Coulomb Displacement Energies and Shell Closure*

ZHU Jian-Yu[1] XU Fu-Rong[1,2,3;1)]

1 (MOE Key Laboratory, School of Physics, Peking University, Beijing 100871, China)
2 (Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, China)
3 (Center for Theoretical Nuclear Physics, National Laboratory for Heavy Ion Accelerator of Lanzhou, Lanzhou 730000, China)

Abstract The mass differences of mirror nuclei have been calculated using the Skyrme-Hartree-Fock model. The deviation of these masses differences from the corresponding experimental results is a longstanding problem and has been studied intensively. In this paper we studied this anomaly systematically. The calculations show that significant changes in the evolution pattern occur around the magic numbers. After the Skyrme force parameters are adjusted to reflect this shell effect, we found that this anomaly is significantly reduced. This shows that the anomaly is dependent on shell closure.

Key words mirror nuclei, Nolen-Schiffer Anomaly, Skyrme-Hartree-Fock model

1 Introduction

With the improvement of experimental techniques, the study of nuclear properties along the N=Z line have been successfully extended to the $A=100$ region. For these nuclei, one of the most attractive topics is studying the properties of mirror pairs[1—6]. A pair of mirror nuclei have the interchanged numbers of proton and neutron. The mass difference of a pair of mirror nuclei is maned the Coulomb displacement energy (CDE). The CDE is related to Coulomb interaction in nuclear many-body systems. However, a longstanding anomaly, which referred to as the Nolen-Schiffer (NS) Anomaly[7], exists in the mean-field studies of CDE.

In the present work, we calculated the CDE systematically using the Skyrme-Hartree-Fock (SHF) model. The calculations show that the NS anomaly also exists in the Skyrme force. We studied the effects of various terms in the Skyrme force on this anomaly and found that the anomaly was sensitive to the x_3 term. This anomaly is significantly reduced with the introduction of a mass dependence into this term, while other properties are not affected.

2 Nolen-Shiffer Anomaly

The CDE is defined as

$$\Delta_E = E_{Z>} - E_{Z<}, \tag{1}$$

where $Z>(Z<)$ indicates the proton(neutron) rich one of mirror nuclei, and $E_{Z>}(E_{Z<})$ are the corresponding binding energy of the mirror nuclei. In this paper, we have preformed self-consistent Hartree-Fock calculation with the Skyrme force[8], which is given as

$$\begin{aligned} V(\boldsymbol{r}_1,\boldsymbol{r}_2) \quad = \quad & t_0(1+x_0\boldsymbol{P}_\sigma)\delta(\boldsymbol{r})+ \\ & \frac{1}{2}t_1(1+x_1\boldsymbol{P}_\sigma)[\boldsymbol{p}'^2\delta(\boldsymbol{r})+\delta(\boldsymbol{r})\boldsymbol{p}^2]+ \\ & t_2(1+x_2\boldsymbol{P}_\sigma)\boldsymbol{p}'\cdot\delta(\boldsymbol{r})\boldsymbol{p}+ \\ & \frac{1}{6}t_3(1+x_3\boldsymbol{P}_\sigma)[\rho(\boldsymbol{R})]^\alpha\delta(\boldsymbol{r})+ \\ & \mathrm{i}\, W_0\sigma\cdot[\boldsymbol{p}'\times\delta(\boldsymbol{r})\boldsymbol{p}], \end{aligned} \tag{2}$$

where

$$\boldsymbol{r}=\boldsymbol{r}_1-\boldsymbol{r}_2,\ \boldsymbol{R}=\frac{1}{2}(\boldsymbol{r}_1+\boldsymbol{r}_2),\ \boldsymbol{p}=\frac{1}{2\mathrm{i}}(\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2),$$

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1) E-mail: frxu@pku.edu.cn

and $\boldsymbol{p}'$ is $\boldsymbol{p}$ acting on the left.

The calculated and experimental CDE[9] of $T = 1/2$ mirror nuclei are shown in Fig. 1. Results calculated by various Skyrme forces are similar, here only the SIII results are shown for simplicity. The overall agreement is satisfactory, which indicate that the SHF calculation could give a good description for the ground states of these nuclei. However, persistent differences still exist. This anomaly could not be removed from the refinement of the Skyrme force parameters, it is also a problem of Skyrme force. The differences are shown in Fig. 2.

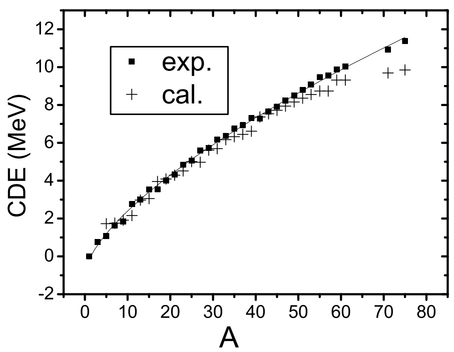

Fig. 1. The calculated CDE and the corresponding experimental data.

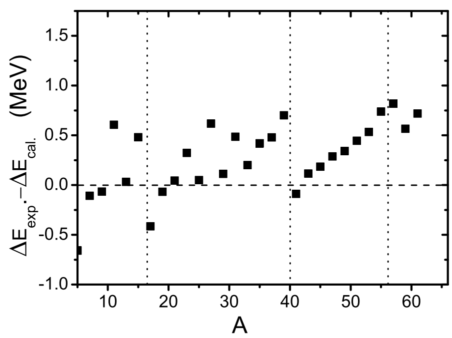

Fig. 2. The differences between the calculated CDE and the corresponding experimental data. The dashed lines indicate the closed shells, with $A = 16, 40$ and 56,respectively.

3 Calculation and discussions

In the past decade, many works have been done trying to remove this difference within the scope of SHF model[10—12]. In Ref. [10] B. A. Brown et al. introduced a charge symmetry-breaking interaction to the Skyrme force and studied the possible contribution from the charge symmetry-breaking effects. In the present work, we evaluated the contributions from various terms in the Skyrme force to this anomaly, and found that the x_3 parameter can affect the anomaly significantly. In usual Skyrme forces, x_3 is a constant,which does not give a good description around magic numbers, as seen in Fig. 2. In the present work, we attempted to give a important description without changing the bulk properties. This could be realized through the introduction of a mass dependent x_3 term. In fp shell, we suggest

$$x_3 = ((N-19)*(Z-19))^a \cdot x_3^0, \quad (3)$$

where x_3^0 is the original value of x_3, and $N(Z)$ is the number of valence neutron (proton) in the major shell. This ensured that the x_3 parameter returns to the original value for the double magic nuclei. The parameter $a = 0.22045$ was derived from minimizing the differences of theoretical and experimental CDE of the $T = 1/2$ mirror nuclei in the $f_{7/2}$ shell. This is a rather phenomenal expression, but its success is very surprising. Besides $T = 1/2$ pairs, this modification also leads to significant reduction of the NS Anomaly in $T = 1, 3/2, 2$ pairs. The CDE calculated with original and modified SIII force for $f_{7/2}$ shell nuclei with $T = 1/2, 1, 3/2$ are shown in Fig. 3. It shows that the Anomaly is possibly related to the shell closure.

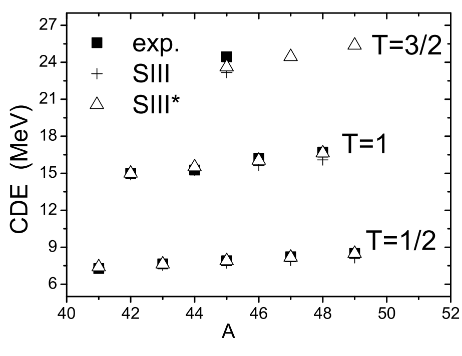

Fig. 3. CDE of the mirror nuclei with $T = 1/2$, 1, 3/2 in the $f_{7/2}$ shell.

Moreover, we also tried to see its validity in other shells. The simple relation given in Eq. (3) is extended as

$$x_3=\left(\frac{N^{\pi}_{f_{7/2}}}{N^{\pi}}(N-19)*\frac{N^{\nu}_{f_{7/2}}}{N^{\nu}}(Z-19)\right)^{a}\cdot x_3^0, \quad (4)$$

where $N^{\nu}(N^{\pi})$ is the maximum number of the neutron (proton) that can be filled in each major shell. The parameter a is left unchanged.

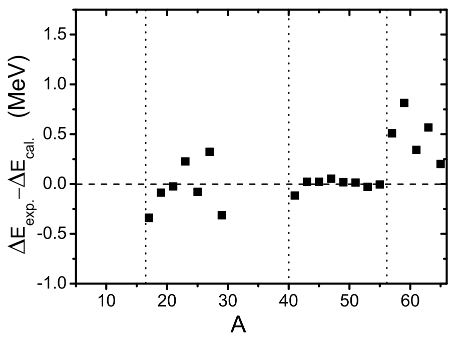

Fig. 4. Same as Fig. 2, but with the modified SIII force.

The differences between calculated CDE using this force and the corresponding experimental data can be significantly reduced, which could be seen in Fig. 4.

4 Conclusion

The mass differences of mirror nuclei have been calculated using the Skyrme-Hartree-Fock model. In the Skyrme force the NS Anomaly also exists. The discrepancies are remarkably large just below the shell closure. We studies this anomaly systematically, and find that it has strong shell closure dependence. When the Skyrme force parameters are modified to reflect this shell effect, this anomaly is significantly reduced.

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库仑置换能的壳结构效应*

朱剑钰[1] 许甫荣[1,2,3;1)]

1 (北京大学技术物理系 北京 100871)

2 (中国科学院理论物理研究所 北京 100080)

3 (兰州重离子加速器国家实验室原子核理论中心 兰州 730000)

摘要 应用Skyrme-Hartree-Fock (SHF) 模型计算了镜像核的结合能差. 计算结果和相应试验值存在差值, 这个问题一直被广泛的研究. 系统地研究了这一异常现象, 计算表明这一异常在幻数附近的原子核当中尤其显著. 在Skyrme力中加入一个和壳效应相关的修正, 可以明显地减小这种异常. 这表明了这一异常具有明显的壳效应.

关键词 镜像核 Skyrme-Hartree-Fock模型 Nolen-Schiffer Anomaly

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1) E-mail: frxu@pku.edu.cn