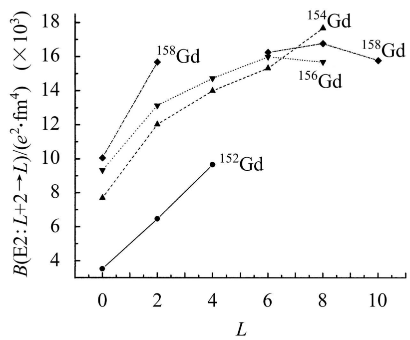

Fig. 6. The values of B (E2:$L+2\rightarrow L$) of the g-band as function of spin L in $^{154-158}$Gd isotopes.

In the $U(5)$ limit, the $\Delta n_\mathrm{d}=1$ selection rule allows a decay from the 0_2^+ state to the 2_1^+ state that can be observed in ^{152}Gd, ^{154}Gd and the decay is strong. In the O (6) limit, the $\Delta\sigma=0$ and $\Delta\tau=1$ selection rule allows a decay from the 0_2^+ state to the 2_2^+ state and the decay from the 0_2^+ state to the 2_1^+ state is forbidden. For all Gd isotopes, the decay from 0_2^+ to 2_2^+ state is not observed. That means $^{140-162}$Gd are the $U(5)$ to $SU(3)$ transitional nuclei.

4 Conclusion

We have made a detailed study of the energy levels and E2 transition in Gd isotopes in the framework of the interacting boson model. The results indicate that $^{140-162}$Gd are the $U(5)$ to $SU(3)$ transitional nuclei. Meanwhile, the discrepancy between the calculated and the experimental data is found, which means that other factors must be introduced into

Hamiltonian, such as pair interacting, isospin effect, high angular momentum boson, IBM-2 and so on.

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$^{140—162}$Gd核的低能谱和电磁跃迁的相互作用玻色子模型*

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摘要　采用相互作用玻色子模型研究了$^{140—162}$Gd偶偶核的低能谱和电磁跃迁, 应用一个$U(5)\rightarrow SU(3)$的简化哈密顿量很好地描述它们的低能谱和电磁跃迁过渡. 结果表明$^{140—162}$Gd同位素核基本上属于$U(5)\rightarrow SU(3)$的过渡核.

关键词　能谱　电磁跃迁　低能正宇称集体态

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