Search for Critical Behaviors by RHIC-PHENIX

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Abstract Integrated two particle correlation functions have been extracted from charge particle multiplicity density fluctuations in pseudorapidity space by analyzing Au+Au collision events at $\sqrt{s_{\rm NN}}=200$GeV taken by RHIC-PHENIX. The correlation lengths as a function of the number of participants $N_{\rm p}$ indicate a non monotonic increase at around $N_{\rm p}=100$ and the corresponding energy density based on the Bjorken picture is $\epsilon_{\rm Bj}\tau\sim 2.5{\rm GeV\cdot fm^{-2}}$. This could be a symptom of a critical behavior.

Key words critical temperature, correlation length, QCD

1 Introduction

For the understanding of the QCD phase diagram, it is important to establish tools to determine critical systems in general. As one of such observables, increases of spatial correlation lengths as a function of system energy density can be a robust signature to determine critical systems whatever the transition order is. Based on the following advocated observables, Au+Au collision events taken by the PHENIX detector[1] at $\sqrt{s_{\rm NN}}=200$GeV have been analyzed and the present results are summarized by focusing on whether critical behaviors of the phase transition exist or not as a function of the number of participants $N_{\rm p}$ which reflects the system energy density[2].

2 Density fluctuation and phase transition

In order to relate the density fluctuations with the phase transition in the simplest form, Ginzburg-Landau(GL)[3] theory with the Ornstein-Zernike picture[4] for a scalar order parameter is briefly reviewed. The first attempt to apply the free energy discussion to nucleus-nucleus collisions can be found in Ref. [5]. GL describes the relation between a free energy density f and an order parameter ϕ as a function of system temperature T. By adding a spatially inhomogeneous term $(\nabla\phi)^2$ and an external field h, the general form is described as follows;

$$f(T,\phi,h) = f_0(T)+\frac{1}{2}A(T)(\nabla\phi)^2+ \frac{1}{2}a(T)\phi^2+\frac{1}{4}b\phi^4+\cdots-h\phi\,, \qquad (1)$$

where terms with odd powers are neglected due to the symmetry of the order parameter and the sign of b is used to classify the transition orders; $b<0$ for first order, $b>0$ for second order and $b=0$ for the critical point. Since the order parameter should vanish above critical temperature $T_{\rm c}$, it is natural for the coefficient $a(T)$ to be expressed as $a(T)=a_0(T-T_{\rm c})$, while b is usually assumed to be constant in the vicinity of $T_{\rm c}$.

In the following analysis, the order parameter corresponds to the multiplicity density fluctuation from the mean density as a function of one dimensional rapidity point y, which is defined as

$$\phi(y)=\rho(y)-\langle\rho\rangle\,, \qquad (2)$$

where a pair of brackets is an operator to take the average. With the Fourier expansion of the density fluctuation $\phi(y)=\sum_k\phi_k{\rm e}^{{\rm i}ky}$ where k is wave number, one can express the deviation of the free energy

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1145 — 1148

density $\Delta F/Y$ due to spatial fluctuations from the equilibrium value as

$$\Delta F/Y = \frac{1}{Y}\int (f-f_0)\mathrm{d}y = \frac{1}{2}\sum_k |\phi_k|^2 (a(T)+A(T)k^2), \tag{3}$$

where Y is the total rapidity range corresponding to one dimensional volume and up to the second order terms are taken into account as an approximation in the vicinity of the critical point in Eq. (1). Given the free energy deviation, one can obtain the statistical weight w for fluctuation $\phi(y)$ to occur in a given temperature T

$$w(\phi(y)) = N\mathrm{e}^{-\Delta F/T}\,. \tag{4}$$

Therefore the statistical average of the square of the density fluctuation with the wave number k is described as

$$\langle|\phi_k|^2\rangle = \int_{-\infty}^{+\infty} |\phi_k|^2 w\left(\sum_k \phi_k \mathrm{e}^{\mathrm{i}ky}\right)\mathrm{d}\phi_k = \frac{NT}{Y}\frac{1}{a(T)+A(T)k^2}\,. \tag{5}$$

Experimentally observable two point density correlation function can be related to the statistical average of the square of the density fluctuation. With a density $\rho(y_i)$ for a given sub volume $\mathrm{d}y_i$, the general two point density correlation G_2 is expressed as

$$G_2(y_1,y_2) = \langle(\rho(y_1)-\langle\rho\rangle)(\rho(y_2)-\langle\rho\rangle)\rangle, \tag{6}$$

where the case that 1 coincides with 2 is excluded to simplify the following discussion. Multiplying $\mathrm{e}^{-\mathrm{i}ky} \equiv \mathrm{e}^{-\mathrm{i}k(y_2-y_1)}$ to the both sides of Eq. (6) and integrating over sub volume $\mathrm{d}y_1$ and $\mathrm{d}y_2$ gives

$$Y\int G_2(y)\mathrm{e}^{-\mathrm{i}ky}\mathrm{d}y = \langle|\int(\rho(y)-\langle\rho\rangle)\mathrm{e}^{-\mathrm{i}ky}\mathrm{d}y|^2\rangle = \langle|\phi_k|^2\rangle. \tag{7}$$

From Eq. (5) and (7), G_2 can be obtained by the inverse Fourier transformation of $\langle|\phi_k|^2\rangle$. Therefore in the one dimensional case G_2 is described as

$$G_2(y) = \frac{NT}{2Y^2A(T)}\xi(T)\mathrm{e}^{-|y|/\xi(T)}\,, \tag{8}$$

where a correlation length $\xi(T)$ is introduced, which is defined as

$$\xi(T)^2 = \frac{A(T)}{a_0(T-T_\mathrm{c})}\,. \tag{9}$$

In general, a singular behavior of $\xi(T)$ as a function of T indicates the critical point of the phase transition.

3 Experimental observables

In the following analysis the density fluctuation is discussed via the charged particle multiplicity distributions as a function of the pseudo-rapidity interval of $\delta\eta$ for each collision centrality. Let us introduce one and two particle inclusive multiplicity densities ρ_1 and ρ_2 based on the inclusive differential cross section with respect to the total inelastic cross section σ_{inel} as follows[6]

$$\frac{1}{\sigma_{\mathrm{inel}}}\mathrm{d}\sigma = \rho_1(\eta)\mathrm{d}\eta\,,$$
$$\frac{1}{\sigma_{\mathrm{inel}}}\mathrm{d}^2\sigma = \rho_2(\eta_1,\eta_2)\mathrm{d}\eta_1\mathrm{d}\eta_2\,. \tag{10}$$

With these densities, a two particle density correlation function is defined as

$$C_2(\eta_1,\eta_2) = \rho_2(\eta_1,\eta_2) - \rho_1(\eta_1)\rho_1(\eta_2)\,. \tag{11}$$

The mathematical connection between second order normalized factorial moment F_2 and the two particle correlation function is expressed as[7]

$$F_2(\delta\eta) = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2} = \frac{\iint^{\delta\eta}\rho_2(\eta_1,\eta_2)\mathrm{d}\eta_1\mathrm{d}\eta_2}{\left\{\int^{\delta\eta}\rho_1(\eta)\mathrm{d}\eta\right\}^2} = \frac{1}{(\delta\eta)^2}\iint^{\delta\eta}\frac{C_2(\eta_1,\eta_2)}{\bar{\rho}_1^2}\mathrm{d}\eta_1\mathrm{d}\eta_2 + 1\,, \tag{12}$$

where n is the number of produced particles, $\delta\eta$ is the rapidity interval which defines the measurable range of $|\eta_1-\eta_2|$, $\bar{\rho_1}$ is the average number density per unit length within $\delta\eta$ which is defined as

$$\bar{\rho_1} = \frac{1}{\delta\eta}\int^{\delta\eta}\rho_1(\eta)\mathrm{d}\eta\,. \tag{13}$$

The two particle correlation function C_2 can be parametrized based on the one dimensional function form obtained in Eq. (8). However, one has to bear in mind that the damping behavior in Eq. (8) is originated only from the spatial inhomogeneity of the system in a fixed temperature. In many experimental conditions, the initial system temperature can not be specified as a point. For instance, corresponding temperature is indirectly discussed by relating it with the collision centrality. The centrality bin has a finite size and it causes fluctuations originating from the finite temperature bin size. In principle this kind of fluctuations must be independent of the spatial fluctuations. Therefore the general function form for the normalized two particle correlation function in the one dimensional analysis can be parameterized as follows which explicitly contains a constant term β;

$$\frac{C_2(\eta_1,\eta_2)}{\bar{\rho_1}^2} = \alpha e^{-\delta\eta/\xi} + \beta\,, \tag{14}$$

where $\bar{\rho_1}$ is proportional to the mean multiplicity in each centrality.

Instead of F_2 itself, we will use an indirect parameter k of the Negative Binomial Distribution (NBD) in the following analysis which is defined as

$$P_{k,\mu}(n) = \frac{\Gamma(n+k)}{\Gamma(n-1)\Gamma(k)}\left(\frac{\mu/k}{1+\mu/k}\right)\frac{1}{1+\mu/k}\,, \tag{15}$$

where μ corresponds to the mean value and k^{-1} reflects deviations from the completely random case i.e. the Poisson distribution which corresponds to $k=\infty$. Intuitively k^{-1} is a measure how strongly particles are correlated. The mathematical relation between k and F_2 is expressed as[8]

$$k^{-1} = F_2 - 1\,. \tag{16}$$

The reason why we adopt NBD rather than F_2 is that NBD can provide an approximate probability distribution which enables us to estimate how inefficient or dead areas of the detector system bias the k parameter and to obtain the true value of k based on the estimation, while factorial moment itself does not provide any specific models on the distribution function which resulted the observed factorial moment.

As the result, the relation between the NBD k parameter and the pseudo-rapidity interval $\delta\eta$ for the parametrization given in Eq. (14) is expressed as

$$k^{-1}(\delta\eta) = F_2 - 1 = \frac{2\alpha\xi^2(\delta\eta/\xi - 1 + e^{-\delta\eta/\xi})}{\delta\eta^2} + \beta\,. \tag{17}$$

4　Analysis result

Figure 1 shows corrected NBD k parameters as a function of pseudo-rapidity interval sizes for centrality classes indicated inside the figure. The upper and lower two panels correspond to 10% and 5% centrality bin width cases, respectively. The vertical error bars show the statistical errors and boxes show the systematic errors which come from correction factors on k due to the possible variation of dead or inefficient areas in the tracking detector. The solid line indicates the fit result by using Eq. (17) with errors of quadratic sum of the statistical and systematic errors. The fit was performed from 0.02 to 0.7 in pseudo-rapidity. The lowest centrality bin was determined as 55%—65%. The fits are remarkably well resulting reduced χ^2 of 0.44 at the worst which corresponds to above 99% confidence level. This guarantees that the parametrization is actually reasonable.

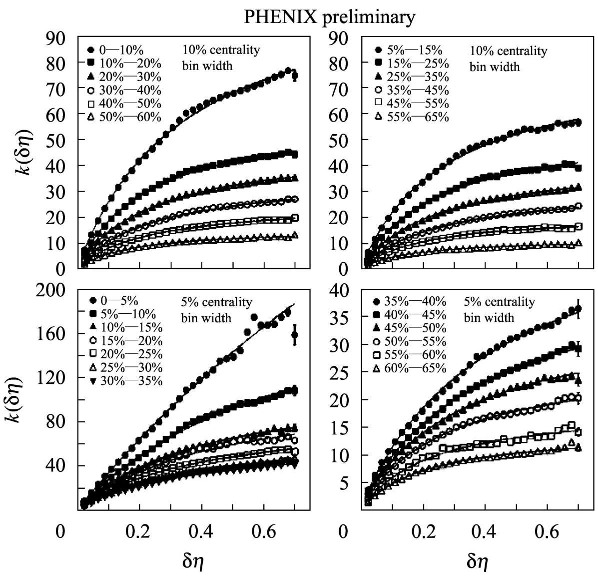

Fig. 1.　k vs $\delta\eta$ in each centrality class.

Figure 2(a), (b) and (c) show obtained fit parameters α, β and ξ as a function of the number of

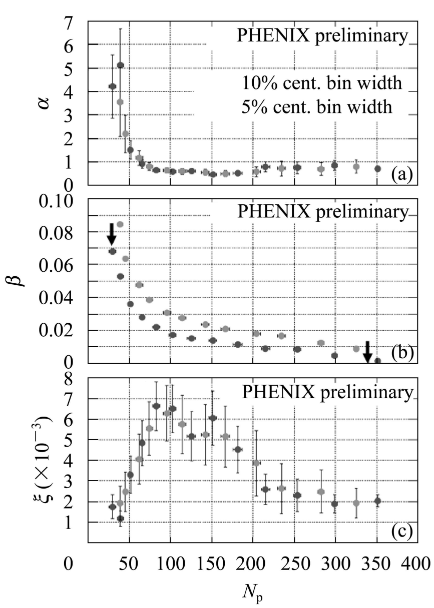

Fig. 2. α, β and ξ as a function of N_p.

participants $N_{\rm p}$ where results for both 10% and 5% centrality bin width cases are plotted as red and blue circles respectively. $N_{\rm p}$ was obtained from the centrality classes based on the Glauber model which is explained in Ref. [9] in detail. The horizontal errors correspond to ambiguities on the mean values of $N_{\rm p}$ when the centralities are mapped upon $N_{\rm p}$. The vertical error bars are obtained from errors on the fitting parameter by the Minuit program.

5 Summary

The multiplicity distributions measured in Au+Au collisions at $\sqrt{s_{\rm NN}}$=200GeV are found to be well described by the negative binomial distribution. The two point correlation lengths have been extracted based on the function form by relating pseudo-rapidity density fluctuations and the Ginzburg-Landau theory up to the second order term in the free energy with the scalar order parameter. The function form can fit k vs. $\delta\eta$ in all centralities remarkably well. The correlation lengths as a function of the number of participants $N_{\rm p}$ indicate a non monotonic increase at around $N_{\rm p}=100$ and the corresponding energy density based on the Bjorken picture is $\epsilon_{\rm Bj}\tau \sim 2.5{\rm GeV{\cdot}fm^{-2}}$ which has been measured by PHENIX[2]. It is interesting to note that the energy density coincides with the one where the first drop of J/ψ suppression from the normal nuclear absorption was observed at SPS[10].

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