Chiral Quark Model Study of $ud\bar{s}\bar{s}$ Tetraquark States*

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Abstract The energies of the low-lying isoscalar and isovector $ud\bar{s}\bar{s}$ configurations with spin-parity $J^P=0^+$, 1^+, and 2^+ are calculated in the chiral $SU(3)$ quark model and the extended chiral $SU(3)$ quark model by using the variational method. The model parameters are determined by the same method as in our previous work, and they still can satisfactorily describes the nucleon-nucleon scattering phase shifts and the hyperon-nucleon cross sections. The s-channel annihilation interaction is fixed by the masses of K and K* mesons, and the configuration mixing is considered. The results show that the $ud\bar{s}\bar{s}$ configuration with $I=0$ and $J^P=1^+$ lies lower than the K*K* threshold, and furthermore, this state has a very small KK* component, thus it can be treated as a possible tetraquark candidate.

Key words tetraquark state, quark model, chiral symmetry

1 Introduction

Since Jaffe predicted the H particle (uuddss) in 1977[1], the research on multi-quark states has always been an attractive topic for nearly three decades in both theoretical and experimental studies. But up to now, there has been no convinced evidence of their existence in experiments. The Θ particle, first reported by LEPS Collaboration in 2003[2], has motivated amounts of theoretical and experimental studies for pentaquarks and further the multi-quark states. Nevertheless its existence is still questioned till now.

Besides dibaryon and pentaquark, the possible $ud\bar{s}\bar{s}$ tetraquark is another interesting multi-quark system, and much work has been devoted to this issue in the past few years[3—9]. Since the studies of the possible $ud\bar{s}\bar{s}$ tetraquark state are presently model dependent, it seems important and necessary to investigate this state via different approaches.

In the past few years, the chiral $SU(3)$ quark model and the extended chiral quark model have proven to be quite successful in reproducing the binding energy of deuteron, the nucleon-nucleon and kaon-nucleon scattering phase shifts, and the nucleon-hyperon cross sections[10, 11]. In this paper, we use these two models to study the structures of the $ud\bar{s}\bar{s}$ configurations. The model parameters are determined by the same method as in our previous work[15—18]. The s-channel quark-antiquark annihilation interaction is fixed by the masses of K and K* mesons, and the configuration mixing is considered. The results show that the $ud\bar{s}\bar{s}$ configuration with $I=0$ and $J^P=1^+$ lies lower than the K*K* threshold, and furthermore, this state has a very small KK* component, thus it can be treated as a possible tetraquark candidate.

2 Formulation

The chiral $SU(3)$ quark model and the extended

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chiral $SU(3)$ quark model have been widely described in the literatures[12—15], and the details can be found in these references. Here we just give the salient features of these two models.

In these two models, the total Hamiltonian of the $\mathrm{ud\bar{s}\bar{s}}$ systems can be written as

$$H=\sum_i T_i - T_G + V_{12} + V_{\bar{3}\bar{4}} + \sum_{\substack{i=1,2\\ j=3,4}} V_{i\bar{j}}\ , \qquad (1)$$

where T_G is the kinetic energy operator for the center-of-mass motion, and V_{12}, $V_{\bar{3}\bar{4}}$ and $V_{i\bar{j}}$ represent the qq, $\bar{q}\bar{q}$ and $q\bar{q}$ interactions, respectively,

$$V_{12}=V_{12}^{\rm OGE}+V_{12}^{\rm conf}+V_{12}^{\rm ch}, \qquad (2)$$

where $V_{12}^{\rm OGE}$ is the OGE interaction, the confinement potential $V_{12}^{\rm conf}$ is taken as the linear form, and $V_{12}^{\rm ch}$ represents the effective quark-quark potential induced by the quark-chiral field coupling. The detailed expressions of these potentials can be found in Refs. [12—15].

$V_{\bar{3}\bar{4}}$ in Eq. (1) represents the antiquark-antiquark interaction,

$$V_{\bar{3}\bar{4}}=V_{\bar{3}\bar{4}}^{\rm OGE}+V_{\bar{3}\bar{4}}^{\rm conf}+V_{\bar{3}\bar{4}}^{\rm ch}, \qquad (3)$$

where $V_{\bar{3}\bar{4}}^{\rm OGE}$ and $V_{\bar{3}\bar{4}}^{\rm conf}$ can be obtained by replacing $\lambda_1^c\cdot\lambda_2^c$ with $\lambda_3^{c*}\cdot\lambda_4^{c*}$ in $V_{12}^{\rm OGE}$ and $V_{12}^{\rm conf}$, and $V_{\bar{3}\bar{4}}^{\rm ch}$ has the same form as $V_{12}^{\rm ch}$.

$V_{i\bar{j}}$ in Eq. (1) represents the quark-antiquark interaction,

$$V_{i\bar{j}}=V_{i\bar{j}}^{\rm OGE}+V_{i\bar{j}}^{\rm conf}+V_{i\bar{j}}^{\rm ch}+V_{i\bar{j}}^{\rm ann}, \qquad (4)$$

where $V_{i\bar{j}}^{\rm OGE}$ and $V_{i\bar{j}}^{\rm conf}$ can be obtained by replacing $\lambda_1^c\cdot\lambda_2^c$ with $-\lambda_i^c\cdot\lambda_{\bar{j}}^{c*}$ in $V_{12}^{\rm OGE}$ and $V_{12}^{\rm conf}$, and $V_{i\bar{j}}^{\rm ch}$ can be obtained from the G parity transformation:

$$V_{i\bar{j}}^{\rm ch}=\sum_k (-1)^{G_k} V_{ij}^{{\rm ch},k}, \qquad (5)$$

with $(-1)^{G_k}$ being the G parity of the kth meson. $V_{i\bar{j}}^{\rm ann}$ denotes the s-channel quark-antiquark annihilation interaction. For the $\mathrm{ud\bar{s}\bar{s}}$ system, $\mathrm{u(d)\bar{s}}$ can only annihilate into K and K^* mesons,

$$V_{i\bar{j}}^{\rm ann}=V_{\rm ann}^{\rm K}+V_{\rm ann}^{\rm K^*}. \qquad (6)$$

Their expressions can be found in the literature[13].

All the model parameters are tabulated in Table 1, where the first set is for the chiral $SU(3)$ quark model, the second and third sets are for the extended chiral $SU(3)$ quark model by taking $f_{\rm chv}/g_{\rm chv}$ as 0 and 2/3, respectively. Here $f_{\rm chv}$ is the coupling constant for tensor coupling of the vector meson fields.

Table 1. Model parameters. The meson masses and the cutoff masses: $m_{\sigma'}=980$MeV, $m_\kappa=980$MeV, $m_\epsilon=980$MeV, $m_\pi=138$MeV, $m_{\rm K}=495$MeV, $m_\eta=549$MeV, $m_{\eta'}=957$MeV, $m_\rho=770$MeV, $m_{\rm K^*}=892$MeV, $m_\omega=782$MeV, $m_\phi=1020$MeV, and $\Lambda=1100$MeV.

	χ-$SU(3)$ QM	Ex. χ-$SU(3)$ QM	
	I	II	III
		$f_{\rm chv}/g_{\rm chv}=0$	$f_{\rm chv}/g_{\rm chv}=2/3$
$b_{\rm u}$/fm	0.5	0.45	0.45
$m_{\rm u}$/MeV	313	313	313
$m_{\rm s}$/MeV	470	470	470
$g_{\rm u}^2$	0.766	0.056	0.132
$g_{\rm s}^2$	0.846	0.203	0.250
$g_{\rm ch}$	2.621	2.621	2.621
$g_{\rm chv}$		2.351	1.973
m_σ/MeV	595	535	547
$a_{\rm uu}^c$/(MeV/fm)	87.5	75.3	66.2
$a_{\rm us}^c$/(MeV/fm)	100.8	123.0	106.9
$a_{\rm ss}^c$/(MeV/fm)	152.2	226.0	196.7
$a_{\rm uu}^{c0}$/MeV	−77.4	−99.3	−86.6
$a_{\rm us}^{c0}$/MeV	−72.9	−127.9	−109.6
$a_{\rm ss}^{c0}$/MeV	−83.3	−174.20	−148.7

The S-wave $\mathrm{ud\bar{s}\bar{s}}$ wave functions can be written as:

$$I=1,\ J^P=0^+ \Longrightarrow \begin{cases} |1\rangle=\left(\{ud\}_1^{\bar{3}}\{\bar{s}\bar{s}\}_1^3\right)_0 \\ |2\rangle=\left(\{ud\}_0^6\{\bar{s}\bar{s}\}_0^{\bar{6}}\right)_0 \end{cases}$$

$$I=0,\ J^P=1^+ \Longrightarrow \begin{cases} |3\rangle=\left([ud]_0^{\bar{3}}\{\bar{s}\bar{s}\}_1^3\right)_1 \\ |4\rangle=\left([ud]_1^6\{\bar{s}\bar{s}\}_0^{\bar{6}}\right)_1 \end{cases}$$

$$I=1,\ J^P=1^+ \Longrightarrow |5\rangle=\left(\{ud\}_1^{\bar{3}}\{\bar{s}\bar{s}\}_1^3\right)_1$$

$$I=1,\ J^P=2^+ \Longrightarrow |6\rangle=\left(\{ud\}_1^{\bar{3}}\{\bar{s}\bar{s}\}_1^3\right)_2$$

Where { } and [ ] represent the flavor symmetry and the antisymmetry, respectively. The superscript is the representation of the color $SU(3)$ group, and the subscript is the spin quantum number. Making a recoupling calculation, we can express these wave functions as:

$$|1\rangle \equiv \left(\{ud\}_1^{\bar{3}}\{\bar{s}\bar{s}\}_1^3\right)_0 = \frac{1}{2}\left((u\bar{s})_0^1(d\bar{s})_0^1\right)_0^1-\sqrt{\frac{1}{12}}\left((u\bar{s})_1^1(d\bar{s})_1^1\right)_0^1- \sqrt{\frac{1}{2}}\left((u\bar{s})_0^8(d\bar{s})_0^8\right)_0^1+\sqrt{\frac{1}{6}}\left((u\bar{s})_1^8(d\bar{s})_1^8\right)_0^1, \quad (7)$$

$$|2\rangle \equiv \left(\{ud\}_0^6\{\bar{s}\bar{s}\}_0^{\bar{6}}\right)_0 = \sqrt{\frac{1}{6}}((u\bar{s})_0^1(d\bar{s})_0^1)_0^1 + \sqrt{\frac{1}{2}}((u\bar{s})_1^1(d\bar{s})_1^1)_0^1 + \sqrt{\frac{1}{12}}((u\bar{s})_0^8(d\bar{s})_0^8)_0^1 + \frac{1}{2}((u\bar{s})_1^8(d\bar{s})_1^8)_0^1, \quad (8)$$

$$|3\rangle \equiv \left([ud]_0^{\bar{3}}\{\bar{s}\bar{s}\}_1^3\right)_1 = \sqrt{\frac{1}{12}}((u\bar{s})_0^1(d\bar{s})_1^1)_1^1 - \sqrt{\frac{1}{12}}((u\bar{s})_1^1(d\bar{s})_0^1)_1^1 + \sqrt{\frac{1}{6}}((u\bar{s})_1^1(d\bar{s})_1^1)_1^1 - \sqrt{\frac{1}{6}}((u\bar{s})_0^8(d\bar{s})_1^8)_1^1 + \sqrt{\frac{1}{6}}((u\bar{s})_1^8(d\bar{s})_0^8)_1^1 - \sqrt{\frac{1}{3}}((u\bar{s})_1^8(d\bar{s})_1^8)_1^1, \quad (9)$$

$$|4\rangle \equiv \left([ud]_1^6\{\bar{s}\bar{s}\}_0^{\bar{6}}\right)_1 = -\sqrt{\frac{1}{6}}((u\bar{s})_0^1(d\bar{s})_1^1)_1^1 + \sqrt{\frac{1}{6}}((u\bar{s})_1^1(d\bar{s})_0^1)_1^1 + \sqrt{\frac{1}{3}}((u\bar{s})_1^1(d\bar{s})_1^1)_1^1 - \sqrt{\frac{1}{12}}((u\bar{s})_0^8(d\bar{s})_1^8)_1^1 + \sqrt{\frac{1}{12}}((u\bar{s})_1^8(d\bar{s})_0^8)_1^1 + \sqrt{\frac{1}{6}}((u\bar{s})_1^8(d\bar{s})_1^8)_1^1, \quad (10)$$

$$|5\rangle \equiv \left(\{ud\}_1^{\bar{3}}\{\bar{s}\bar{s}\}_1^3\right)_1 = \sqrt{\frac{1}{6}}((u\bar{s})_0^1(d\bar{s})_1^1)_1^1 + \sqrt{\frac{1}{6}}((u\bar{s})_1^1(d\bar{s})_0^1)_1^1 - \sqrt{\frac{1}{3}}((u\bar{s})_0^8(d\bar{s})_1^8)_1^1 - \sqrt{\frac{1}{3}}((u\bar{s})_1^8(d\bar{s})_0^8)_1^1, \quad (11)$$

$$|6\rangle \equiv \left(\{ud\}_1^{\bar{3}}\{\bar{s}\bar{s}\}_1^3\right)_2 = \sqrt{\frac{1}{3}}((u\bar{s})_1^1(d\bar{s})_1^1)_2^1 - \sqrt{\frac{2}{3}}((u\bar{s})_1^8(d\bar{s})_1^8)_2^1, \quad (12)$$

where $(u\bar{s})(d\bar{s})$ represents $\sqrt{1/2}[(u\bar{s})(d\bar{s})+(d\bar{s})(u\bar{s})]$ for Eqs. (7), (8), (11) and (12), and denotes $\sqrt{1/2}[(u\bar{s})(d\bar{s})-(d\bar{s})(u\bar{s})]$ for Eqs. (9) and (10).

3　Results and discussions

We calculate the energies for six low configurations of $ud\bar{s}\bar{s}$ system in the chiral quark models. The size parameter for K and K^* is taken to be 0.4fm, which is smaller than that for baryons. The calculated results (without configuration mixing) are given in Table 2. In this table, the first set is for the chiral $SU(3)$ quark model, the second and third sets are for the extended chiral $SU(3)$ quark model.

Table 2.　Energies (in MeV) of $ud\bar{s}\bar{s}$ six single states in various chiral quark models.

	χ-$SU(3)$ QM	Ex. χ-$SU(3)$ QM		Threshold
	I	II	III	
		$f_{chv}/g_{chv}=0$	$f_{chv}/g_{chv}=2/3$	
$J^P=0^+$				
$\left(\{ud\}_1^{\bar{3}}\{\bar{s}\bar{s}\}_1^3\right)_0$	1744	1683	1681	KK (990)
$\left(\{ud\}_0^6\{\bar{s}\bar{s}\}_0^{\bar{6}}\right)_0$	1753	1834	1808	KK (990)
$J^P=1^+$				
$\left([ud]_0^{\bar{3}}\{\bar{s}\bar{s}\}_1^3\right)_1$	1641	1682	1675	KK* (1387)
$\left([ud]_1^6\{\bar{s}\bar{s}\}_0^{\bar{6}}\right)_1$	1722	1784	1770	KK* (1387)
$\left(\{ud\}_1^{\bar{3}}\{\bar{s}\bar{s}\}_1^3\right)_1$	1771	1754	1745	KK* (1387)
$J^P=2^+$				
$\left(\{ud\}_1^{\bar{3}}\{\bar{s}\bar{s}\}_1^3\right)_2$	1821	1872	1852	K*K* (1784)

In Ref. [8], Cui et al. argued that the strong attractive color-magnetic interaction could reduce the energies of the $ud\bar{s}\bar{s}$ systems, and they found a $I=0$ and $J^P=1^+$ $ud\bar{s}\bar{s}$ tetraquark state with a mass around 1347MeV. In our chiral $SU(3)$ quark model calculation, the color-magnetic interactions are attractive both in the isovector $J^P=0^+$ $\left(\{ud\}_1^{\bar{3}}\{\bar{s}\bar{s}\}_1^3\right)_0$ state and the isoscalar $J^P=1^+$ $\left([ud]_0^{\bar{3}}\{\bar{s}\bar{s}\}_1^3\right)_1$ state, while they are repulsive in the other four configurations. In the extended chiral $SU(3)$ quark model, the OGE is largely reduced and the color-magnetic attractions are almost replaced by ρ exchange. Furthermore, both in the chiral $SU(3)$ quark model and the extended chiral $SU(3)$ quark model, the σ and π exchanges provide more attractive interactions in the isovector $J^P=0^+$ $\left(\{ud\}_1^{\bar{3}}\{\bar{s}\bar{s}\}_1^3\right)_0$ state and the isoscalar $J^P = 1^+$ $\left([ud]^{\bar{3}}_0\{\bar{s}\bar{s}\}^3_1\right)_1$ state than in the other four configurations. Thus the energies of the $\left(\{ud\}^{\bar{3}}_1\{\bar{s}\bar{s}\}^3_1\right)_0$ and $\left([ud]^{\bar{3}}_0\{\bar{s}\bar{s}\}^3_1\right)_1$ states are respectively the lowest one in $J^P = 0^+$ and $J^P = 1^+$ cases in various models. However, due to the high kinetic energies, the attractive interactions are not strong enough to reduce the energies of these two states to be lower than the corresponding meson-meson thresholds (see Table 2).

Further we consider the configuration mixing between different states. The results are shown in Table 3. Comparing it with Table 2, one can see that the configuration mixing can shift the energies over 60MeV for most configurations. And we also find that in the chiral $SU(3)$ quark model one energy of the $I=0$ and $J^P=1^+$ state is 1768MeV, lower than the threshold of K^*K^*, and the corresponding root mean square radius is about 0.57fm. The wave function of this state is

$$|4\rangle' = -0.17\left((u\bar{s})^1_0(d\bar{s})^1_1\right)^1_1 + 0.17\left((u\bar{s})^1_1(d\bar{s})^1_0\right)^1_1 + 0.71\left((u\bar{s})^1_1(d\bar{s})^1_1\right)^1_1 - 0.47\left((u\bar{s})^8_0(d\bar{s})^8_1\right)^1_1 + 0.47\left((u\bar{s})^8_1(d\bar{s})^8_0\right)^1_1 + 0.0056\left((u\bar{s})^8_1(d\bar{s})^8_1\right)^1_1, \quad (13)$$

where it is clear to see that besides 43.8% part of two color octet $q\bar{q}$ pairs, the component of two color singlet $q\bar{q}$ pairs is 50.4% for K^*K^* and 5.8% for KK^*, which means the K^*K^* component is dominate and comparatively the KK component is very small, thus this state has a few possibility decaying into K and K^*. Furthermore, since the energy of this configuration, 1768MeV, is lower than the K^*K^* threshold (1784MeV), it cannot decay into K^*K^* final state. This means that this state will possibly have a narrow width, and can be treated as a good candidate for the $ud\bar{s}\bar{s}$ tetraquark state.

Table 3. Energies (in MeV) of $ud\bar{s}\bar{s}$ states with configuration mixing considered.

	χ-$SU(3)$ QM	Ex. χ-$SU(3)$ QM	
	I	II	III
$I=1, J^P=0^+$	1602	1573	1572
	1857	1909	1882
$I=0, J^P=1^+$	1577	1623	1618
	1768	1833	1817
$I=1, J^P=1^+$	1771	1754	1745
$I=1, J^P=2^+$	1821	1872	1852

4 Summary

The structures of $ud\bar{s}\bar{s}$ states with $J^P=0^+$, 1^+, and 2^+ are studied in the chiral $SU(3)$ quark model and the extended chiral $SU(3)$ quark model. We calculate the energies of six low-lying $ud\bar{s}\bar{s}$ configurations using the variational method. The configuration mixing is considered, and the model parameters are determined by the same method as in our previous work. With the size parameter for mesons taken to be 0.4 fm, the $ud\bar{s}\bar{s}$ configuration with $I=0$ and $J^P=1^+$ is found to lie lower than the K^*K^* threshold, and furthermore, this state has a very small KK^* component, thus it can be treated as a possible tetraquark candidate. A dynamical calculation would be done in future work.

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手征夸克模型下 $ud\bar{s}\bar{s}$ 四夸克态的研究*

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摘要　在手征 $SU(3)$ 夸克模型和扩展的手征 $SU(3)$ 夸克模型的框架下, 用变分的方法系统地研究了同位旋为 0、1, 自旋宇称为 0^+、1^+ 和 2^+ 的 $ud\bar{s}\bar{s}$ 四夸克系统 6 个低组态的能量. 模型的参数取自以前的工作, 它能很好地描述核子–核子散射相移以及核子–超子散射截面. S 道相互作用的参数由拟合 K 介子和 K^* 介子的质量定出, 并且考虑了具有相同量子数的态之间的态混合效应. 结果表明, 同位旋为 0 且自旋宇称为 1^+ 的 $ud\bar{s}\bar{s}$ 能量低于相应的 K^*K^* 的阈能, 且该组态中 KK^* 的成分相当小, 因此该组态的宽度可能较小, 可视为一个可能的四夸克态的候选者.

关键词　四夸克态　夸克模型　手征对称性

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