Re-fit of τ Mass Based on Improved Theoretical Calculations^{*}

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Abstract Recently many theoretical calculations have been made for the cross section of $e^+e^- \rightarrow \tau^+\tau^-$ near the threshold, the accuracy of which is up to 10^{-4} . Based on one of the calculations, the e µ-contained final state data are refitted by utilizing the technique once adopted by BES collaboration for τ mass measurement. The systematic uncertainties are analyzed again and the final result of τ mass is $1776.98^{+0.44+0.12}_{-0.51-0.13}$ MeV.

Key words τ mass, cross section, likelihood maximization

1 Introduction

The mass of τ lepton is a fundamental parameter in the Standard Model. The high accurate knowledge of it, for example, can be used in the test of the lepton universality and the verification of the experimental values of the leptonic decay branching ratio $Br(\tau^- \to e^- \nu_{\tau} \bar{\nu}_e)$ and the τ lifetime^[1].

The dramatic improvement in the accuracy of the measured mass of the τ lepton, achieved in the BES experiment^[2-4], has demonstrated one of the advantages of studying the production of the $\tau^+\tau^-$ pair in e⁺e⁻ annihilation in the immediate vicinity of the threshold. In view of the special kinematic and background advantages of the threshold region^[5] one may expect that the τ mass measurement at the threshold could be continued for high luminosity experiment at tau-charm region, such as CLEO-c^[6] at present and BESIII^[7] in the near future.

Some theoretical calculations have been made recently and claimed the achievement of the accuracy at the level of 10^{-4} for the production cross section $\sigma(e^+e^- \rightarrow \tau^+\tau^-)$ near the threshold^[8-10]. The motivation of the present paper is to re-fit the experiment data using the high accurate theoretical formula provided by Ref. [8], hoping to obtain a τ mass value with better accuracy.

In the following parts, we begin with compiling the formulas needed for evaluating the production and the observed cross sections, then compare the present and the previous theoretical results. By virtue of PDG04 values^[11], we scaled again the fore experiment data^[2], and fit the rescaled data following the maximization scheme adopted by BES collaboration^[3, 4]. At last, we analyze all possible uncertainties based on our new fitting and information contained in pertinent references.

2 Cross section formula

One high accurate calculation about τ cross section is provided by P. Ruiz-Femenía and A. Pich, who analyzed the threshold behavior of the cross section $\sigma(e^+e^- \rightarrow \tau^+\tau^-)$ based on an adaptation of the methods of non-relativistic effective field theories of QCD^[12]. They claimed to take into account the known higher-order corrections and determined the production cross section to next-to-next-to-leading order (NNLO) in a combined expansion in powers of α_s and fermion velocities. The dominant NNLO

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corrections can be incorporated to the numerical predictions, which could provide a theoretical precision better than 0.1%.

However, in one of M. B. Voloshin papers^[8], he pointed out the complicatedness and plausibility in Ref. [12], and at the same time adopted certain "on shell" renormalization scheme to treat the radiative and the vacuum polarization corrections to Coulomb potential. In his paper^[8], Voloshin presented an evaluation of various corrections which include (a) radiation from the initial electron and positron; (b) vacuum polarization in the time-like photon; (c) corrections to the special density of the electromagnetic current of the tau leptons; and (d) the interference between the effects (a)—(c) which starts from the relative order α^2 . He argued that the known $\mathcal{O}(\alpha)$ corrections provide the accuracy of the description of the production cross section close to 10^{-4} , which is sufficient for a measurement of the τ mass down to at least \mathcal{O} (1keV).

Since Voloshin's formula was once adopted by BES collaboration for τ mass measurement^[2-4], we now adopt improved theoretical calculation by Voloshin to treat the same data from BES, so that it is also easy to see the effect due to the accuracy of cross section calculation. In this section, we first compile the formula presented in Ref. [8] for production cross section calculation, then for comparison, we also listed the formula once used by BES. In addition, the initial state radiation, vacuum polarization and energy spread are also taken into consideration in order to acquire the experiment cross section.

2.1 Production cross section $\bar{\sigma}$

According to Refs. [8] and [9], the production cross section $\bar{\sigma}(e^+e^- \rightarrow \tau^+\tau^-)$ (shortened as $\bar{\sigma}$ hereafter) has the form

$$\bar{\sigma}(v) = \frac{2\pi\alpha^2}{3s}v(3-v^2)F_{\rm c}(v)\left(1+\frac{\alpha}{\pi}S(v)-\frac{\pi\alpha}{2v}+h(v)\right),$$
(1)

where $v = \sqrt{1 - 4m_{\tau}^2/s}$ is the velocity of each of the τ leptons in center-of-mass (C.M.) frames, which is subject to aforementioned corrections (a)—(d); \sqrt{s} is the C.M. energy; $F_c(v)$ is the so-called Coulomb

factor, which is defined as

$$F_{\rm c}(v) = \frac{\pi \alpha/v}{1 - \exp(-\pi \alpha/v)} \ . \tag{2}$$

The description of correction function S(v) can be found in Schwinger's textbook^[13], which reads

$$S(v) = \frac{1}{v} \left\{ (1+v^2) \left[\frac{\pi^2}{6} + \ln\left(\frac{1+v}{2}\right) \ln\left(\frac{1+v}{1-v}\right) + 2Li_2\left(\frac{1-v}{1+v}\right) + 2Li_2\left(\frac{1+v}{2}\right) - \left(\frac{1-v}{2}\right) + 2Li_2\left(\frac{1-v}{2}\right) - 4Li_2(v) + Li_2(v^2) \right] + \left[\frac{11}{8}(1+v^2) - 3v + \frac{1}{2}\frac{v^4}{(3-v^2)} \right] \ln\left(\frac{1+v}{1-v}\right) + 6v \ln\left(\frac{1+v}{2}\right) - 4v \ln v + \frac{3}{4}v\frac{(5-3v^2)}{(3-v^2)} \right\}, \quad (3)$$

with

$$Li_{2}(x) = -\int_{0}^{x} \ln(1-t) dt/t = \sum_{n=1}^{\infty} x^{n}/n^{2}$$

The correction function h(v) is expressed in terms of a double integral^[8, 9]:

$$h(v) = \frac{2\alpha}{3\pi} \left[-2\lambda \mathrm{Im} \int_0^\infty \mathrm{d}t \int_1^\infty \mathrm{d}x \left(\frac{1+t}{t}\right)^{\mathrm{i}\lambda} \times \frac{(t+\mathrm{i}zxv^{-1})^{\mathrm{i}\lambda-1}}{(t+1+\mathrm{i}zxv^{-1})^{\mathrm{i}\lambda+1}} \left(1+\frac{1}{2x^2}\right) \frac{\sqrt{x^2-1}}{x^2} \right] ,$$

$$(4)$$

with

$$z = m_{\rm e}/m_{ au}, \quad \lambda = \frac{lpha}{2v}$$

Function h(v) contains the corrections of two sources: from the so-called hard correction due to a finite radiative effect in the τ electromagnetic vertex at the threshold, and from the modification of the Coulomb interaction due to running of the coupling α , which is described by the Uehling-Serber radiative correction to the potential^[14]. The derivation of h(v) can be found in Refs. [8] and [9], also according to the suggestion of which, *Mathematica* is used to carry out the numerical calculation. The plot of h(v) as a function of velocity v is shown in Fig. 1, along with the functions of $F_c(v)$ in Eq. (2) and $\frac{\alpha}{\pi}S(v)$ in Eqs. (1) and (3), whose values are about 10³ times larger than those of h(v). Their values are denoted by the scale in the right side of the figure.



Fig. 1. The variation of the correction function h(v) (solid line) against the velocity of the τ leptons in the C.M. system. For comparison, also drawn are the Coulomb factor $F_c(v)$ (dashed line) and the function $\frac{\alpha}{\pi}S(v)$ (dotted line). Their values have different scales which are drawn in the right side of the figure.

In the previous τ mass experiment preformed at BES, the production cross section of $\tau^+\tau^-$ near the threshold is divided into three parts^[3], viz.

$$\bar{\sigma}^*(v) = \sigma_0(v) F_{\rm r}(v) F_{\rm c}(v) , \qquad (5)$$

where $\sigma_0(v)$ is the bare cross section for producing a fermion pair in e⁺e⁻ annihilation process without any correction,

$$\sigma_0(v) = \frac{2\pi\alpha^2}{3s}v(3-v^2)$$

 $F_{\rm c}(v)$ is the Coulomb interaction factor, which is given in Eq. (2); and $F_{\rm r}(v)$ is the final state radiative correction and spin correction factor and can be expressed as,

$$F_{\rm r}(v) = 1 + \frac{\alpha}{\pi} S(v) - \frac{\alpha \pi}{2v} - \frac{\alpha \pi v}{2}$$

where S(v) is defined in Eq. (3).

The difference between $\bar{\sigma}$ and $\bar{\sigma}^*$ can be easily figured out from Eqs. (1) and (5), that is

$$\Delta \sigma \equiv \bar{\sigma}(v) - \bar{\sigma}^*(v) = \sigma_0(v) F_{\rm c}(v) \left(h(v) + \frac{\alpha \pi v}{2} \right).$$
(6)

Since v approaches zero near the threshold, the relative order of the correction is dominated by the term h(v), whose value is at the level of 10^{-3} . Nevertheless, with the increase of C.M. energy, v becomes larger, the correction $\Delta \sigma$ will be dominated by the term $\alpha \pi v/2$.

2.2 Experiment cross section σ_{exp}

The experimentally measured cross section has the form $^{[15]}$

$$\sigma_{\exp}(s, m_{\tau}, \Delta) = \int_{0}^{\infty} d\sqrt{s'} G(\sqrt{s'}, \sqrt{s}) \int_{0}^{1 - \frac{4m_{\tau}^{2}}{s'}} dx F \times (x, s') \frac{\bar{\sigma}(s'(1-x), m_{\tau})}{|1 - \Pi(s'(1-x))|^{2}}, \quad (7)$$

where F(x,s) is the initial state radiation factor^[15], Π is the vacuum polarization factor^[12,16,17], and $G(\sqrt{s'},\sqrt{s})$, which is usually treated as a Gaussian distribution^[18], depicts the energy spread of the e⁺e⁻ collider. The production cross section $\bar{\sigma}(\bar{\sigma}^*)$ can be expressed by Eq. (1) (Eq. (5)). Notice v is the function of s and m_{τ} , we explicitly indicate the dependence of the cross section on s and m_{τ} in Eq. (7).

Figure 2 shows the cross sections considering different kinds of corrections.



Fig. 2. The $\tau^+\tau^-$ cross sections near the threshold as function of C.M. energy $W = \sqrt{s}$. The dotted curve indicates the production cross section $\bar{\sigma}$ [Eq. (1)]. The solid curve indicates the final experiment cross section $\sigma_{\rm exp}$ [Eq. (7)]. The dashed curve indicates the cross section without correction of vacuum polarization and energy spread. The inset shows the variation of different cross sections near the threshold. In calculation, $m_{\tau} = 1.77699 \text{GeV}^{[11]}$ while $\Delta = 0.16203 \times 10^{-3} \times W^2/4 + 0.89638 \times 10^{-3} \text{GeV}$, the experienced function obtained by fitting the data given in Table 1.

By virtue of $\bar{\sigma}$ and $\bar{\sigma}^*$ defined in Eq. (1) and Eq. (5) together with Eq. (7), we obtain experiment cross section σ_{\exp} and σ_{\exp}^* respectively. The comparison of these two kinds of cross sections is shown in Fig. 3, where the solid and dashed lines correspond to the difference of the experiment and production cross sections respectively.



Fig. 3. The difference of cross sections calculated with different accuracy. The solid and dashed lines correspond to the difference of the experiment and production cross sections respectively. In (a) the difference $\Delta \sigma = \sigma - \sigma^*$ is given while in (b) the relative difference $\Delta \sigma / \sigma$ is given. Here σ and σ^* can be σ_{exp} (denoted by the solid line) or $\bar{\sigma}$ (denoted by the dashed line). In calculation, the values of m_{τ} and Δ are the same as in Fig. 2.

Notice the behavior of h(v) and Eq. (6), the difference at the threshold is mainly due to the correction function h(v). It is noticeable that in the vicinity of the threshold the difference between two kinds of cross sections is less than 2pb although the relative difference is larger; on the contrary, when away from the threshold the relative difference is less than 0.3% although the difference $\Delta\sigma$ enlarges with the increase of energy. Under such condition, we expect the effect involving the accuracy of theoretical calculation may be small. The following study indeed conforms with our expectation.

3 Maximization

3.1 Scheme

The mass of the τ lepton is obtained from the $\tau^+\tau^-$ candidate sample by means of a maximum likelihood fit to the C.M. energy dependence of the observed τ pair cross section. The likelihood function is a product of a serial Poisson distributions^[4], with the expected number of events given by

$$\mu_i(m_{\tau}, \epsilon) = [\epsilon B \sigma_{\exp}(m_{\tau}, W_i, \Delta_i) + \sigma_{\mathrm{BG}}] \cdot \mathscr{L}_i , \qquad (8)$$

where $W_i = \sqrt{s_i}$ is the C.M. energy at *i*th scan point; ϵ is the overall efficiency for identifying $\tau^+\tau^$ events through e μ -contained final state, which includes trigger efficiency and event selection efficiency; $B=0.06194^{[11]}$ is the combined branching ratio for decays $\tau^+ \to e^+ \bar{\nu}_{\tau} \nu_e$ and $\tau^- \to \mu^- \bar{\nu}_{\mu} \nu_{\tau}$, or the corresponding charge conjugate mode; $\sigma_{\rm BG}=0.024 {\rm pb}^{[2]}$, which is the effective background cross section estimated from the J/ ψ data sample; Δ_i is the energy spread at scan point i; \mathscr{L}_i is the integrated luminosity at the corresponding point, and $\sigma_{\rm exp}(m_{\tau}, W_i, \Delta_i)$ is the experiment cross section as given in Eq. (7). The product of ϵ , B and $\sigma_{\rm exp}$ is the actual observed cross section.

Table 1. A summary of the $\tau^+\tau^-$ threshold scan data; W denotes the corrected C.M. energy, W^0 denotes the fore-scaled C.M. energy, and Δ the energy spread of C.M. energy.

Scan	W/2	$W^{0}/2$	$(W - W^0)/2$	$(W-W^0)/W$	Δ
point	$/{\rm MeV}$	$/{\rm MeV}$	$/{\rm MeV}$	$(\times 10^{-5})$	$/{\rm MeV}$
1	1784.23	1784.19	0.04	2.2	1.34
2	1781.02	1780.99	0.03	1.7	1.33
3	1772.12	1772.09	0.03	1.7	1.36
4	1776.60	1776.57	0.03	1.7	1.37
5	1778.52	1778.49	0.03	1.7	1.44
6	1775.98	1775.95	0.03	1.7	1.43
7	1776.78	1776.75	0.03	1.7	1.47
8	1777.01	1776.98	0.03	1.7	1.47
9	1776.48	1776.45	0.03	1.7	1.44
10	1776.65	1776.62	0.03	1.7	1.40
11	1799.55	1799.51	0.04	2.2	1.44
12	1789.59	1789.55	0.04	2.2	1.43

With experimental inputs W_i , \mathscr{L}_i , Δ_i $(i = 1, 2, \dots, n)$, as listed in Table 1 and given in Ref. [2], maximizing the quantity of likelihood function, we can obtain the needed parameters.

3.2 Data rescale

Before fitting, some words about experiment data are needed. The experimentally measured energy usually need to be scaled by a well known nominal energy. It is fortunate that the experiment points of τ mass measurement just fall in the interval of two resonance peaks of J/ ψ and ψ' . The determination of resonance masses can be realized by several scans of J/ ψ and ψ' resonances performed during τ mass measurement^[4]. Then assuming a linear relation between measured energy $W_{\rm M}$ and the corrected value W, it is readily to have

$$W^{0} = T_{\psi}^{0} + (W_{\rm M} - M_{\psi}) \frac{T_{\psi'}^{0} - T_{\psi}^{0}}{M_{\psi'} - M_{\psi}} . \qquad (9)$$

In Eq. (9) the meaning of symbols is expounded in Table 2, and the superscript 0 of the symbol indicates the value used in previous energy scale^[2]. Since PDG04^[11] presents the more accurate world average values for resonances of J/ψ and ψ' , we can write out an expression similar to Eq. (9), and the ratio of them yields

$$W = T_{\psi} + \frac{T_{\psi'} - T_{\psi}}{T_{\psi'}^0 - T_{\psi}^0} \cdot (W^0 - T_{\psi}^0) . \qquad (10)$$

So with the values of previous scaled energy and those of PDG04, we can acquire the rescaled energy values, which are also listed in Table 1.

Table 2. Information relevant to energy scale.

Symbol	Meaning	Value	Error
Symbol	Wieannig	$/{\rm MeV}$	$/\mathrm{MeV}$
$W_{\rm M}{}^{[2, 4]}$: BEPC measured C.M. energy		0.10
W^0	: fore-scaled energy values		0.22
W	: rescaled energy values		0.24
M_{ψ}	: BES value for J/ψ mass	3097.20	0.18
$M_{\psi'}$: BES value for ψ' mass	3686.88	0.15
$T_{\psi}^{0}^{[19]}$: fore nominal value for J/ψ mass	3096.93	0.09
$T^{0}_{\psi'}{}^{[19]}$: fore nominal value for ψ' mass	3686.00	0.10
$T_{\psi}^{[11]}$: new nominal value for ${\rm J}/\psi$ mass	3096.916	0.011
$T_{\psi'}^{[11]}$: new nominal value for ψ' mass	3686.093	0.034

3.3 Fit

In the actual maximizing likelihood fit, m_{τ} and ϵ are allowed to vary. The fit to the data with rescaled energy W is performed by using the program package MINUIT^[20]. The best fit curve together with measured points are shown in Fig. 4 and the maximized solution corresponds to the parameters

$$m_{\tau} = 1776.98^{+0.44}_{-0.51} \text{MeV} ,$$

$$\epsilon = 14.2^{+4.7}_{-3.9} \% .$$
(11)

If fixing the efficiency to be the fitted value $\epsilon = 14.2\%$ and fit m_{τ} only, the result is $m_{\tau} = 1776.97^{+0.40}_{-0.43}$ MeV, which consists with that from twoparameter-fitting. Here the smaller error is due to the more input information¹⁾.



Fig. 4. The measured points and the best fitted cross section curve. The error bars are only statistic uncertainty calculated according to the one standard deviation of Poisson distribution.

To see the accuracy effect, the $\bar{\sigma}^*$ in Eq. (5) is adopted in the fit, which leads to the results:

$$m_{\tau} = 1776.97^{+0.43}_{-0.51} \text{MeV} ,$$

$$\epsilon = 14.3^{+4.7}_{-3.9} \%.$$
(12)

The difference for m_{τ} between Eq. (11) and Eq. (12) is actually at the level of 0.001MeV, smaller enough to be neglected²⁾.

As a fit check, utilizing the formula in Eq. (5) and the data with fore-scaled energy W^0 , we repeat the fitting procedure and obtain the fitted m_{τ} to be $1776.9^{+0.4}_{-0.5}$ MeV, which is the same as the value acquired in Ref. [2] up to the significant digits shown here.

4 Systematic uncertainty

In this section, we analyze all possible uncertainties based on our study and information contained in the related literature.

The systematic errors due to uncertainties in the C.M. energy scale have been studied in detail in Ref. [4], some relevant data are contained in Table 2. In Eq. (9), the error of W^0 is obtained from the errors of T_{ψ}^0 and $W_{\rm M}$ with the value $\delta W_{\rm M} = 0.10 \text{MeV}$. Then this error is put into Eq. (10). Together with the errors of T_{ψ}^0 and T_{ψ}^0 , the systematic uncertainty

¹⁾ The fix of efficiency indicates that more data analysis has been performed in order to determine ϵ .

²⁾ According to the output of the fitting program, the read out central values of m_{τ} in Eq. (11) and Eq. (12) are 1776.975 and 1776.974 MeV respectively.

of rescaled C.M. energy W is determined as $\delta W = 0.24$ MeV. In addition, our fitting study indicates that the uncertainty of energy scale does linearly and completely transfer to the final results of τ mass fitting. Furthermore, the accuracy of energy scale is almost fully determined by the accelerator running state. In another word, the uncertainty due to energy scale is a detector-independent factor, which could constrain the precision improvement for τ mass measurement in future high accuracy experiment. Whatever, in this analysis, $\Delta m_{\tau} = \delta W/2 = 0.12$ MeV is used as the final scaling uncertainty.

Fits to the two resonances are used not only to provide the scaled energy, but also to measure the beam energy spread and its variation with C.M. energy and beam current^[2, 4]. The resulting uncertainty in C.M. energy spread is estimated to be ± 0.08 MeV. By varying the energy spread parameter over this range and repeating the likelihood fits, the corresponding uncertainties for the τ mass value are found to be $\Delta m_{\pi} = {}^{+0.02}_{-0.03}$ MeV.

Then we consider the error from $\sigma_{\rm BG}$. The null and doubled background fittings are preformed to find the uncertainty of $\sigma_{\rm BG}$ and this yields an uncertainty $\Delta m_{\tau} = {}^{+0.01}_{-0.02} \text{MeV}.$

At last, we consider the errors due to quantities ϵ , B and \mathscr{L} . Since they appear in the expression of μ_i in the form of product, ϵ , B and \mathscr{L} are strongly correlated in the fitting. Moreover, because ϵ is a free parameter in the actual fitting, the fluctuation of B and \mathscr{L} will transform to ϵ , so the error of ϵ can

also contain the uncertainty of B and \mathscr{L} . For the second term of the production $\sigma_{BG}\mathscr{L}$, here the error due to uncertainty of \mathscr{L} should be considered. Its fluctuation is $3\%^{[21]}$. However, similar to the product of $\epsilon B\mathscr{L}$, the uncertainty of σ_{BG} and \mathscr{L} is correlated and the impact of 3% fluctuation due to \mathscr{L} is negligable comparing to the variation of σ_{BG} .

In short, three sources of systematic uncertainties are considered: the C.M. energy scale, the energy spread, and the effective background cross section $\sigma_{\rm BG}$. Assuming the independence between them, added the errors from all the sources in quadrature, we get a total systematic uncertainty for τ mass: $\Delta m_{\tau} = ^{+0.12}_{-0.13} \text{MeV}.$

5 Summary

Based on the high accurate theoretical calculation, the result

$$m_{\tau} = 1776.98^{+0.44+0.12}_{-0.51-0.13} \text{MeV}$$

is obtained by using the maximum likelihood fit to the data once used by BES collaboration. In our analysis, the energy of the old data is rescaled by virtue of the values from PDG04 and the systematic uncertainties are studied as well. The present result is consistent with BES previous analysis in Ref. [2]. Moreover, we notice the effect of theoretical accuracy on the measured uncertainty is comparatively small. To improve the precision of measured τ mass value, more researches are needed¹⁾.

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¹⁾ Recently, based on sampling technique, a study involving high accurate measurement of τ mass has been made, the interesting results are summarized in Ref. [22].

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基于高精度理论截面计算的τ质量的再拟合^{*}

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摘要 对于e⁺e⁻ $\rightarrow \tau^+\tau^-$ 过程在阈值附近的截面,目前的理论计算精度已经达到10⁻⁴. 基于理论截面的高精度计算,利用极大似然函数方法,重新拟合由e μ 末态标记的 $\tau^+\tau^-$ 事例的扫描数据,得到 τ 质量的测量结果为1776.98^{+0.44+0.12}MeV,其中第一项是统计误差,由拟合程序给出;第二项是系统误差,主要包含能量刻度,能散及本底估计等三方面的不确定性.

关键词 て质量 截面 极大似然法

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