Effect of the Incident Energy on the Observables of Proton Elastic Scattering on Halo Nucleus^{*}

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Abstract The proton elastic scatterings on 14 Be, 16 O and 12 C at the intermediate incident energies of 200, 400 and 800MeV, are investigated within the relativistic impulse approximation (RIA). The effect of the incident energy on three observables, the differential cross section, the analyzing power and the spin rotation function, is discussed. It is shown that in the region of small scattering angle, the halo neutron effect on the observables exhibits an unchanged tendency with the variation of the incident energy of proton in the intermediate energy region.

Key words proton elastic scattering, relativistic impulse approximation (RIA), halo nuclei, differential cross section, analyzing power, spin rotation function

1 Introduction

For decades, the study of exotic nuclei that are near the drip lines on N-P plane has drawn great attention of nuclear physicists around the world^[1]. Halo nucleus is one kind of such nuclei, whose properties have been studied ever since Tanihata and his coworker's work in 1985^[2]. Near the neutron drip line, the low neutron binding energy makes the wave function of the peripheral neutrons extend far beyond the range of the strong force. This gives rise to the neutron halo phenomenon, which has already been observed in some light nuclei such as ¹¹Li, ¹⁴Be, ⁶He and ⁸He, etc.^[3-13]. Near the proton drip line, the proton-rich nuclei, such as ⁸B, ¹⁷F, ¹⁷Ne, ²³Al and ^{26–28}P, show some proton halo structures^[14–19]. Halo nuclei could be detected by a variety of experimental methods^[20, 21], such as the beta-decay measurement following in-beam polarization and the inves-

tigation of the momentum distributions of reaction products produced in bombarding a nuclear target with a halo nucleus^[9]. As is known to all, the elastic proton scattering on nuclei at intermediate energies could provide important information of the nucleonnucleon (NN) interaction, the radial structure of the target nucleus and reaction mechanisms $^{[10, 22, 23]}$. The differential cross section $d\sigma/d\Omega$, the analyzing power A_{y} and the spin rotation function Q are three important observables in describing proton elastic scattering on target. The relativistic impulse approximation $(RIA)^{[24-27]}$ is one of the useful schemes to describe proton elastic scattering off nuclei at intermediate incident energies between 100 and 1000MeV, microscopically. Murdock and Horowitz^[28] calculated the obsevables for proton elastic scattering off ¹²C, ¹⁶O, ⁴⁰Ca and ²⁰⁸Pb around 200MeV and reproduced the data remarkably well within the RIA formalism. Ray^[29] combined RIA with the distorted wave Born

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approximation (DWBA) and used the RIA-DWBA scheme to study proton elastic scattering on nuclei with non-zero-spin. Sakaguchi et al^[30] employed RIA to analyze the elastic scattering data of the p-⁵⁸Ni process at the incident energies of 192, 295 and 400MeV, respectively. Recently, Baldini-Neto et al^[31] extended the RIA scheme to describe proton elastic scattering off exotic nuclei such as the p-⁶He and p-⁸He processes. In the present paper, by using the RIA formalism, we study the elastic scatterings in the p-¹⁴Be, p-¹⁶O and p-¹²C processes at $E_{lab}=200, 400$ and 800MeV, respectively, and compare the resultant $d\sigma/d\Omega$, A_{y} and Q for these processes at each energy. We find that excessive neutrons in ¹⁴Be most probably affect $d\sigma/d\Omega$, A_y and Q at the small scattering angles, and the affected angular range is narrowed as the projectile energy increases. These phenomena may help us to understand some of the properties of the neutron-riched nuclei.

2 The theoretical formalism

In RIA, the NN scattering operator, denoted as $\Im(q, E; n)^{[32, 33]}$, is represented by a set of Lorentz invariants

$$\Im(q, E; n) = \sum_{\mathrm{L}} \Im^{\mathrm{L}}(q, E; n) \lambda_{(0)}^{\mathrm{L}} \cdot \lambda_{(n)}^{\mathrm{L}} , \qquad (1)$$

where $\lambda_{(i)}^{\text{L}}$ stands for the Dirac operator (or Dirac matrix^[28]) for the incident nucleon (i = 0) and struck nucleon (i = n with n being the n-th nucleon in the target nucleus), the dot product implies that all the Lorentz indices are contracted, L stands for the scalar (S), vector (V), pseudoscalar (PS), tensor (T), axial vector (A) fields, respectively. $\mathfrak{S}^{\text{L}}(q; E)$ is a complex function of momentum transfer q and laboratory energy E.

If applying this approximation to the proton elastic scattering off a spherical nucleus, the first order Dirac optical potential which deals with singlescattering approximation and spin-saturated nucleus, contains only the scalar and vector potentials, and can be written as

$$U_{\rm opt} = U^{\rm S} + \gamma^0 U^{\rm V} , \qquad (2)$$

$$U^{\rm L} = U^{\rm L}_{\rm D}(r; E) + U^{\rm L}_{\rm X}(r; E) , \qquad (3)$$

where γ^0 is the gamma matrix^[34],

$$U_{\rm D}^{\rm L}(r;E) = -\frac{4\pi i p}{M} \int d^3 x' \rho^{\rm L}(\boldsymbol{x}') t_{\rm D}^{\rm L}(|\boldsymbol{x}-\boldsymbol{x}'|;E) , \quad (4)$$
$$U_{\rm X}^{\rm L}(r;E) = -\frac{4\pi i p}{M} \int d^3 x' \rho^{\rm L}(\boldsymbol{x},\boldsymbol{x}') t_{\rm X}^{\rm L}(|\boldsymbol{x}-\boldsymbol{x}'|;E) \times j_0(p |\boldsymbol{x}-\boldsymbol{x}'|) , \quad (5)$$

are direct and exchange optical potentials, respectively, with j_0 being the zeroth order spherical Bessel function, and $t^{\rm L}(|\mathbf{x}|, E)$ functions having the forms as those in Refs. [27—29]. The densities in Eqs. (4) and (5) can be expressed as

$$\rho^{\mathrm{L}}(\boldsymbol{x}',\boldsymbol{x}) = \sum_{\kappa}^{\mathrm{occupied}} \bar{u}_{\kappa}(x')\lambda^{\mathrm{L}}u_{\kappa}(x) , \qquad (6)$$

$$\rho^{\rm L}(\boldsymbol{x}) \equiv \rho^{\rm L}(\boldsymbol{x}, \boldsymbol{x}) , \qquad (7)$$

where u_{κ} is the single-particle wave function, κ runs over all the occupied states of the target nucleus. The off-diagonal one-body density is approximated by the local-density^[35]

$$\rho^{\mathrm{L}}(\boldsymbol{x}',\boldsymbol{x}) \approx \rho^{\mathrm{L}}\left(\frac{1}{2}(\boldsymbol{x}'+\boldsymbol{x})\right)\left(\frac{3}{sk_{\mathrm{F}}}\right)j_{1}(sk_{\mathrm{F}}),\qquad(8)$$

where $s \equiv |\mathbf{x}' - \mathbf{x}|$ and $k_{\rm F}$ is related to the nuclear baryon density by $\rho_{\rm B}\left(\frac{1}{2}(\mathbf{x}' + \mathbf{x})\right) = \frac{2k_{\rm F}^3}{3\pi^2}$, j_1 denotes the first order spherical Bessel function.

For intermediate incident energies, it is necessary to correct the optical potentials with the Pauli blocking effect,

$$U^{\rm L}(r;E) \to \left[1 - a(E) \left(\frac{\rho_{\rm B}(r)}{\rho_0}\right)^{2/3}\right] U^{\rm L}(r;E), \quad (9)$$

where $\rho_{\rm B}(r)$ is the local baryon density of the target with $\rho_0=0.1934 {\rm fm}^{-3}$, and a(E) the "Pauli blocking factor"^[28]. All the above densities are calculated by using the relativistic mean field theory (RMF)^[36-38] and then are applied to calculate Dirac optical potentials with the equations discussed above.

The Dirac equation for the projectile is then written as

$$\left[-\mathrm{i}\boldsymbol{\alpha}\cdot\nabla + U^{\mathrm{V}}(r;E) + \beta(M+U^{\mathrm{S}}(r;E))\right]|\psi\rangle = E|\psi\rangle,$$
(10)

where M is the rest mass of the projectile, E the projectile energy in the center of mass (c.m.) frame. By selecting correct asymptotic boundary conditions, we can solve Eq. (10) numerically and consequently obtain the results of the observables $d\sigma/d\Omega$, A_y and Q, respectively, in the elastic proton scattering process.

3 The results and discussions

By using the RIA formalism, we obtain $d\sigma/d\Omega$, A_{ν} and Q for the proton elastic scattering off the nuclei ¹⁴Be, ¹⁶O and ¹²C, respectively. As the Dirac optical potentials U^{L} which are given in Eqs. (4), (5) and (9) contain the densities of scalar and vector fields, we firstly have to calculate the proton and neutron density distributions of target nuclei ¹⁴Be, ¹⁶O and ¹²C, respectively, based on the RMF theory with the parameter set NL3^[33]. The binding energy is adjusted by the center of mass correction with the HS scheme for light nuclei^[33]. The density distributions of the target nucleus and the Dirac optical potentials $U^{\rm L}$ for the proton elastic scattering off above three nuclei at $E_{\text{lab}}=200 \text{MeV}$ have been presented in Ref. [33]. In the present work, Dirac optical potentials for the same scattering systems at E_{lab} =800MeV are calculated and plotted in Fig. 1. It is shown that the Dirac optical potentials for the p-¹⁴Be system, in comparison with other two systems, exhibit the same 'long-tail' phenomenon as that in Ref. [33] at $E_{\text{lab}}=200 \text{MeV}$. This kind of 'long-tail' in Fig. 1 is considered as the result of the extended neutron distribution in the halo nucleus ¹⁴Be. In Table 1 some resultant quantities in the RMF theory and the corresponding data, which characterize nuclear properties, are tabulated. It is shown that the RMF results (marked by Theo.), such as the average binding energy^[39], rms radius^[40] and the rms charge radius^[41], agree with the experimental data (marked by Expt.) very well. In the columns for ¹⁴Be(Expt.), the rms charge radius of ¹⁴Be is obtained by the empirical formula $R_{\rm e} = r_{\rm e} A^{\rm e}$, where $r_{\rm e} = 1.153(7)$ and $e = 0.2938(12)^{[41]}$. Finally, the observables $d\sigma/d\Omega$, A_y and Q for the elastic scatterings of p-¹⁴Be, p-¹⁶O and p-¹²C at $E_{lab}=200, 400$ and 800MeV are evaluated, respectively. The results are demonstrated in Figs. 2—4, respectively, where $\theta_{\rm cm}$ is the scattering angle in the c.m. frame, σ stands for $\mathrm{d}\sigma/\mathrm{d}\varOmega$ in the present work and $\sigma_{\rm M}$ denotes the Mott cross section^[42]. In order to check the validity of RIA in describing the proton elastic scattering on the nucleus at the intermediate energy, we calculate the elastic scattering for p-¹²C and p-¹⁶O at $E_{\rm lab}=200$ MeV, respectively. The theoretical results and the data are illustrated in Fig. 5 and Fig. 6, respectively. From Fig. 5 and Fig. 6, one sees that the RIA results at $E_{\rm lab}=200$ MeV are in good agreement with the data especially in the small angular regions. Other proofs can also be found in literatures^[25-28, 30-32]. Therefore, RIA is a useful approach to describe the elastic scattering for p-nucleus at intermediate energies and to investigate the properties of the target nucleus.





The corresponding figures for very small angle behaviors are also presented in Figs. 2—4. From Fig. 2, we note that the positions of the first minimum of $\sigma/\sigma_{\rm M}$ for the p- $^{14}{\rm Be},$ p- $^{16}{\rm O}$ and p- $^{12}{\rm C}$ processes appear at angles in an ascending order, and the order will keep unchanged with different incident energies. These phenomena are probably due to the influence of the extended matter distribution, the halo, in ¹⁴Be. In Fig. 3 and Fig. 4 we find that in the very small angular region, the values of the spin-related observables A_y and Q in the p-¹⁴Be process are larger than those in the p-¹⁶O and p-¹²C processes no matter what incident energy is. These unique phenomena could be explained by the fact that the spin-dependent interaction between the projectile and the target more likely appears in p-¹⁴Be than in p-¹⁶O and p-¹²C in the small angular region. Because A_y and Q are spin observables that reflect the spin-dependent NN interaction and are much more sensitive to the target structure,

Table 1. Average Binding Energies (E/A) in MeV, rms Radii (R_m) in fm and Charge (R_{ch}) in fm for ¹⁶O, ¹²C and ¹⁴Be.

	$^{16}O(Theo.)$	$^{16}O(Expt.)$	$^{12}C(Theo.)$	$^{12}C(Expt.)$	14 Be(Theo.)	14 Be(Expt.)
E/A	-7.733	-7.976(0)	-7.049	-7.680(0)	-4.906	-4.994(9)
$R_{\rm m}$	2.594	2.54(2)	2.35	2.35(22)	3.34	3.20(30)
$R_{\rm ch}$	2.7315	2.7013(55)	2.4986	2.4703(22)	2.4645	2.5036(232)



Fig. 2. The $\sigma/\sigma_{\rm M}$ of the elastic scatterings for p-¹²C (dot), p-¹⁶O (dash) and p-¹⁴Be (solid) at $E_{\rm lab} = 200 \text{MeV}$ (a), 400MeV (b) and 800MeV (c), respectively.



Fig. 4. The same as Fig. 2 but for Q.



Fig. 5. The observables, $d\sigma/d\Omega$ (a), A_y (b) and Q (c), of the elastic scatterings for p-¹²C (solid) at $E_{\text{lab}} = 200$ MeV. The data (circles) are taken from Ref. [28].



Fig. 6. The same as Fig. 5 but for $p-^{16}O$ (solid).

especially to the nucleon distributions in the target^[43], we could use A_y and Q, in addition to σ , of the proton elastic scattering at intermediate energies as two new probes to study halo nuclei.

4 Summary

Based on the RIA formalism, we investigate the elastic scattering observables $d\sigma/d\Omega$, A_y and Q for the exotic system p-¹⁴Be, as well as two normal systems p-¹²C and p-¹⁶O, at the intermediate energies $E_{\rm lab} = 200, 400$ and 800MeV, respectively. We find that at different intermediate energies, $d\sigma/d\Omega$, A_y and Q behave consistently, namely the positions of

the first minimum of $\sigma/\sigma_{\rm M}$ appear at the angles in an ascending order for the p-¹⁴Be, p-¹⁶O and p-¹²C, and the values of the spin-related observables A_y and Q in the p-¹⁴Be process are larger than those in the p-¹⁶O and p-¹²C processes. Therefore, in comparison with those in the p-¹⁶O and p-¹²C processes, the behaviors of the differential cross section, the analyzing power and the spin rotation function in the p-¹⁴Be process in the small angular region exhibit an unchanged tendency with the variation of the incident energy in the intermediate energy region and the values of $d\sigma/d\Omega$, A_y and Q in the elastic scattering process in the intermediate energy region could be used to investigate the properties of neutron-riched nuclei.

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入射质子的能量对质子与晕核弹性散射观测量的影响*

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摘要 用相对论脉冲近似(RIA)对不同的中能质子(200MeV,400MeV和800MeV)与¹⁴Be,¹⁶O和¹²C的弹性散 射进行了研究,讨论了不同的入射能量对3种观测量,即微分散射截面、分析本领和自旋转动函数的影响.研究 发现在小散射角的区域晕中子对观察量的影响趋势保持一致,不随入射质子能量的改变而改变.

关键词 质子弹性散射 相对论脉冲近似 晕核 微分散射截面 分析本领 自旋旋转函数

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