A Generator for Multipole Analysis of $\psi' \to \gamma' \chi_{cJ} \to \gamma' \gamma J/\psi$ *

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Abstract To experimentally study the charmonium ψ' cascade radiative decay, $\psi' \to \gamma' \chi_{cJ} \to \gamma' \gamma J/\psi$, a generator which describes correctly all angular distributions is a necessity. We can determine the photon multipolarities (or equivalently the helicity couplings) by analyzing the angular correlations. In this work, a generator with fulls description of angular distributions for such process will be introduced, as well as the measurement method of multipolarities will be discussed, which can be implemented at BESIII/BEPCII or CLEOc.

Key words generator, multipolarity, helicity amplitude, angular distribution

1 Introduction

The charmonium ψ' electromagnetic decays of the type:

$$\psi'(\lambda') \to \gamma'\ (\mu') + \chi_{cJ}(\nu'), \tag{1}$$

and P-wave spin triplet charmonium decays:

$$\chi_{cJ}(\nu) \to \gamma(\mu)\ +\ J/\psi(\lambda), \tag{2}$$

are very interesting, in which J/ψ can be tagged with lepton pair. The electric dipole(E1) radiative decays of these processes have been studied extensively, both theoretically[1—8] and experimentally[9—13], but so far most of the experimental measurements have focused on the decay rates and the branching ratios. To relative order v^2/c^2 the higher multipoles do not contribute to the total one photon decay rates but they contribute in a significant way to the angular distributions of the photons in the decay of the charmonium states[6].

It is especially interesting that the study of angular distributions will give the information of mixing coefficient between $\psi(2S)$ and $\psi(1D)$. Refs. [6,8] point out that the E3 multipole transition amplitude of $\psi' \to \gamma' \chi_{c2}$ will be zero if ψ' is a pure S-state and the E3 amplitude in $\psi' \to \gamma' \chi_{c2}$ is directly proportional to the D-state mixing coefficient of ψ' (and/or F-state mixing coefficient of χ_{c2}). In principle, the mixing of χ_{c2} with F-state also can contribute to the E3 amplitude. For the detail, see Ref. [6].

Another interesting aspect of the study of angular distribution is that any deviation from the pure E1 distribution is a hint that the relativistic corrections to the E1 transition operator are important[8]. When the first order relativistic corrections to the radiative transition operator are introduced, the transition amplitude involved is not pure E1, but a coherent mixture of E1, M2 and E3 amplitudes. As a result the angular distributions change considerably. The relative strengths of these M2 and E3 parts in the transition amplitude, and hence in the angular distributions, depend on the dynamics, in particular on the potential used and how the initial and final states

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are constructed. A precise determination of the angular distributions in cascade decay (1) and (2) will give us an estimate for the relative strengths of these relativistic corrections.

The format of the rests of the paper is as follows. In Sec. Ⅱ we introduce the angular distributions of such processes, including the relation between the multipole and helicity amplitudes, and how the generator are realized. In order to compare our results with Ref. [5] easily, a special convention is used. In Sec. Ⅲ some projected anglular distributions are introduced. These projected angular distributions will be used to check the generator. Then in the Sec. Ⅳ, we will discuss the measurement of multipolarites and in the end a short summary will be given.

2 Generator

2.1 Angular distribution

Many works[5, 6] have calculated the angular distributions of $\psi' \to \gamma'\chi_{cJ} \to \gamma'\gamma J/\psi$. In Ref. [7], the author also gave the joint angular distributions with J/ψ further decaying to lepton pair, which will be adopted in this work.

All the angular variables which describe the decay process (1) and (2) including the J/ψ leptonic decay are defined as following (see Fig. 1):

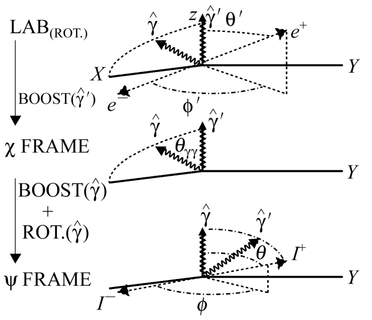

Fig. 1. Vectors and frames describing the cascade decay $\psi' \to \gamma'\chi_c$, $\chi_c \to \gamma J/\psi$, $J/\psi \to l^+l^-$.

θ' and ϕ': the polar and azimuthal angles of the incident e^+(by definition) in the laboratory frame, with the polar axis taken along the direction of γ', and with $\hat{y}$ orthogonal to the two photon directions.

$\theta_{\gamma\gamma}$: angle between γ' and γ in χ rest frame. In this frame the azimuthal angle of the photon γ' is $\phi = \pi$.

θ and ϕ: the polar and azimuthal angles of l^+ in $J/\psi \to l^+l^-$ relative to a rotated J/ψ rest frame in which γ defines the z axis and γ', γ define the x-z plane.

The vectors used to obtain these angles are measured in different frames accordant with the calculation for the correlated angular distribution; they can be defined using the unit vectors $\hat{e}^+$(the incident positron) and $\hat{\gamma}'$ in the ψ' rest frame; $\hat{\gamma}'_\chi$ and $\hat{\gamma}_\chi$ in χ frame; and $\hat{l}^+$(the final positive lepton) and $\hat{\gamma}_\psi$ in the J/ψ rest frame. Details concerning the boosts and rotations can be found in Refs. [10,11]:

$$\cos\theta' = \hat{e}^+ \cdot \hat{\gamma}' \ , \quad \tan\phi' = \frac{\hat{e}^+ \cdot (\hat{\gamma}'_\chi \times \hat{\gamma}_\chi)}{\hat{e}^+ \cdot [(\hat{\gamma}'_\chi \times \hat{\gamma}_\chi) \times \hat{\gamma}']} \ , \tag{3}$$

$$\cos\theta_{\gamma\gamma} = \hat{\gamma}_\chi \cdot \hat{\gamma}'_\chi \ , \tag{4}$$

$$\cos\theta = \hat{l}^+ \cdot \hat{\gamma}_\psi, \quad \tan\phi = \frac{\hat{l}^+ \cdot (\hat{\gamma}'_\chi \times \hat{\gamma}_\chi)}{\hat{l}^+ \cdot [(\hat{\gamma}'_\chi \times \hat{\gamma}_\chi) \times \hat{\gamma}'_\chi]} \ . \tag{5}$$

The joint angular distribution of cascade decay processes (1), (2) and final state leptons can be expressed as:

$$W(\theta',\phi',\theta_{\gamma\gamma},\theta,\phi) = \sum_{\substack{\nu'\tilde{\nu}';\mu'=\pm1 \\ \nu\tilde{\nu};\mu=\pm1}} \rho^{(\mu'-\nu',\mu'-\tilde{\nu}')}(\theta',\phi') \times$$
$$B_{|\nu'|}B_{|\tilde{\nu}'|}d^1_{-\nu'\nu}(\theta_{\gamma\gamma})d^1_{-\tilde{\nu}'\tilde{\nu}}(\theta_{\gamma\gamma})A_{|\nu|}A_{|\tilde{\nu}|}\rho^{*(\nu-\mu,\tilde{\nu}-\mu)}(\theta,\phi), \tag{6}$$

where $\rho^{(\mu'-\nu',\mu'-\tilde{\nu}')}(\theta',\phi')$ and $\rho^{(\nu-\mu,\tilde{\nu}-\mu)}(\theta,\phi)$ are the ψ' and J/ψ density matrix elements(see Table 1), B_ν and A_ν are the helicity amplitudes for processes (1) and (2), respectively.

Table 1. Density-matrix elements $\rho^{\lambda\tilde{\lambda}(\theta,\phi)}$, defined with $\mathrm{Tr}\rho = 2$.

$\rho^{(11)}(\theta,\phi) = \frac{1+\cos^2\theta}{2}$,	$\rho^{(00)}(\theta,\phi) = \sin^2\theta$
$\rho^{(10)}(\theta,\phi) = \frac{\sin\theta\cos\theta}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\phi}$,	$\rho^{(1-1)}(\theta,\phi) = \frac{\sin^2\theta}{2}\mathrm{e}^{-2\mathrm{i}\phi}$
$\rho^{(\tilde{\lambda}\lambda)} = \rho^{(\lambda\tilde{\lambda})*} = (-1)^{\lambda+\tilde{\lambda}}\rho^{(-\lambda,-\tilde{\lambda})}$,	$\rho^{(\lambda\tilde{\lambda})} = \sum_{k=\pm1} D^1_{\lambda k}(\phi,\theta,-\phi)D^{1*}_{\tilde{\lambda}k}(\phi,\theta,-\phi)$

2.2 Helicity amplitudes and multipole amplitudes

The helicity amplitudes $A_{|\nu|}$ (and $B_{|\nu|}$) are in definite ratios to one another for transitions of definite multipolarities. We have

$$A_{|\nu|}^{(J)}=\sum_{J_\gamma} a_{(J)}^{J_\gamma}\left(\frac{2J_\gamma+1}{2J+1}\right)^{\frac{1}{2}}\langle J_\gamma,1,1,|\nu|-1\Big|J,|\nu|\rangle, \quad (7)$$

$$B_{|\nu'|}^{(J)}=\sum_{J_\gamma} b_{(J)}^{J_\gamma}\left(\frac{2J_\gamma+1}{2J+1}\right)^{\frac{1}{2}}\langle J_\gamma,1,1,|\nu'|-1\Big|J,|\nu'|\rangle, \quad (8)$$

where J is the spin of χ and $a_{(J)}^{J_\gamma}$ and $b_{(J)}^{J_\gamma}$ are arbitrarily normalized. When we adopt certain normalization, the mulitpole amplitudes and the helicity amplitudes have simple relations which are shown below.

For J=2:

$$\rho=\frac{b_2}{b_1}=\frac{M2}{E1}, \qquad \rho'=\frac{b_3}{b_1}=\frac{E3}{E1}\ , \quad (9)$$

$$r=\frac{a_2}{a_1}=\frac{M2}{E1}, \qquad r'=\frac{a_3}{a_1}=\frac{E3}{E1}\ . \quad (10)$$

Then the helicity amplitudes will get the forms[1)]:

$$x=\frac{B_1}{B_0}=\sqrt{\frac{1}{3}}\cdot\frac{3+\sqrt{5}\rho-4\rho'}{1+\sqrt{5}\rho+2\rho'}\ ,$$
$$y=\frac{B_2}{B_0}=\sqrt{\frac{2}{3}}\cdot\frac{3-\sqrt{5}\rho+\rho'}{1+\sqrt{5}\rho+2\rho'}\ , \quad (11)$$

and

$$\xi=\frac{A_0}{A_1}=\sqrt{3}\cdot\frac{1+\sqrt{5}r+2r'}{3+\sqrt{5}r-4r'}\ ,$$
$$\eta=\frac{A_2}{A_1}=\sqrt{2}\cdot\frac{3-\sqrt{5}r+r'}{3+\sqrt{5}r-4r'}\ . \quad (12)$$

For J=1, $\rho'=r'=0$,

$$\rho=\frac{b_2}{b_1}=\frac{M2}{E1}\ , \quad (13)$$

$$r=\frac{a_2}{a_1}=\frac{M2}{E1}\ . \quad (14)$$

The helicity amplitudes take the forms:

$$x=\frac{B_1}{B_0}=\frac{1-\rho}{1+\rho}\ , \quad (15)$$

and

$$\xi=\frac{A_0}{A_1}=\frac{1+r}{1-r}\ . \quad (16)$$

For J=0, there is only E1 contribution, we have the relation: $b^1=B_0$ and $a^1=A_0$.

A generator named as ggjpsi is coded to simulate the whole decay processes $\psi'\rightarrow\gamma'\chi_{cJ}\rightarrow\gamma'\gamma J/\psi$, $J/\psi\rightarrow l^+l^-$, based on all the Eqs. (6)—(16). First, we can simulate the processes (1), (2) and final state leptons just according to phase space, then we use Eq. (6) to sample the angular distributions with the multipole ratios ρ, ρ', r and r' as input parameters.

3 Checking the angular distributions

In order to check the simulation results, one should know the angular distribution of each angle.

Table 2. Angular distributions of $\psi'\rightarrow\gamma'\gamma J/\psi$.

angle	J=2	J=1
θ'	$\alpha_1=\frac{1-2x^2+y^2}{1+2x^2+y^2}$	$\alpha_1=\frac{1-2x^2}{1+2x^2}$
ϕ'	$\alpha_1'=\frac{\sqrt{6}(3\eta^2-2-2\xi^2)y}{12(1+x^2+y^2)(\xi^2+1+\eta^2)}$	$\alpha_1'=0$
$\theta_{\gamma\gamma}$	$\beta=6\frac{(\eta^2-2\xi^2)(y^2-2)-2(1-2\xi^2)(x^2-2)}{(\eta y)^2+4(\eta x)^2+4y^2+6\eta^2+6(\xi y)^2+4x^2+4\xi^2}$,	$\beta=\frac{(2\xi^2-1)(2-x^2)}{x^2+2(x\xi)^2+2}$
	$\beta'=\frac{(\eta^2-4+6\xi^2)(y^2-4x^2+6)}{(\eta y)^2+4(\eta x)^2+4y^2+6\eta^2+6(\xi y)^2+4x^2+4\xi^2}$	$\beta'=0$
θ	$\alpha_2=\frac{\xi^2-2+\eta^2}{\xi^2+2+\eta^2}$	$\alpha_2=\frac{\xi^2-2}{\xi^2+2}$
ϕ	$\alpha_2'=\frac{\sqrt{6}(3y^2-2x^2-2)\xi\eta}{12(1+x^2+y^2)(\xi^2+1+\eta^2)}$	$\alpha_2'=0$

1) We adopt this convention for multipole ratios in order to check our results with the formulas in Ref. [5].

In Ref. [6], we can easily get the angular distributions of θ'_γ, ϕ'_γ, θ_γ and ϕ_γ, just by integrating over all other angle variables. From Ref. [6], we can conclude that the helicity angles of final state l^+, θ and ϕ, have the same distributions with those of initial e^+, respectively, if there is only E1 contribution. Generally the angular distributions of θ' and ϕ'(or θ and ϕ) will take the forms $(1+\alpha\cos^2\theta^{(')})$ and $(1+\alpha'\cos2\phi^{(')})$, and the $\theta_{\gamma\gamma}$ will take the form $(1+\beta\cos^2\theta_{\gamma\gamma}+\beta'\cos^4\theta_{\gamma\gamma})$. In Table 2 we list all the expressions of α,β and α',β' in helicity amplitudes. The χ_{c0} case is trivial, because there could be only E1 contribution due to angular momentum conservation, which is not listed in Table 2.

If there is only E1 transition in (1) and (2), say $\rho=\rho'=r=r'=0$, the helicity amplitudes will take simple ratios, for $J=2$, $B_0{:}B_1{:}B_2=A_0{:}A_1{:}A_2=1{:}\sqrt{3}{:}\sqrt{6}$, for $J=1$, $B_0{:}B_1=A_0{:}A_1=1{:}1$. In this case the nontrivial angular distributions should be:

$$W_0(\theta')=1+\cos^2\theta' , \tag{17}$$

$$W_1(\phi_{\gamma\gamma})\propto1+\frac{1}{5}\cos^2\phi_{\gamma\gamma},\quad W_1(\theta')=1-\frac{1}{3}\cos^2\theta', \tag{18}$$

$$W_2(\theta_{\gamma\gamma})\propto1+\frac{21}{73}\cos^2\phi_{\gamma\gamma},\quad W_2(\theta')=1+\frac{1}{13}\cos^2\theta' ,$$
$$W_2(\phi)=1+\frac{1}{5}\cos2\phi . \tag{19}$$

We can fit all these distributions to get α, α' and β to examine if the Monte Carlo event samples have been generated correctly.

In Fig. 2, the histograms show the angular distribution of pure E1 transition process (1) and (2) generated by ggjpsi, the fit results(solid curves) are also plotted in the figures. The upper raw is $(1+\alpha_1\cos^2\theta)$, $\alpha=1.0$, -0.3333 and 0.0769 for $\chi_{c0,1,2}$ respectively; the center raw is $(1+\alpha'_1\cos2\phi)$ distribution, all the α'_1s are zero except that the last one is 0.2; the bottom raw is $(1+\beta\cos^2\theta_{\gamma\gamma})$ distribution, and the $\beta=0$, 0.2 and 0.2877 for $\chi_{c0,1,2}$ respectively. We can see that the parameters $\alpha_1^{(')}$ and β are well consistent with the expectation when considering the errors.

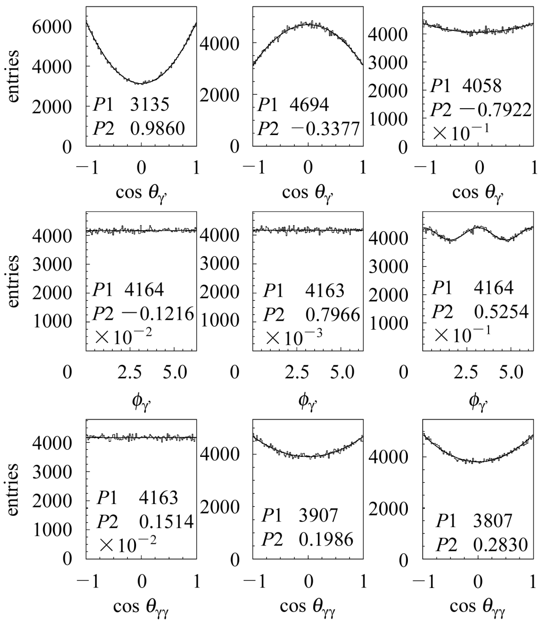

Fig. 2. Angular distributions of pure E1 transition process $\psi'\to\gamma'\chi_c\to\gamma'\gamma J/\psi(\mu^+\mu^-)$. The left column is for χ_{c0}, the center is for χ_{c1} and the right is for χ_{c2}, the parameter $P1$ is a normalized factor, the $P2$ is α_1(the upper), α'_1(the middle) and β(the bottom) of each column.

Table 3. Check of angular distributions(theory/fitted).

J=1		J=2			comments
α_1	β	α_1	β	β'	$\rho=r(=\rho'=r')$
−0.4985/ −0.5033±0.0030	0.0312/ 0.0304±0.0049	0.0081/ 0.0093±0.0048	0.4057/ 0.3952±0.0202	0.1169/ 0.0882±0.0221	−0.10
−0.4197/ −0.4236±0.0033	0.0950/ 0.0927±0.0051	0.0388/ 0.0403±0.0050	0.3679/ 0.3613±0.0200	0.0337/ 0.0270±0.0214	−0.05
−0.3333/ −0.3377±0.0036	0.2000/ 0.1986±0.0057	0.0769/ 0.0792±0.0051	0.2877/ 0.2849±0.0189	0.0000/ −0.0021±0.0208	0.00
−0.2416/ −0.2460±0.0039	0.3538/ 0.3577±0.0062	0.1220/ 0.1234±0.0053	0.1557/ 0.1506±0.0179	0.0388/ 0.0347±0.0196	0.05
−0.1449/ −0.1524±0.0043	0.5664/ 0.5703±0.0070	0.1732/ 0.1745±0.0055	−0.0323/ −0.0416±0.0168	0.1588/ 0.1570±0.0189	0.10

When there not only E1 transition but also M2 and E3 have contribution to the transition amplitude, the angular distributions will change significantly and we also can fit the distributions to check the generator. Table 3 shows the fitting results of some projected angular distributions and the theoretical expectation with different input multipole parameters, we can see that most of them are consistent well.

We also adopt the angular distribution formula from Ref. [5] to perform an independent check. In work of P.K. Kabir et al., they expressed the angular distribution with helicity angles, say $\theta_1, \phi_1, \theta_2$ and ϕ_2 which are the helicity angles of γ' and γ in laboratory frame and χ_c rest frame, respectively. When different input parameters were used, the fitting results are also consistent with theoretical expectation well.

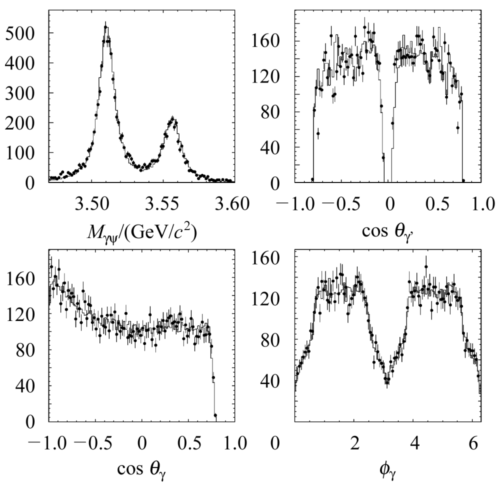

Fig. 3. Angular distributions of $\psi' \to \gamma'\chi_{c1,2} \to \gamma'\gamma J/\psi$, in which error bar is for data and the histogram is for Monte Carlo. The background of $\psi' \to \pi^0\pi^0 J/\psi$ is subtracted by Monte Carlo method for data, and the Monte Carlo generated events adopt assumption of pure E1 transition.

For the check of a physical generator, the comparison between data and Monte Carlo are also needed. After the $\psi' \to \gamma'\chi_{cJ} \to \gamma'\gamma' J/\psi$, $J/\psi \to l^+l^-$ events generated by the generator ggjpsi which is incorporated into the SIMBES[14] , BESⅡ detector simulation package, we can get all the related distributions for this process. For data, however, we can not select pure samples of $\psi' \to \gamma'\chi_{c1}$, $\gamma'\chi_{c2}$ events respectively, since $\psi' \to \pi^0\pi^0 J/\psi$ background can not be removed completely and also the χ_{c1}, χ_{c2} peaks overlap each other on the $\gamma J/\psi$ invariant mass distribution due to a large mass resolution. Fig. 3 shows the data and Monte Carlo comparison, in which the data are selected from 14M ψ' events data and $\psi' \to \pi^0\pi^0 J/\psi$ background is subtracted with Monte Carlo, which are generated according to the branching ratios from PDG[15]. From Fig. 3, we can see that the data and Monte Carlo are consistent, the difference are mainly due to the survived backgrounds that can not be removed and the limited statistics of data.

4 Measurement of the multiplicities

The keypoints of the measurement of the multipolarites are to select a clean data sample with low background, to use more information of the whole event and to perform the efficiency correction properly. From Table 2, we can see that all the the multipole parameters can not be determined with any projected angular distribution when J=2. The correlation between γ' and γ must be considered, and the joint angular distributions are needed. The best way is to use joint angular distribution and to use more information as much as possible. The efficiency correction and background rejection are very important for measurement of the multipolaritis, since the higher multipole amplitudes are much smaller than E1 amplitude. The efficiency correction depends on Monte Carlo simulation, so a generator with full description of angular distributions are needed to study the detector simulation and to check the consistency between the input and output which help us to choose a effective fit scheme. In this measurement, the backgrounds mainly come from $\psi' \to \pi^0\pi^0 J/\psi$, $\psi \to \eta J/\psi$ and $\psi' \to \pi^0 J/\psi$. In order to remove such backgrounds, an electromagnetic calorimeter with excellent energy resolution are needed.

5 Summary and discussion

In this work, we introduced a generator with full angular distribution for process $\psi' \to \gamma'\chi_c \to \gamma'\gamma J/\psi \to \gamma'\gamma l^+l^-$. In order to determine the multipole ratios $\rho(\rho')$ and $r(r')$, we must consider the correlated angular distribution of this decay process. At BESⅡ, we can not perform such a work because the background is hard to be removed completely due to the poor energy resolution of electromagnetic calorimeter. But at BESⅢ or CLEOc, we can expect that such $\chi_{c0,1,2}$ states can be reconstructed well with very low background, and the measurement may be implemented beautifully with high precision.

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用于$\psi' \to \gamma'\chi_{cJ} \to \gamma'\gamma J/\psi$过程中多极分析的产生子*

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摘要　为了实验研究粲偶素ψ'级联衰变过程 $\psi' \to \gamma'\chi_{cJ} \to \gamma'\gamma J/\psi$, 一个用于正确描述全部末态角分布的产生子是必要的. 可以通过分析角分布的关联来确定这一过程的多极展开参数(或者等效的螺旋度振幅), 而从中可以知道高阶电磁多极矩和相对论修正的贡献, 同时可以检验ψ'是不是$\psi(2S)$和$\psi(1D)$的混合. 介绍了一个能完全描述这一过程的角分布的产生子, 同时讨论了测量多极参数的测量方法. 这些都可以被用在BESⅢ或者CLEOc的分析当中.

关键词　产生子　多极展开振幅　螺旋度振幅　角分布

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