# New chromaticity compensation approach and dynamic aperture increase in the SSRF storage ring\*

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**Abstract** Strong chromatic sextupoles used to compensate natural chromaticities in the third generation light source storage ring usually reduce dynamic aperture drastically. Many optimization methods can be used to find solutions that provide large dynamic apertures. This paper discusses a new optimization approach of sextupole strengths with step-by-step procedure, which is applied in the SSRF storage ring, and a better solution is obtained. Investigating driving terms generated by the sextupoles in every step can analyze their convergences and guide the weight setting among different terms in object function of the single resonance approach based on the perturbation theory.

**Key words** SSRF storage ring, chromaticity compensation, nonlinear optimization, dynamic aperture, sextupole

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## 1 Introduction

In the third generation light source, strong sextupoles are usually introduced in order to correct the large natural chromaticities, and corresponding nonlinearity limits dynamic aperture drastically. Commonly, harmonic sextupoles added in straight sections are utilized to optimize the nonlinearity. And detailed optimizations of the sextupole families, strengths, positions and phase advances are indispensable processes<sup>[1]</sup>. Both analytical and numerical methods can devote to nonlinear optimization, and they are usually iterated each other. For a lattice with decided structure, strength scheme of the sextupoles dominates dynamic aperture with a given linear optics. Two possible approaches dedicating to increasing the dynamic aperture may be mentioned. The first one is the strength scan of the harmonic sextupoles, which can be carried out in a simple lattice with one or two harmonic sextupole families, and the solution providing the largest dynamic aperture can be easily obtained, such as ELETTRA<sup>[2]</sup> and  $ASP^{[3]}$ . But, for a complex lattice with several harmonic sextupole families, this method requires a lot

This paper discusses a new optimization approach of the sextupole strengths, in which the best pair of focusing and defocusing sextupoles is used to compensate little chromaticities released by chromatic correc-

of computing time. The second one is the single resonance approach based on the perturbation theory. In this method, object function constituted with driving terms is analytically converged to a valley, which is expected to provide a larger dynamic aperture. At present, it has been extensively applied in many third generation light sources, such as DIAMOND<sup>[4]</sup>, SLS<sup>[5]</sup> and NSLS-II<sup>[6]</sup>. However, the weight factors for many terms are ambiguous to the largest dynamic aperture. More precisely, these driving terms can be expanded to harmonics, and a specific harmonics is assumed to drive a relevant resonance. More elaborate optimization of the sextupoles devotes to suppressing the harmonics, which are affirmed as the dangerous ones, and increase the dynamic aperture consequently. It has been applied in an assumed ultimate storage ring. But the dynamic aperture obtained by particle tracking is not consilient with the analytical  $one^{[7]}$ . This indicates limitation of the single resonance approximation.

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tional sextupoles in arc cells. One can repeat this process in order to increase the dynamic aperture stepby-step. This method is numerical but does not require excessive running time. Generally, several hours time-consumption is enough. Moreover, investigating the driving terms of the sextupole strength scheme resulting from each step can reveal the relationship between the convergence of driving terms and the dynamic aperture increase. Though, this method requires the harmonic sextupoles to compensate small chromaticities, one can change the strengths of the best pair harmonic sextupoles step-by-step instead of chromaticity correction in order to obtain a good solution for dispersion-free mode.

# 2 Algorithm descriptions

This method can be carried out in the following consecutive steps. The first step is to set the initial strength values of all the sextupoles to zero, and then correct the natural chromaticities to small quantities, such as  $\xi_N/100$ , with chromatic sextupoles in arc cells. The second step is to correct the small chromaticities to zero (or slightly positive values for realistic operating) with all the possible pairs of focusing and defocusing sextupoles respectively, then track and compare all the dynamic apertures, and select the best one, whose harmonic sextupole strength scheme is regarded as the initial value of the next step, and then, repeat the above process so as to obtain the biggest dynamic aperture and the optimum solution of sextupole strengths.

The lattice of the SSRF storage ring contains eight sextupole families, including four focusing families (S1 S3 S5 SF) and four defocusing ones (S2 S4 S6 SD). SD and SF are the chromatic sextupoles in arc cells<sup>[8]</sup>. There are 15 possible pairs, except for the pair of SF and SD. Each iteration step must track the dynamic aperture for 15 times, and the time-consumption may be excessive, so some improved method should be employed. For example, the non-zero initial values of the sextupoles can be set empirically to deduce the iterative times, and the tracking method can be predigested to save the running time.

## 3 Application in SSRF storage ring

#### 3.1 Lattice parameters

The method has been applicated to the SSRF storage ring. The results and some details are presented in the following sections. Table 1 shows the main parameters of the lattice, and Table 2 shows the strengths of the quadrupoles.

Table 1. Lattice parameters.

parameter	value
tune $Q_x, Q_y$	22.28, 11.32
$\beta_x, \ \beta_y, \ \eta_x/m$ in the center of long straight	10,  6.0,  0.15
section and the short one	3.6, 2.5, 0.10
natural emittance/(nm·rad)	3.92
natural chromaticity $\xi_x$ , $\xi_y$	-55.66, -17.93
momentum compactor	$4.1636 \times 10^{-4}$
damping factors (	0.9969, 1, 2.0031
natural energy spread/(rms)	$9.8959 \times 10^{-4}$

Table 2. Strength of quadrupoles.

quadrupoles	strength/K
QL1, QL2, QL3	-1.0414, 1.3642, -1.2345
QL4, QL5	-1.0648, 1.3966
QM1, QM2, QM3	-1.5402, 1.5341, -1.0543
QM4, QM5	-1.3506, 1.4609

#### 3.2 Dynamic aperture increase

The procedure has carried out 50 steps with the initial sextupole strengths of zero, and the tracking of dynamic apertures lasted a definite number of 500 turns. The seeds providing the biggest dynamic apertures are fixed in lattice after each step. The dynamic aperture increases step-by-step until the 40th step, which can be selected as optimum solution, and then decreases in the following steps, for the chromatic sextupoles weaken gradually, and the harmonic sextupoles strengthen too much. Fig. 1 reveals the dynamic aperture increasing with step, in which the dots denote the results of other possible pairs. We consider the area of 1 mm×1 mm as the dynamic aperture unit, showed on y-axis. Fig. 2 shows the



Fig. 1. Dynamic apertured increasing with step.



Fig. 2. The biggest dynamic apertures of all the steps.

biggest dynamic apertures of all the steps. The selected times of the sextupole pairs in the former 40 steps are depicted in Fig. 3, in which all the possible pairs of the sextupoles are arranged on x-axis and the selected times are denoted on y-axis. One can see that some pairs have never been chosen as the best ones while others occurred rather frequently, such as S1 and S2, S3 and S4, S5 and S6.



Fig. 3. The selected times of 15 sextupole pairs in the former 40 steps.

#### 3.3 Solution of the sextupole strengths

Generally, one can deduce more than one solution from different results of other sextupole pairs in some steps, which still provide a large enough dynamic aperture. As an example, one of the solutions is reported here. The strength integrals of the eight sextupole families are summarized in Table 3, and all of them satisfy the hardware maximum gradient restriction with 300 T/m<sup>2</sup> for S1 460 T/m<sup>2</sup> for S5 S6 and 400 T/m<sup>2</sup> for the others. The dynamic apertures for on and off momentum particles are obtained by tracking 1000 turns, shown in Fig. 4. The phase maps of the two transverse directions reveal that the horizontal dynamic aperture is affected by some resonances, including the 7th order and the 5th order and the 3rd order, while the vertical one is regular relatively, shown in Fig. 5.

Table 3. Strength integral of the sextupoles.

sextupole	strength integral
S1, S2, S3	1.5916, -2.7130, 2.9771
S4, S5, S6	-3.4285, 3.5528, -4.4261
SF, SD	3.5176, -2.4815



Fig. 4. Dynamic apertures for on and off momentum particles.



Fig. 5. Phase maps of the two transverse directions.

### 4 Convergences of the driving terms

In the single resonance approach, the Hamiltonian can be written as a series of driving terms  $(h_{jklmp})$ of different orders in field gradients<sup>[1, 5]</sup>. It is supposed that the convergence of the driving terms correspond with the increase of the dynamic aperture. In the previous work, some acceptable results for the SSRF storage ring have been obtained by applying the HARMON module based on this theory in OPA<sup>[9]</sup> and AT<sup>[10]</sup> codes. Whereas, weight setting among driving terms in the object function is uncertain for the optimization, no theory can guide the choice of the weight directly, and the setting is empirical completely.

Here, the separate driving term considered in the object function at present, which includes five geometric terms of the first order and three amplitude dependent tune shift terms of the second order, is computed with the sextupole strength scheme obtained

in every step. Table 4 shows the driving terms and their effects. The computational results are depicted in Fig. 6.

Table 4. The driving terms and their effects.

order	driving term	effect
the first	$ h_{10110} $	$ u_x$
	$ h_{10200} $	$\nu_x + 2\nu_y$
	$ h_{10020} $	$\nu_x - 2\nu_y$
	$ h_{21000} $	$ u_x$
	$ h_{30000} $	$3\nu_x$
the second	$ h_{22000} $	$\partial \nu_x / \partial J_x$
	$ h_{11110} $	$\partial \nu_{x,y} / \partial J_{y,x}$
	$ h_{00220} $	$\partial  u_y / \partial J_y$

It is revealed in Fig. 6 that three geometric terms  $(h_{10020} h_{10110} h_{21000})$  and three tune shift terms  $(h_{22000} h_{11110} h_{00220})$  converge to minimums within the vicinity of the 40th step, where the biggest dynamic aperture is provided, while the other two geometric terms  $(h_{10200} h_{30000})$  can't converge. The reason may be that the values of  $h_{10200}$  and  $h_{30000}$  are too small, and the dynamic aperture can't respond to the minichange. Therefore, the two terms shouldn't contribute much to the object function, even they can be ignored, and the weights of the two terms can be set to small values, even zero. With the same method, high order driving terms can be calculated, and analysis of the convergence can guide the weight setting

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of object function in further optimization process.



Fig. 6. Driving terms of all the 50 steps.

## 5 Conclusions

The new chromaticity compensation approach is discussed and applied to the SSRF storage ring. An acceptable solution of the sextupole strengths is obtained, which proves the feasibility of this method. Analysis of the convergences of the driving terms can guide the weight setting in the object function of the single resonance approach.

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