Three-body model for neutron-halo nuclei*

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Abstract The neutron-halo nuclei, ^{11}Li, ^{14}Be, and ^{17}B, are studied in the three-body model. The Yukawa interaction is used to describe the interaction of the two-body subsystem. For given parameters of the two-body interaction, the properties of these neutron-halo nuclei are calculated with the Faddeev equations and the results are compared with those in the variational method. It is shown that the method of the Faddeev equations is more accurate. Then the dependencies of the two- and three-body energies on the parameters are studied. We find numerically that two- and three-body correlations differ greatly from each other with the variation of the intrinsic force range.

Key words neutron-halo nuclei, three-body model, Faddeev equation

PACS 21.10.Dr, 21.60.Gx, 27.20.+n

1 Introduction

It has been more than twenty years since the neutron halo of the nucleus ^{11}Li was reported by Tanihata in 1985[1, 2]. The nucleus ^{11}Li is a very neutron-rich nucleus, whose two neutron separation energy is approximately 300 keV and whose valence neutrons have a large probability in the s-wave component. Thus it has an abnormally extended distribution in space and this leads to narrow momentum distribution after fragmentation. The discovery of ^{11}Li's exotic characters stimulates much work on the study of the nuclei far from stability[3—10]. In experiment, with the development of radioactive nuclear beam facilities, more and more nuclei very far away from stability line, i.e., very neutron-rich or proton-rich nuclei, are produced in laboratory. This provides much opportunity to study neutron haloes. Accompanied by the improvement of experimental techniques, great progress has also been made in theory. The shell model, the Hartree-Fock method, the relativistic mean field (RMF) method and the cluster model have been developed and applied to study such nuclei. Up to now, a lot of neutron-halo nuclei, such as ^{6}He, ^{8}He, ^{11}Li, ^{11}Be, ^{14}Be, ^{17}B, ^{19}B, ^{19}C and ^{22}C, have been suggested and investigated[11—17]. The cluster model[18—22] has an advantage in investigating the properties of neutron-halo nuclei, for example, the binding energy, matter distribution, root-mean-square radius, and so on. The three-body model is very suitable to study two-neutron haloes. This kind of neutron-halo nuclei can be treated as a system composed of three-particles, an inert core and two outer correlated neutrons. Usually the three-body system is bound while each two-body subsystem is unbound, so this kind of neutron-halo nuclei are called the Borromean nuclei.

2 Two-body interactions and the three-body model

There are various forms for two-body interactions, such as Gaussian potentials, Woods-Saxon potentials, exponential potentials and Yukawa potentials. Each of them should reproduce the low-energy properties of the two-body subsystems. Three-body Schrödinger equations and Faddeev equations are equivalent in describing the quantum three-body systems, and are

Received 9 May 2008

* Supported by National Natural Science Foundation of China (10535010, 10775068), 973 National Major State Basic Research and Development of China (2007CB815004), CAS Knowledge Innovation Project (KJCX2-SW-N02), Research Fund of Education Ministry under contract RFDP (20070284016), and Green-Blue Project of Jiangsu Province

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used widely. In particular, the Faddeev equations are very suitable to describe a three-body system and the technique to solve them has also been developed greatly[18—20]. Besides, the variational Schrödinger equation method has been improved by Z. Z. Ren et al.[21—23] to study neutron haloes, which is more smart and can give reasonable physical explanation on some aspects of neutron haloes.

In the present article, a kind of two-body Yukawa interaction is adopted for both neutron-core interaction and neutron-neutron interaction. The method of the Faddeev equations is applied to study the two-neutron haloes, and the results are compared with those calculated with the variational method[22]. Based on these, the nuclei ^{11}Li, ^{14}Be, and ^{17}B are studied. By developing the computer code of solving the Faddeev equations, we get the two-body resonant energy and the three-body energy for different parameters.

The nuclei, ^{11}Li, ^{14}Be, and ^{17}B, are treated in the same way here. Therefore, we'll take ^{11}Li for example to give an explanation on the details of the calculation. The nucleus ^{11}Li is treated as a three-body system, i.e., an inert ^{9}Li core and two valence neutrons. As the first step to develop the Faddeev code, the relative angular momenta of all the two-body subsystems are assumed to be zero, i.e., only s-wave two-body interactions are included. This agrees with the fact that the valence neutrons of the neutron-halo nuclei usually have a rather large s-wave component.

The neutron-neutron interaction is chosen as a Yukawa potential that can reproduce the low-energy properties of the two-neutron subsystem in a singlet s-state

$$V(r) = -147.484 s b^{-2} \frac{b}{r} \exp\left(-2.1196 \frac{r}{b}\right). \quad (1)$$

where r is the distance between the two valence neutron, s=0.949 is the well-depth parameter, and b=2.06±0.21 (fm) is the intrinsic force range[24].

The neutron-core interaction is in the form of empirical central interaction[22]

$$V(r) = -147.484 s_{\rm c} \frac{N_{\rm c}+1}{2N_{\rm c}} b_{\rm c}^{-2} \frac{b_{\rm c}}{r} \exp\left(-2.1196 \frac{r}{b_{\rm c}}\right). \quad (2)$$

where $s_{\rm c}$ is the well-depth parameter, $b_{\rm c}$ is the intrinsic force range and $N_{\rm c}$ is the mass number of the core. Here $s_{\rm c}$ <1 in order to guarantee that there is no bound state for the two-body subsystems, and $b_{\rm c}$ should be adjusted to reproduce the experimental two-body resonant energy approximately.

The Faddeev equations for the three-body systems write

$$\begin{pmatrix} T_1+V_1-E & E & V_1 \\ V_2 & T_2+V_2-E & V_2 \\ V_3 & V_3 & T_3+V_3-E \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \end{pmatrix} = 0. \quad (3)$$

where $V_i = V_{jk}$ are the two-body interactions, T_i is the kinetic operator expressed in a given Jacobi coordinate set, E is the eigenenergy of the three-body system and Ψ_i is the eigenfunction in a given Jacobi coordinate set. In this article, $S_{2n} = -E$ is the two-neutron separation energy of the neutron-halo nucleus as a three-body system. The full three-body wavefunction is $\Psi = \Psi_1 + \Psi_2 + \Psi_3$, where each component wavefunction Ψ_i is defined by the above equations. The coupled equations are solved with hyperspherical harmonics expansion method which has been stated detailedly in the previous reference[20].

3 Numerical calculations and results

The ground state properties of ^{11}Li, ^{14}Be, and ^{17}B have been studied for given parameters $s_{\rm c}$, $b_{\rm c}$ with the variational method[22]. The calculations with the same parameters are performed here by solving the Faddeev equations. The results are listed in Table 1. The experimental energies $E_0^{\rm exp}$ of the three-body systems are cited from Ref. [25], and $E_0^{\rm exp} = -S_{2n}$ (S_{2n} is the two-neutron separation energy). $E_0^{\rm th1}$ and $E_0^{\rm th2}$ are the theoretical eigenenergies calculated with the variational method and with the method of Faddeev equations respectively. $r_{\rm c}$ is the root-mean-square (RMS) matter radius of the core, which comes from experiment, and R_m is the RMS matter radius of the neutron-halo nuclei. $R_m^{\rm exp}$, $R_m^{\rm th1}$ and $R_m^{\rm th2}$ are the experimental value, theoretical value computed with the variational method and with the method of Faddeev equations respectively. It is clearly seen from Table 1 that the three-body energies of the ground states here are about 0.17 MeV smaller than those in Ref. [22]. Because the three-body systems are more bound here, the corresponding RMS matter radii are smaller than those in Ref. [22]. The difference between the two calculations is not accidental. The Faddeev calculation is more accurate than the variational calculation.

Table 1. Numerical results for ^{11}Li, ^{14}Be, and ^{17}B with the same parameters as those in Ref. [22].

	$s_{\rm c}$	$b_{\rm c}$	$E_0^{\rm exp}$/MeV	$r_{\rm c}$/fm	$R_m^{\rm exp}$/fm	$E_0^{\rm th1}$/MeV	$R_m^{\rm th1}$/fm	$E_0^{\rm th2}$/MeV	$R_m^{\rm th2}$/fm
^{11}Li	0.89	5.0	−0.300(±0.019)	2.31	3.10(±0.17)	−0.36	3.18	−0.54	2.95
^{14}Be	0.99	5.0	−1.26(±0.13)	2.57	3.10(±0.30)	−0.90	2.90	−1.07	2.85
^{17}B	0.98	5.0	−1.34(±0.17)	2.50	3.00(±0.40)	−0.84	2.81	−1.01	2.76

Thus, the wavefunction calculated from the coupled Faddeev equations is closer to the real one, and the eigenenergy of the three-body system is lower.

There are two parameters, s_c and b_c, in the Faddeev calculation. We will take ^{11}Li for example in order to study the dependence of the theoretical results on the two parameters. First we will study $E_{\rm res.}$, the lowest resonant energy of the two-body subsystem. The two-body system composed of a ^{9}Li core and a neutron is unbound. According to experiment, the one-neutron separation energy $S_n = -25 \pm 15$ keV, i.e., the lowest resonant energy of the system is 25±15 keV. s_c is varied from 0.8 to 1.0 and b_c is varied from 3 fm to 8 fm in order to study the dependence of $E_{\rm res.}$ on them. Three dimensional plot of $E_{\rm res.}$ against s_c and b_c is displayed in Fig. 1. It is obviously seen that $E_{\rm res.}$ decreases with the increase of b_c for a given s_c and $E_{\rm res.}$ decreases with the increase of s_c for given b_c.

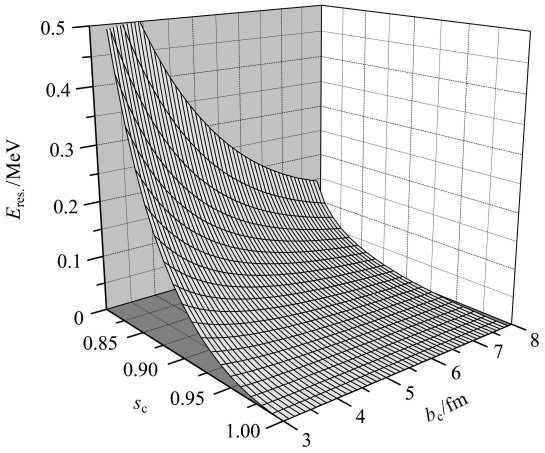

Fig. 1. The lowest two-body resonant energy against s_c and b_c for the subsystem of the ^{9}Li core and the neutron.

Then we study the dependence of the three-body energy E_0 on s_c and b_c. The three dimensional plot of E_0 against s_c and b_c is displayed in Fig. 2. It is seen that E_0 increases with the increase of b_c for a given s_c and E_0 decreases with the increase of s_c for a given b_c. Thus $E_{\rm res.}$ and E_0 have similar dependence on s_c, but have completely different dependence on b_c. The opposite dependence on b_c comes from the discrepancy of the nature of the two-body and the three-body correlations. The two-body resonant energy is determined by the behavior of the Yukawa potential in the distance, while the three-body energy has a strong correlation with the well-depth of the Yukawa potential near the origin.

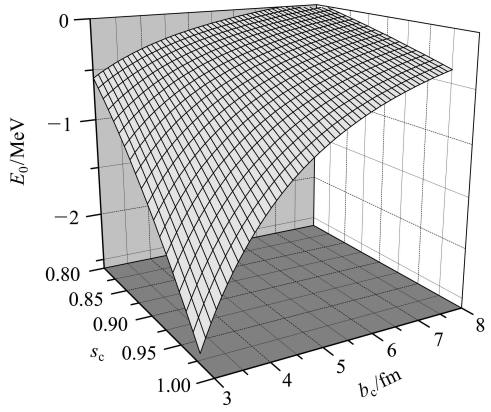

Fig. 2. The three-body energy against s_c and b_c for ^{11}Li.

Considering both the two-body resonant energy and the three-body energy, we get a better set of parameters to describe ^{11}Li as a three-body system, s_c=0.88, b_c=6.1 fm. The RMS matter radius of ^{11}Li is consistent with the experimental value. The RMS matter radius of ^{11}Li is much larger than that of the ^{9}Li core, thus the halo character of ^{11}Li is displayed through the computation.

^{14}Be and ^{17}B are treated in the same way with ^{11}Li. s_c =0.91 and b_c =3.5 is better to describe ^{14}Be. The calculated RMS matter radius of ^{14}Be is 2.75 fm which is smaller than the experimental value but still in the acceptable range. s_c =0.94 and b_c =3.8 is preferred for ^{17}B whose calculated RMS matter radius is 2.66 fm. All the three preferred sets of parameters for ^{11}Li, ^{14}Be and ^{17}B are listed in Table 2 respectively. The theoretical values, $E^{\rm th}_{\rm res.}$, $E^{\rm th}_0$ and $R^{\rm th}_{\rm m}$, are listed in the table. It is clearly seen that the well-depth parameter s_c of ^{11}Li is smaller than that of ^{14}Be and ^{17}B, while the intrinsic force range parameter b_c of ^{11}Li is bigger than that of the other two. It is reasonable to conclude that the core-neutron potential for ^{11}Li is shallower and has a larger force range. Hence, ^{11}Li has more loosely bounded valence neutrons and more extended matter distribution in space.

Table 2. New calculation for ^{11}Li, ^{14}Be, and ^{17}B with suitable parameters.

	s_c	b_c/fm	$E^{\rm exp}_{\rm res.}$/keV	$E^{\rm exp}_0$/MeV	$E^{\rm th}_{\rm res.}$/keV	$E^{\rm th}_0$/MeV	$R^{\rm th}_{\rm m}$/fm
^{11}Li	0.88	6.1	25±15	−0.300(±0.019)	36	−0.30	3.17
^{14}Be	0.91	3.5	100±70	−1.26(±0.13)	51	−1.26	2.75
^{17}B	0.94	3.8	40±60	−1.34(±0.17)	17	−1.34	2.66

4 Summary

In summary, we have investigated the neutron haloes in ^{11}Li, ^{14}Be, and ^{17}B. These nuclei are assumed to be with a three-body structure. The interactions of all the subsystems are chosen as Yukawa interactions which can reproduce the corresponding low-energy properties. By solving the coupled Faddeev equations, we get the three-body energy and the root-mean-square matter radius of the system. With the same parameters as those of Ref. [22], we perform calculations for the above nuclei, and get more accurate results than those in the variational method. We vary the parameters s_c and b_c and investigate the dependence of the two- and three-body information on the parameters. We find numerically that the two- and three-body correlations differ greatly from each other with the variation of b_c. To fit the two-body resonant energy, three-body energy and RMS matter radius, a preferred set of parameters is given. We find that the suitable parameters can reasonably reproduce the properties of ^{11}Li, ^{14}Be, and ^{17}B. The development of the codes solving the Faddeev equations is still in progress.

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