Emittance-dominated long bunches in dual harmonic RF system*

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Abstract The storage of long bunches for long time intervals needs flattened stationary buckets with a large bucket height. The longitudinal motion of the initially mismatched beam has been studied for both the single and dual harmonic RF systems. The RF amplitude is determined to be r.m.s wise matched. The bucket height of the single harmonic system is too small even for shorter bunch with only 20% increased energy spread. The Halo formation and even debunching can be seen after a few synchrotron periods for single particles with large amplitude. In the case of small energy spread for a cooled beam, Coulomb interaction cannot be ignored. The external voltage has to be increased to keep the r.m.s bunch length unchanged. The new voltage ratio $R(N)$ simplifies physics for the emittance-dominated bunches with modest particle number N. For the single harmonic system, substantial amount of debunching occurs without increasing the external voltage, but very little if the RF amplitude is doubled. Results from the ORBIT tracking code are presented for the 1 GeV bunch in the HESR synchrotron, part of the GSI FAIR project.

Key words bucket height, mismatch, space charge effect, debunching

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1 Introduction

In the dual harmonic RF systems long bunches with phase extension of more than $\pm\pi/6$ can be exactly matched only for the self-consistent phase space distributions, compensating the synchrotron tune spread, but leading to non-elliptical phase space boundaries. Short bunches with elliptical phase space distributions can be matched in every RF system by keeping the linear voltage unchanged. The bucket height or energy acceptance here is always much larger than the maximal energy spread.

Using distributions with elliptical boundaries, long bunches can be matched r.m.s wise in the dual and the single harmonic systems, but synchrotron tune spread cause filamentation. For shorter bunches with 20% larger energy spread and zero intensity, the bucket height is too small for a single harmonic system. Halo formation and even debunching can be seen after a few oscillation periods.

Coulomb interactions are evaluated by analyzing the longitudinal envelope equation, including the linear space charge (SC) forces. For long bunches with modest particle number N, enlarging the RF amplitude by the new voltage factor $R(N)$, which is smaller than 2 for emittance-dominated bunches, keeps the r.m.s bunch length unchanged and simplifies physics. Motion of test particles and phase space distributions are presented to evaluate the halo formation and debunching. Without enlarging the RF amplitude, substantial debunching occurs in the single harmonic systems.

The results for coherent and incoherent motion due to mismatch are consistent with the analysis of envelope and single particle equation, including the contributions from SC forces.

Findings here are valid in general, they are only dependent on the new voltage factor $R(N)$ and the synchrotron oscillation period.

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2 Tracking results without SC forces

2.1 r.m.s wise matching for long bunches

Single particle motion within a stationary bucket, formed by the dual harmonic RF system, and in smooth approximation is given by Ref. [1—3]:

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}t^2}+\omega_{s0}^2E(\phi)=0\,. \tag{1}$$

With $\omega_{s0}^2=\dfrac{qV_0\eta}{2\pi\beta^2\gamma m_0c^2}\omega_0^2$ and the waveform $E(\phi)$ is defined by:

$$E(\phi)=\sin\phi-d\sin 2\phi\,, \tag{2}$$

where ω_0 is the angular revolution frequency and V_0 is the amplitude of the RF voltage per turn. $\phi=z/R$ is the phase value relative to the position of the bunch center, m_0c^2 is the particle rest mass, c is the velocity of light and q is the elementary charge. β, γ are relativistic factors. Phase slip factor η is defined as $\eta=1/\gamma^2-1/\gamma_t^2$ with $\gamma_t^2=1/\alpha_C$, where α_C is the momentum compaction factor. γ_t is real for the positive α_c value, but imaginary for the negative α_c value.

For positive momentum compaction factor, $\gamma_t m_0c^2$ is called transition energy, leading to η to change its sign at transition energy. Staying below transition or $\eta>0$ requires the positive amplitude V_0 to be focused in the longitudinal plane.

For a bunching factor of $B>0.17$, which means the phase extension of more than $\pm\pi/6$, the matched voltage V_0 is determined to keep the r.m.s phase values unchanged. As the synchrotron tunes are amplitude dependent[1], the emittance dilution cannot be avoided.

1D tracking results by ORBIT[4] are shown in Figs. 2 and 3 for a 1 GeV bunch in the HESR ring[5] with 574 m in circumference and the imaginary γ_t value of 6.5i. A time gap of 10% improves the lifetime of anti-protons. Either the single or dual harmonic RF system with d=0 or d=0.31 is used. The Gaussian bunch has the length of 400 m or the bunching factor B=0.7, the r.m.s phase width σ_ϕ of 0.78 rad, and the r.m.s energy spread σ_E of 0.6 MeV or the r.m.s momentum spread of 4×10^{-4}. The total number of 5000 macro-particles and 64 frequency bins are used.

The resulting RF voltages are shown in Fig. 1(a) for different d values, determined from the r.m.s wise matching. The single harmonic RF system means d=0. For small phase values, all particles oscillate with synchrotron frequency ω_{s0}. The dual harmonic RF system reduces the linear voltage by a factor of 1-2d, but leads to flattened buckets with increased bucket height, see Fig. 1(b). Choosing d=0.31 results in a somewhat reduced slope, but leads to stronger RF focusing from 1 rad on, the maximal value is reached at about 2.2 rad. As the synchrotron tune distributions are different for both systems[1], the RF amplitude is larger for the single harmonic system to compensate greater tune spread.

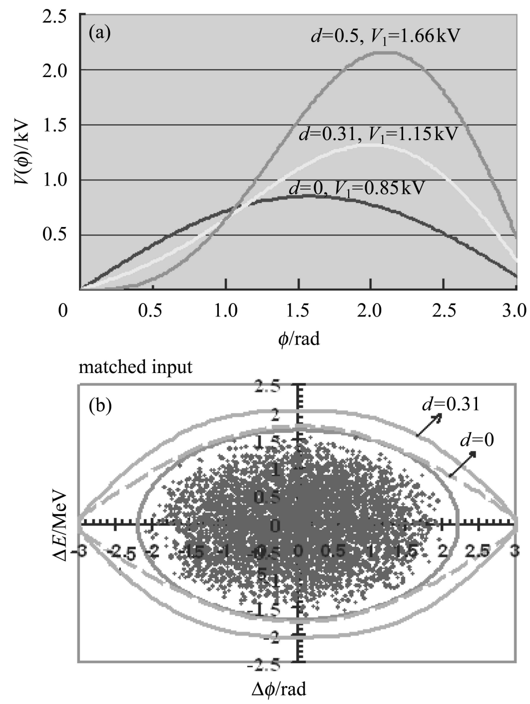

Fig. 1. (a) RF voltages for different d values; (b) RF acceptances, matched Gaussian input distribution with elliptical boundary.

For completeness in Fig. 1(a), the dual harmonic system with d=0.5 is also shown, having the largest voltage. But the RF acceptance and bucket height are almost identical with the system for d=0.31. But for small phases, the RF voltages are drastically reduced, as the slope goes to zero. As adding particle number requires linear voltage to compensate SC voltage, the RF systems with d=0.5 cannot be used.

2.2 RF acceptances for long bunches

The resulting RF acceptances are shown in Fig. 1(b) for single, d=0 and dual harmonic RF system, d=0.31. The dual harmonic system leads to flattened RF acceptance with a larger bucket height $\sim f(d)\times\sqrt{V_0}$, 2 MeV for dual, but only 1.77 MeV for the single one. Factor $f(d)$ can be calculated analytically[1], ratio $f(d{=}0.31)/f(d{=}0)$ between the dual and single harmonic systems is 0.97. For the dual harmonic system, the bucket height is 20% larger than the maximum energy spread for the matched Gaussian input distribution truncated at about $3\sigma_E$. But the single harmonic system leads to a bucket height about $3\sigma_E$. In Fig. 1(b) also shown are the matched Gaussian input distribution together with the elliptical phase space boundary, 8 times the r.m.s emittance.

2.3 Quadrupole oscillation due to mismatch

Figure 2 shows the r.m.s phase oscillation for the single and dual harmonic systems, demonstrating the r.m.s matching in both cases. Injecting shorter bunch with 20% increased energy spread, but with unchanged r.m.s emittance, excites the coherent quadrupole oscillations. 17% shorter bunch leads to 22% longer bunch after 2000 turns or about 4 ms, predicted from an harmonic potential $P(N,x)$, see Section 3.3. The coherent quadrupole mode is undamped for the dual harmonic RF system, but damped for the single harmonic one, as the tune spread is larger but the energy acceptance is smaller here. The tune spread leads to decoherence time of only $1.5{\times}10^4$ turns for single harmonic, but $7.2{\times}10^4$ turns for dual harmonic, obtained from the dipole oscillation, excited by the initial shift of 0.05 MeV.

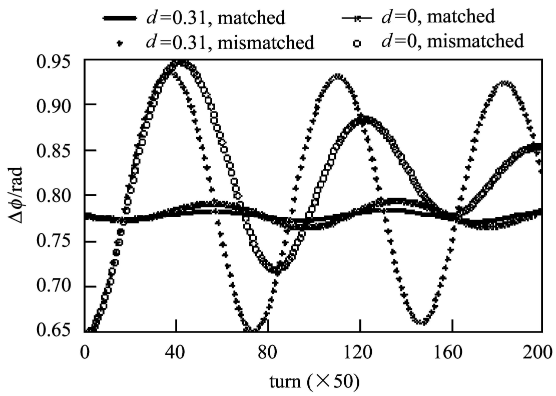

Fig. 2. The r.m.s phase oscillations, matched and mismatched cases.

2.4 Debunching & halo formation by mismatch

The phase space filamentation after 10^4 turns is shown in Fig. 3 for shorter bunch, 20% increased energy spread, but unchanged emittance. Less than 10^{-3} particles are outside the RF acceptance for the dual harmonic system, but about 1% for the single harmonic one. As these particles are leaving their RF buckets, debunching will occur after some turns. Due to different coherent oscillation periods, see Fig. 2, there is an upright phase elliptical boundary, 8 times the r.m.s emittance, for single harmonic, but a tilted one for dual harmonic.

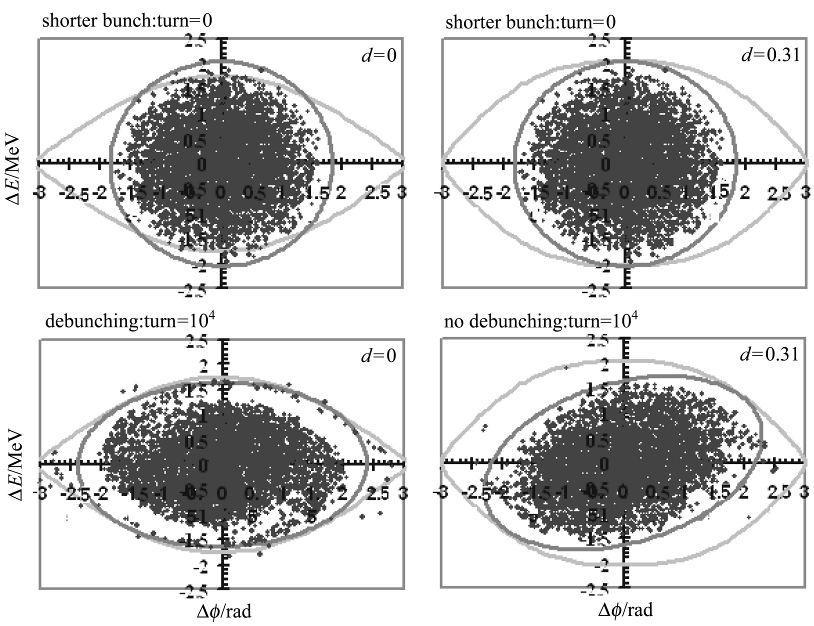

Fig. 3. The initial and final distributions with the elliptical boundary and RF acceptance.

As the matched voltage amplitude scales like $(\sigma_{\rm E})^2$ for the unchanged phase values, the energy acceptance is always 20% larger than the maximal energy spread for Gaussian input distribution. For short bunches with unchanged r.m.s. emittance, the matched RF amplitude scales like $1/\sigma_\Phi^4$: the r.m.s phase value σ_Φ which is smaller by a factor of 2 requires the matched voltage to be increased by a factor of 16, thus making the energy acceptances 4 times larger than before, whereas $\sigma_{\rm E}$ is only increased by a factor of 2.

Using different energies and other lattice parameters will change only the quadrupole mode oscillation time but not the underlying physics.

3 Longitudinal envelope equation with SC

3.1 General case for single particle equation

The single particle motion for a stationary bucket, but with any kind of external voltage $V_{\rm rf}(z/R)$ and for an arbitrary line density $\lambda(z)$ is given by Ref. [1—3]:

$$z''+\alpha\left[V_{\rm rf}\left(\frac{z}{R}\right)+V_{\rm SC}(z)\right]=0\,, \tag{3}$$

$$\alpha=\frac{q\omega_0^2\eta R}{2\pi\gamma m_0c^2\beta^4c^2}\,, \tag{4}$$

$$V_{\rm SC}(z)=Nq\beta cR\left(\frac{gZ_0}{2\beta\gamma^2}\right)\frac{\partial\lambda(z)}{\partial z}\,, \tag{5}$$

where $z'\equiv {\rm d}z/{\rm d}s$, $\phi=z/R$, $R=C/2\pi$, C for the ring circumference, N stands for the particle number, z for the longitudinal coordinate of particles relative to the bunch center, $Z_0=377\Omega$ for free space impedance, g for the geometry factor $\left(g=1+2\ln\frac{b}{a}\right)$, a for twice the average r.m.s beam radius, and b for the average pipe radius. Eq. (5) is correct for particle well inside the bunch, but breaks down nearby the edges[3].

For arbitrary complex impedances, the SC voltage $V_{\rm SC}(\phi)$ in single particle Eq. (3) has to be replaced by retarding the voltage $V_{\rm r}(\phi)$[3].

For the dual harmonic RF system, the external voltage is $V_{\rm rf}(z/R)=V_0E(z/R)$. The SC voltage $V_{\rm SC}(\phi)$ for Gaussian line density with r.m.s phase value σ_ϕ is

$$V_{\rm SC}(\phi)=-\frac{1}{\sqrt{2\pi}}\frac{NqcZ_0}{2R}\frac{g}{\gamma^2}\frac{\phi}{\sigma_\phi^3}\exp\left(\frac{-\phi^2}{2\sigma_\phi^2}\right). \tag{6}$$

For long bunches, the RF and SC voltages have different phase dependence. For real density, only particles in the core receive linear external and SC forces.

3.2 Envelope equation for short bunches

In case of short bunches with parabolic line density, single particle Eq. (3) can be rewritten as:

$$z''+k_0^2(N)z-\frac{K_{\rm L}(N)}{z_{\rm m}^3(s)}z=0 \quad \text{or} \quad z''+k_z^2(s,N)z=0\,, \tag{7}$$

where $z_{\rm m}(s)$ is the oscillating r.m.s bunch length and $K_{\rm L}(N)$ is the longitudinal perveance:

$$K_{\rm L}(N)=\frac{3}{10\sqrt{5}}\cdot\frac{gNr_{\rm p}}{\beta^2\gamma^3}\eta\,, \tag{8}$$

where $r_{\rm p}=1.54\times10^{-18}$ m is the classical proton radius.

Without intensity, $K_{\rm L}(N)$=0. For a dual harmonic RF system:

$$k_0^2(N=0)=(\omega_{\rm s0}/\beta c)^2\sim V_0(1-2d)\,.$$

With intensity, the effective voltage $k_z^2(s,N)$ is determined by the external voltage $k_0^2(N)$, which may be increased to compensate SC voltage, and the oscillating r.m.s bunch length $z_{\rm m}(s)$.

Equation (7) can be converted into an envelope equation for the r.m.s bunch length $z_{\rm m}(s)$[2]:

$$z_{\rm m}''(s)+k_0^2(N)z_{\rm m}(s)-\frac{K_{\rm L}(N)}{z_{\rm m}^2(s)}-\frac{\varepsilon_z^2}{z_{\rm m}^3(s)}=0\,, \tag{9}$$

where the r.m.s emittance $\varepsilon_z=\sqrt{\langle z^2\rangle\langle z'^2\rangle-\langle zz'\rangle^2}$ is the conserved quantity. The matched r.m.s bunch length $z_{\rm m}(N)$ is given by $z_{\rm m}(N)=\sqrt{\varepsilon_z/k_z(N)}$.

The matched r.m.s bunch length $z_{\rm m}(N)$ for a given particle number N is kept unchanged by increasing the external RF voltage or $k_0^2(N)$ by the factor $R(N)$. $R(N)<2$ describes the emittance-dominated bunches, whereas $R(N)>2$ describes the SC dominated bunches. For $R(N)$=2, the SC and emittance terms are equal in the longitudinal envelope equation, leading to 0.7 synchrotron tune depression.

Varying r.m.s bunch length $z_{\rm m}(N)$ means the change of RF amplitude like $\varepsilon_z^2/z_{\rm m}^4(N)$, but for realistic line densities, SC voltage scales like $N/z_{\rm m}^3(N)$, see Eqs. (6) and (7). Shorter bunch requires higher RF amplitude, but reduces the voltage factor $R(N)$, leading to larger full RF acceptance. Longer bunch needs less RF amplitude, voltage factor $R(N)$ is increased, but full RF acceptance is decreased. Larger full RF acceptance hinders debunching and the tracking results are shown in Section 5.1.

3.3 Coherent, incoherent motion due to mismatch

For small initial mismatch, the coherent quadrupole mode frequency $\omega_{\rm Q}^2(N)$, derived from the envelope Eq. (9), is given by:

$$\omega_{\rm Q}^2(N)/\omega_{\rm s,0}^2(N)=1+3R(N)\,. \tag{10}$$

This is a special case of discussions in Ref. [6], where the r.m.s bunch length is dependent on the particle number N. In general, coherent mode frequencies are expressed by full and zero current synchrotron tunes.

Also derived from the envelope Eq. (9), is a harmonic potential $P(N,x)$, describing the oscillation of relative mismatch x (x=−0.2 for a 20% shorter bunch) also for large values

$$P(N,x)\;=\;\frac{Rx^2}{2}+(R-1)\left(\frac{1}{1+x}+x\right)+\left(\frac{0.5}{(1+x)^2}+x\right)-(R-0.5), \tag{11}$$

almost given by $(3R(N)+1)f(x)$, where $f(x)$ is positive, but asymmetric function: 20% shorter bunch leads to 30% longer bunch afterwards.

Results from ORBIT tracking code for quadrupole oscillations are shown in Fig. 2 and Fig. 4, demonstrating the asymmetric oscillation and increase of the coherent mode frequency with $R(N)$.

Core particles with small phase values receive linear forces, strongly affected by mismatch. The effective voltage slope $\alpha(N,x)$ in single particle Eq. (6) can be expressed as

$$z''+k_0^2\alpha(N,x)z=0\,, \tag{12}$$

where $\alpha(N,x)=R(N)-\dfrac{R(N)-1}{(1+x)^3}$.

20% shorter bunch or x=−0.2 changes the effective slope from focusing to defocusing for $R(N)>2$. Incoherent motions of test particles are shown in Fig. 5, they are consistent with the prediction of Eq. (12).

4 Coherent & incoherent motion with SC

4.1 Coherent motion for $R(N)<3$

For long bunches with realistic phase space distributions, the envelope Eq. (9) is no longer valid. However quite good r.m.s wise matching is possible for modest intensities, $R(N)$=2, where the external voltage is only increased by a factor of 2, inspite of having different phase dependence for RF and SC voltages.

For a matched bunch with ±2.2 rad phase extension, the r.m.s phase values obtained from 1D ORBIT tracking code including Coulomb interaction, are shown in Fig. 4. A cooled HESR beam with N=10^{13} and b/a=75 leads to $R(N)$=2. R.m.s wise matching is possible for both the dual and the single harmonic systems, whereas the RF amplitude differs by a factor of 1.35.

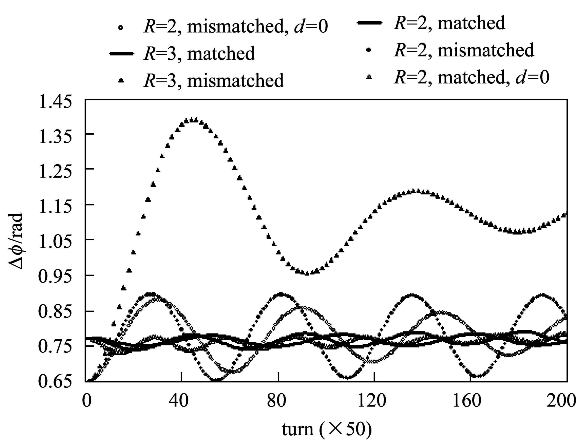

Fig. 4. The r.m.s phase oscillations for the matched and shorter bunches.

Dual harmonic system is used, if single harmonic is not indicated: The voltage factor $R(N)$ depends only on the particle number N, but not on the parameter d. Also the case with $R(N)$=3, is shown, representing twice the particles for $R(N)$=2.

The coherent quadrupole oscillations are excited by using shorter bunches of Fig. 3. The mode frequencies increase like $\sqrt{1+3R(N)}$, Eq. (10), and initial short bunch leads to somewhat longer one, as predicted by Eq. (11).

Very large r.m.s phase values are obtained in the single harmonic system for the modest particle number N=10^{13} and shorter bunch, if RF voltage is kept unchanged. The resulting phase space distribution after 10^4 turns is shown in Fig. 7, and substantial amount of debunching is visible.

Further increasing the particle number to N=3×10^{13} leads to the voltage factor $R(N)$=4. Such high intensity can be handled only by the dual harmonic system, but not by the single one, even for the matched input[7], as the full RF acceptance is too small. The bucket height for full voltage, but without SC, increases like $f(d)\sqrt{R(N)V_0}$, where the amplitude V_0 is determined from the r.m.s wise matching.

For short bunches, the factor $R(N)$ can be calculated analytically from Eq. (9), but only numerically for long bunches. Varying parameter d leads to different matched voltages for zero intensity, but for the same particle number, the voltage factor $R(N)$ is unchanged. $R(N)$ is also independent of input distributions if the r.m.s values are kept the same[7].

Compared with the situation of short bunches, increasing the initial phase values without changing the RF amplitude does not always lead to the matched situation, see Section 5.

4.2 Incoherent motion, affected by mismatch

Figure 5 shows the motions of test particles after every 50 turns, initially placed at zero phase value, but with energy deviation of either 1.8 MeV or 2 MeV, interacting with all other particles in the bunch. The test particles start at the bunch edge of either matched or mismatched Gaussian input distribution, see Figs. 2 and 3. Dual harmonic RF system is used for all shown cases. Results for matched and shorter bunch, with and without the SC forces, are shown. Only for zero intensity, synchrotron motion leads to continuous increase of energy.

For the r.m.s wise matched case with $R(N)$=2, the particle loses energy, indicating an effective negative voltage slope. Afterwards, non-linear components of either RF or SC voltages, see Eq. (3), leads to energy gain at larger phase values. By using the single harmonic system with its larger voltage slope, the test particle loses almost no energy.

For the mismatched case, $R(N)$=2, the particle starting at 1.8 MeV loses more energy compared with the matched case, which is consistent with Eq. (11). But starting at the bunch edge with 2 MeV energy deviation, the particle loses little energy for about 400 turns, as large phase values are reached quite soon.

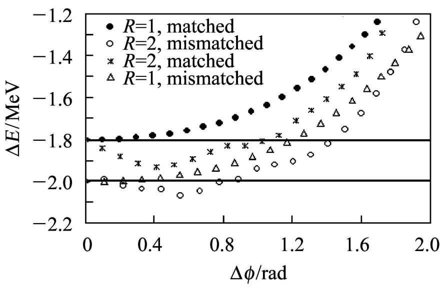

Fig. 5. Motions of test particles after every 50 turns, initial energies are indicated by solid lines.

5 Distributions for $N=10^{13}$, $R(N)=1,2$

5.1 Halo formation for $R(N)=2$, $d=0$

As shown before, the r.m.s phase values are quite similar for both the single and the dual harmonic RF systems. But movement of the large amplitude particles is quite different, as the bucket height is much smaller for the single harmonic RF system. Full RF acceptance is not exact boundary with SC[7], but allows quantifying amount of debunching.

Figures 6 & 7 show the phase space distributions after 10^4 turns for injecting shorter Gaussian bunch, see Fig. 3, with $N=10^{13}$ in the dual and the single harmonic RF systems, respectively. The elliptical phase space boundaries at 8 times the r.m.s emittance are shown together with the full RF acceptance. RF voltages are increased by a factor of 2 for Fig. 6, leading to almost no change of the r.m.s emittances, little halo formation occurs for single harmonic system.

In the case of the dual harmonic RF system, the bunch is tilted after 10^4 turns, caused by the coherent quadrupole oscillation, see Fig. 6, no debunching occurs, as the RF acceptance is large enough. All the particles are staying well inside the full RF acceptance. Less than 0.01 % of the particles have phase values above ± 2.5 rad after some turns. Both incoherent effects are not much enlarged by adding small resistive impedances at low frequencies[7].

In the case of the single harmonic system, initially there are no particles outside the full RF acceptance with the bucket height of about 2.5 MeV. The phase space ellipse is upright after 10^4 turns, see Fig. 6. Particles are outside 8 times the r.m.s emittance due to the mismatch and SC effect. About 0.2% of the particles have phase values above ± 2.5 rad after some turns. Large amplitude particles suffer from debunching, as they are outside the full RF acceptance. Adding small resistive impedances at low frequencies enlarges both incoherent effects, and also emittance dilution occurs[7].

Injecting 25% longer bunch with 10^{13} particles in a dual harmonic system with amplitude V_0 of only 1.15 kV, leads to matching without exciting coherent quadrupole oscillations. This is consistent with scaling laws for external and internal RF voltages, see Section 3.2, leading to the voltage factor $R(N)$ of 2.25. Due to the reduced full RF acceptance, debunching process occurs for the large amplitude particles.

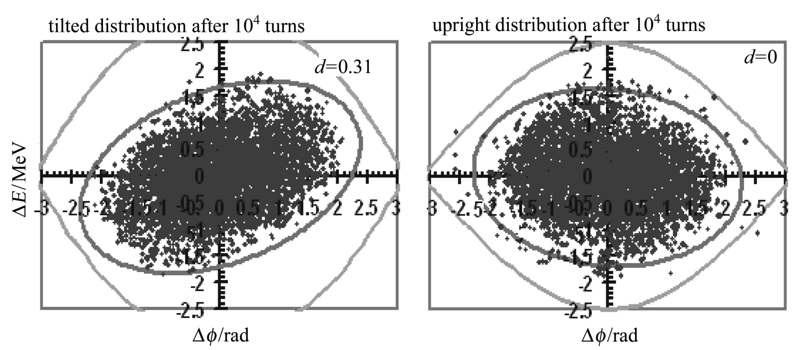

Fig. 6. Phase space distribution with elliptical boundary and full RF acceptance.

5.2 Debunching for $R(N)=1$, $d=0$

Not increasing RF voltage for shorter Gaussian bunch, see Fig. 3, with 10^{13} particles in single harmonic RF system, amplitude V_0 of 0.85 kV, leads to phase space filamentation and substantial debunching after 10^4 turns, shown in Fig. 7. The resulting r.m.s phase values are shown in Fig. 4. The final r.m.s emittance is increased by 58%. The elliptical phase boundary is shown at 8 times the initial r.m.s emittance. The maximal energy spread is a little above 2 MeV, the largest initial value due to the initial mismatch, see Fig. 3.

Injecting 25% longer bunch in the single harmonic system, but with zero intensity, excites coherent quadrupole oscillation, see Fig. 2. However for 10^{13} particles, the bunch length is increased to the maximal value, causing debunching. Complete different behavior of the same bunch in either the dual or the single harmonic system is caused by rapid decay of RF voltage at large phase values for d=0.

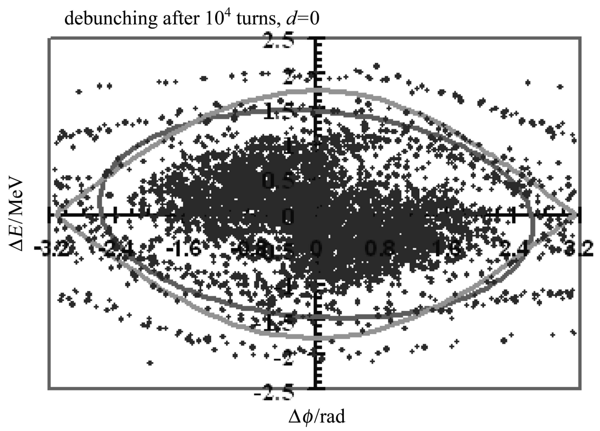

Fig. 7. Phase space distribution with elliptical boundary and full RF acceptance.

5.3 Validity of 1D simulations with SC

1D ORBIT calculations ignore radial-axial coupling, as SC impedance only depends on b/a ratio, where pipe radius b & beam radius a are the averaged values. Even for the SNS facility with a planned multi MW power upgrade, longitudinal studies for keeping time gap are done with 1D tracking only. However, to get the transverse distribution at the stripper foil requires time consuming 2D tracking for the detailed lattice, as beam radii are oscillating[8].

For short linac bunches, where all 3-beam radii are similar, the radial-axial mode coupling due to high intensity cannot be ignored[9], requiring full 3D tracking. Exciting similar radial-axial modes in circular accelerators is not very likely.

6 Summary

Long bunches with more than ±2 rad in the total phase extension or 0.78 rad in r.m.s phase can be r.m.s wise matched with both the single and the dual harmonic systems.

RF acceptance and bucket height are larger for a dual harmonic system with d=0.31.

Longitudinal SC forces can be compensated by increasing the external RF voltage by a new factor $R(N)$ to keep the r.m.s bunch length unchanged, simplifying physics. For $R(N)$=2, SC and emittance terms are equal in longitudinal envelope equation. Bucket height for full voltage, without intensity, increases like $\sqrt{R(N)}$.

Coherent and incoherent motions together with the resulting phase space distributions are shown for long bunches with $R(N) \leqslant 3$. Injecting shorter bunches only little halo formation occurs for the dual harmonic, whereas the same halo formation and even debunching can be seen for the single harmonic system after few synchrotron periods. Adding small resistive impedances at low frequencies enlarges incoherent effects, and also emittance dilution occurs[7]. High intensity, $R(N)>4$, can be handled only by the dual, but not by the single harmonic system, even for matched input and no resistive impedances[7], as the full RF acceptance is too small.

Different from the situation of short bunches, increasing initial phase values without changing RF amplitude does not always lead to the matched situation, especially in the single RF system. Complete different behavior of the same bunch in either dual or single harmonic system is caused by the rapid decay of RF voltage at large phase values for d=0.

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