

# Analysis of charged hadron multiplicity distributions at RHIC<sup>\*</sup>

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**Abstract** It is demonstrated that with Heinz's collective flow model charged particle distributions at AGS and lower SPS energies (less than 20 GeV/n), can successfully be analyzed, but that the model fails for the RHIC data. Heinz's model calculation underestimates the tails of the charged particle distributions from RHIC, the discrepancy becoming bigger as the energy increases. To study the multiplicity distributions at RHIC we develop the so-called "Thermalization Component Model", which is based on Heinz's collective flow model. It is realized that the limitation of phase space of collective flow can be reflected in that of the thermalization region. By comparing the contributions of particle production from the thermalization regions at different energies and different centralities, we can deepen our understanding of the features of collective motion at RHIC.

**Key words** thermalization, collective flow, thermalization component model

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## 1 Introduction

A central question at RHIC is the extent to which the particles produced in the collision interact and thermalize<sup>[1, 2]</sup>. Nuclear collisions generate an enormous amount of particles and large transverse energy. An interesting question is, to what extent does the collision generate matter in local equilibrium which can be characterized by thermodynamic parameters, such as temperature, pressure, and energy density. Only if thermalization has been established can more detailed questions be asked about the equation of state of the matter.

Recently it has been realized that the study of collective flow is one of the important tools in investigating multi-hadron production in relativistic heavy-ion collisions. This is because the longitudinal and transverse flow includes rich physics, and the collective flow is closely related to nuclear stopping and the early evolution of the system. Collective flow is often utilized to express the thermalization degree of a relativistic heavy-ion collision system. Detailed studies of the observed final state flow pattern will deepen

our understanding of the dynamic mechanism in relativistic heavy-ion collisions.

Heinz's Collective flow model<sup>[3, 4]</sup> (HCFM) was developed based on a pure thermal model. It achieved great success in the discussion of charged particle distributions at AGS and lower SPS energies (20 GeV) and became an indicator for the existence of collective flow<sup>[3, 4]</sup>. But a detailed analysis of the experimental data at SPS and RHIC energies with HCFM has shown that with increasing collision energy, the two tails of  $\pi$  or charged hadron distributions show a (symmetric) discrepancy between the data and the calculation. This phenomenon led us to reconsider Heinz's collective flow theory for higher collision energies.

As shown in Fig. 1, HCFM fails to reproduce the charged hadron distribution at the RHIC energy of 200 AGeV. With the increase of the collision energy, the tails of the distribution of the experimental data are strongly underestimated by the calculation with HCFM. A simple explanation would be, that due to the increased collision energies also the phase space available for the produced particles increases and the

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thermalization in this enlarged phase space becomes more difficult. Therefore a detailed analysis of the thermalization process and its relation to the centralities and energies at RHIC is needed.

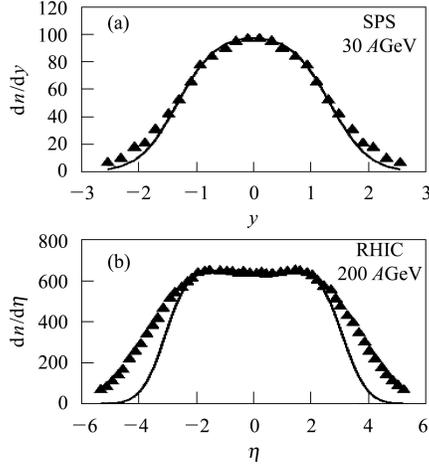


Fig. 1. (a)  $\pi$  rapidity distribution of 30 AGeV Pb+Pb Collision at SPS<sup>[5]</sup>; (b) Charged hadron pseudo-rapidity distribution for 200 AGeV at RHIC. The triangles are from the experimental data<sup>[6–11]</sup>.

The main topic of this paper is to investigate the question of how to simulate the data of charged hadron distributions at higher SPS and RHIC energy regions. The PHOBOS-collaboration used three Gaussian distributions to simulate the distribution of charged hadrons successfully<sup>[6–11]</sup>. Georg Wolschin<sup>[12]</sup> et al also discussed the charged hadron distribution by using three component distribution functions to construct a Fokker Plank equation. Both models assumed three emitting sources with a random Gaussian distribution. The Three Gaussian sources represent the target, projectile and central source, respectively.

In the following we will concentrate on the thermalization features of multi-particle production in heavy ion collisions at high energy in the framework of the collective flow theory. We restrict ourselves here to the basic features and essential results of the HCFM approach. A complete survey of the assumptions and results, as well as of the relevant references, is available in Refs. [3, 4, 13–17].

In Sec. 2 the details of the analysis based on the thermalization component model (TCM) are described. A comparison of the TCM calculations with the experimental data is presented in Sec. 3. The summary is given in Sec. 4.

## 2 Thermalization component model

The evolution of the hot and dense matter produced in relativistic heavy ion collisions may be de-

scribed by the following scenario: pre-equilibrium, thermal (or chemical) equilibrium of partons, possibly the formation of a QGP or a QGP-hadron gas mixed state, a gas of hot interacting hadrons, and finally, a freeze-out state when the produced hadrons no longer strongly interact with each other. Since the produced hadrons carry information on the entire space-time evolution of the system from the initial to the final stage of collisions, in particular on the collision dynamics, a precise analysis of the multiplicity distributions of charged hadrons is essential for the understanding of the dynamics and properties of the created matter.

Our model is based on three assumptions; some of them are rather different from those usually made in other flow models.

(i) The size of phase space of the particle distribution increases with the increase of collision energy. It seems more difficult to realize thermalization in the whole phase space of particle production at SPS and RHIC data. It is assumed that the Gaussian distributions were fit to the distributions of the produced charged hadrons in the two fragmentation regions, and that for the SPS and RHIC data in question thermalization prefers to occur at the central rapidity region.

(ii) The collective flow in the central rapidity region carries information on the early time steps of the heavy-ion collision. The system expands not only in the longitudinal direction, but also in the transverse direction. This two dimensional collective flow is used to study the thermalization process at RHIC energies.

(iii) The phase space is divided into an equilibrium region (thermalization region) and non-equilibrium regions (non-thermalization regions). The non-thermalization regions correspond to the two fragmentation regions. The total multiplicity distribution is the sum of the contributions from the target fragmentation region, projectile fragmentation region and central region, respectively

$$\frac{dn}{dy} = n_1 F_1 + n_2 F_2 + n_3 F_3 = \sum_i n_i F_i. \quad (1)$$

Here  $i = 1, 2, 3$  denote target, projectile and central region, and  $n_i$  and  $F_i$  are the corresponding particle numbers and normalized distribution functions.

As mentioned before, the distributions of the target and projectile fragmentation regions are given by Gaussians:

$$F_1 = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y+y_1)^2}{2\sigma^2}}, \quad (2)$$

$$F_2 = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-y_2)^2}{2\sigma^2}}. \quad (3)$$

Here  $\sigma$  is the width of Gaussian distribution, and  $y_1$  and  $y_2$  are the locations of centers of the target and projectile emitting sources.

$F_3$  is the distribution function of the two dimensional flow, which is given by<sup>[3, 4]</sup>:

$$F_3 = \frac{g\tau_f R_f^2 K}{8\pi} \int_{m_t^{l_o}}^{m_t^{h_i}} dm_t^2 m_t I_0(\alpha) \times \int_{-\eta_0}^{\eta_0} d\eta_1 \cosh(y - \eta_1) e^{\mu/T} e^{-\tilde{\alpha} \cosh(y - \eta_1)}. \quad (4)$$

Here  $m_t^{l_o}$  and  $m_t^{h_i}$  are the experimental limits in which the spectrum is measured, and  $R_f$  is the freeze-out radius. The longitudinal extend of the fireball is fixed via the finite interval  $(-\eta_0, \eta_0)$ ,  $I_0$  is the modified Bessel function.  $\tilde{\alpha} = (m_t/T) \cosh \eta_t$  and  $\alpha = (p_t/T) \sinh \eta_t$ ,  $\eta_t = \tanh^{-1} \beta_t$  is the average rapidity of transverse flow, and  $\beta_t$  is the average velocity of transverse flow (here we give  $\beta_t = 0.5c$ ).

Related to the two dimensional flow, short explanation is necessary. The geometry of the freeze-out hyper-surface  $\sigma_f$  of the two dimensional flow is fixed as follows: we take a surface of constant proper time  $\tau = \tau_f$  in the time direction. In the  $\eta_1$  direction, the freeze-out volume extends only to a maximum space-time rapidity  $\eta_0$ , which is required by the finite available total energy and breaks the longitudinal boost-invariance proposed by Bjorken<sup>[14]</sup>. In the transverse direction the boundary is given by  $R_f$ , which describes a cylindrical fireball in the  $(\eta, r)$  space. A detailed discussion can be found in Refs. [3, 4].

### 3 Comparison with experimental data

HCFM describes experimental charged particle distribution for Au-Au central collisions in the AGS energy region rather well. The contributions from the fragmentation regions can be ignored, and Eq. (1) can be simplified to:

$$\frac{dn}{dy} = n_3 F_3. \quad (5)$$

The results from HCFM are consistent with the experimental data for Au-Au collisions in the AGS energy region (momenta of 4, 6, 8, 11.6 AGeV/c). This indicates that our thermalization component model (TCM) reduces in this energy region to Heinz's collective flow model (HCFM). The reason seems to be that at AGS the nucleus stopping power is very strong and the phase space small, so the particles can be almost completely thermalized in the whole phase space. The same situation is true for Pb+Pb interactions in the SPS energy region below 30 AGeV.

With increasing collision energies (above

30 AGeV), the HCFM calculations systematically underestimate the experimental data at the two symmetric tails of the distribution (see Fig. 1). This phenomenon can be explained by the nuclei's penetrability. The higher the collision energies, the more transparent the nuclei, and the larger the extension of the phase space of the produced particles. After thermalization collective flow is formed at the central rapidity region and becomes one part of the whole phase space. The distributions of the non-thermalized charged hadrons are represented by Gaussian distributions.

Since June 2000, the Relativistic Heavy-Ion Collider (RHIC) has opened a new energy region for the study of multi-hadron production. We have analyzed the experimental charged particle distributions in Au-Au collisions in the RHIC energy region from 19.6 to 200 AGeV.

The rapidity distribution of the charged particles can be calculated with Eq. (1). It can be transformed into the pseudo-rapidity distribution just by multiplying it with a factor (the Jacobian)<sup>[12]</sup>:

$$\frac{dn}{d\eta} = \frac{dn}{dy} \sqrt{1 - \left(\frac{m}{m_T \cosh y}\right)^2}. \quad (6)$$

We fitted the available experimental data for the RHIC energy region with our TCM model and present in Fig. 2(a) comparison of the measured and calculated distributions. One can see that the TCM calculations (shown by the solid lines) are in accordance with the experimental results. The contributions from the three components are given by dotted lines. The percentage of the charged hadron production from the thermalization regions in the AGS, SPS and RHIC energy regions is presented in Fig. 3. It is obvious that the percentage of the produced particles from the thermalization region decreases with the energy increase. Most of the produced particles at AGS come from the thermalization region. The reduction trend becomes weaker and seems to reach saturation when  $\sqrt{s}$  reaches 62.5 GeV in the RHIC energy region. The detailed fit-results of our TCM with experimental data are shown in Table 1.

Table 1. The fit results of TCM with the experimental data from the SPS and RHIC energy regions.

	$E_{\text{lab}}$	$\eta_0$	$y_{1,2}$	$n_1 + n_2$	$n_3$	$n_3/(n_1 + n_2 + n_3)$
SPS	30	1.33	2.1	16	256	94.13%
	40	1.4	2.05	22	301	93.19%
	80	1.4	2.0	64	392	85.97%
	158	1.38	2.0	100	507	83.52%
RHIC	$\sqrt{s}$	$\eta_0$	$y_{1,2}$	$n_1 + n_2$	$n_3$	$n_3/(n_1 + n_2 + n_3)$
	19.6	1.85	$\pm 2.6$	370	1310	77.99%
	62.4	2.47	$\pm 3.15$	670	2157	76.30%
	130	2.62	$\pm 3.45$	1100	3016	73.28%
	200	2.8	$\pm 3.62$	1320	3639	73.38%

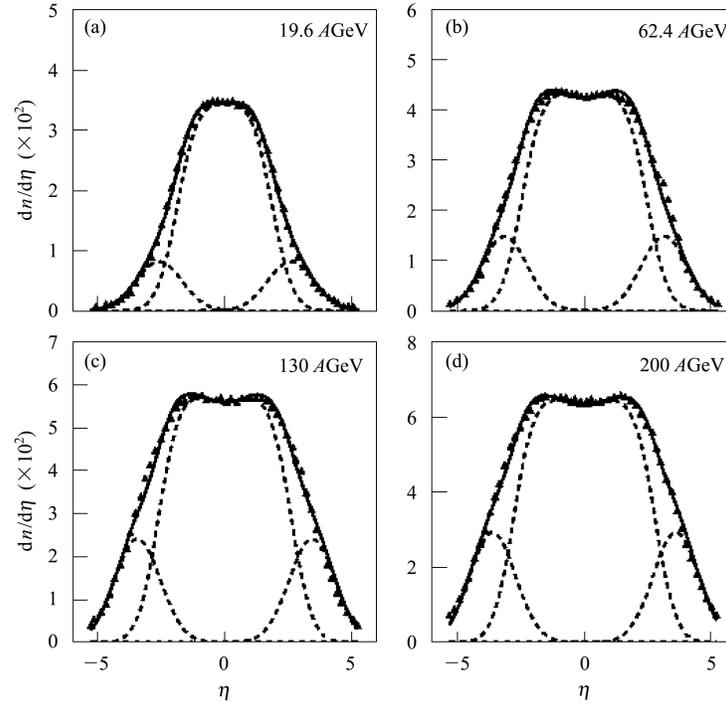


Fig. 2. The pseudo-rapidity distributions at  $\sqrt{s}=19.6, 62.4, 130, 200$  GeV. for Au+Au collisions. Experimental data are given by triangles. The solid lines are the results of the TCM, which is the sum of the contributions of the three components.

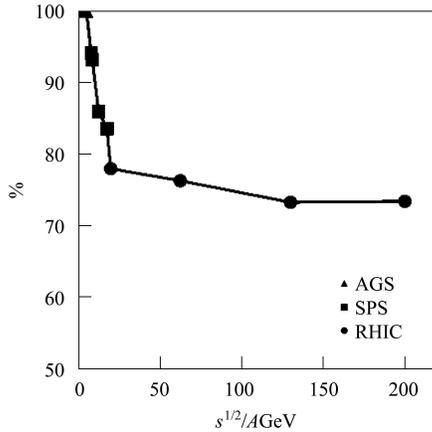


Fig. 3. The dependence of the percentage of the charged hadron production from the thermalization regions on the collision energies.

The PHOBOS Collaboration, working at RHIC, has presented many experimental data<sup>[6–11]</sup> at different centralities, including Au-Au collisions and Cu-Cu collisions at  $\sqrt{s}=62.4$  and 200 GeV, respectively. It is found that the calculational results from TCM are consistent with the experimental data at different energies and different centralities. The results are presented in Fig. 4 and Table 2. The experimental data are taken from Refs. [6–11].

It is shown in Table 2 and Fig. 5 that the percentage of the particle production from the thermalization regions increases with the increase of the centralities at RHIC. From Fig. 5(a) it can be seen that

for  $\sqrt{s} = 62.4$  GeV the contribution from the thermalization region is appreciably larger in the smaller collision system (Cu+Cu) than that in the larger collision system (Au+Au). However, at  $\sqrt{s} = 200$  GeV the percentage of particle production from the thermalization region is almost independent of the size of the collision system.

Table 2. Fit results with the TCM at different centralities and different energies for the RHIC.

	$\sqrt{s}$	centrality	$\eta_0$	$y_{1,2}$	$n_3/(n_1+n_2+n_3)$
Au+Au	62.4	0–6	2.47	$\pm 3.15$	76.30%
		6–15	2.47	$\pm 3.2$	75.52%
		15–25	2.55	$\pm 3.3$	74.99%
		25–35	2.5	$\pm 3.3$	72.86%
		35–45	2.5	$\pm 3.3$	72.53%
		45–55	2.6	$\pm 3.5$	72.58%
	200	0–6	2.8	$\pm 3.62$	73.29%
		6–15	2.78	$\pm 3.62$	72.11%
		15–25	2.76	$\pm 3.62$	71.28%
		25–35	2.78	$\pm 3.7$	70.67%
Cu+Cu	62.4	35–45	2.82	$\pm 3.8$	70.96%
		45–55	2.84	$\pm 3.8$	70.63%
		0–6	2.65	$\pm 3.45$	79.08%
		6–15	2.64	$\pm 3.45$	78.50%
		15–25	2.61	$\pm 3.45$	77.21%
		25–35	2.6	$\pm 3.45$	75.91%
	200	35–45	2.56	$\pm 3.45$	73.98%
		0–6	2.8	$\pm 3.68$	73.82%
		6–15	2.8	$\pm 3.68$	72.13%
		15–25	2.8	$\pm 3.68$	71.09%
200	25–35	2.8	$\pm 3.68$	70.53%	
	35–45	2.8	$\pm 3.7$	69.74%	
	45–55	2.85	$\pm 3.75$	69.48%	

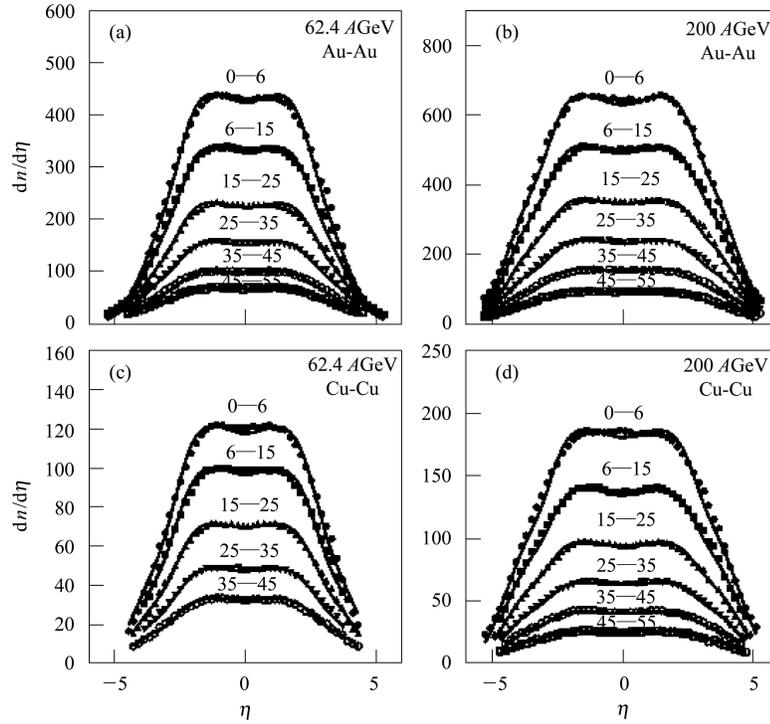


Fig. 4. The charged hadron pseudo-rapidity distributions at different centralities, at  $\sqrt{s} = 62.4$  GeV and 200 GeV for Au+Au and Cu+Cu collisions. Solid lines are the results from TCM, the experimental data are from PHOBOS-collaboration<sup>[6–11]</sup>.

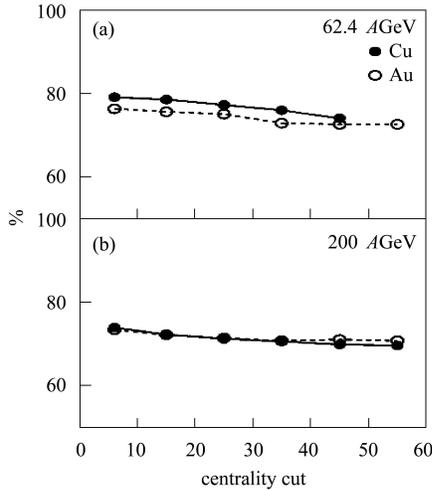


Fig. 5. The dependence of the percentage from the thermalization region on different centralities for  $\sqrt{s} = 62.4$  and 200 GeV.

In our TCM, the free parameters are the extension of collective flow,  $\eta_0$ , and the emission source positions of the two fragmentation areas  $y_{1,2}$ . In the case of symmetric collisions we have  $y_1 = -y_2$ . The values of velocity and temperature of the collective flow have been discussed in Refs. [3, 4, 16]. The values of  $n_i$  ( $i = 1, 2, 3$ ) are the numbers of particles from the fragmentation and the thermalization regions, respectively.

A detailed study shown in Fig. 6 leads to a linear relationship between the extension of the collective

flow  $\eta_0$  and  $\ln\sqrt{s}$ . Fitting the data from the SPS and RHIC energy regions one obtains:

$$\eta_0 = 0.40 \ln\sqrt{s} + 0.71. \quad (7)$$

One can then predict the extension of the thermalization region with the increased collision energy at LHC.

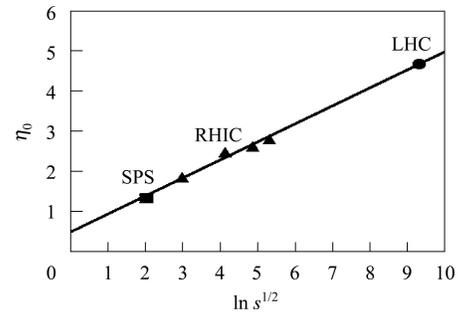


Fig. 6. The linear dependence of the extension of the thermalization region on  $\ln\sqrt{s}$ .

## 4 Summary and conclusions

Hadron multiplicities and their distributions are observables which can provide information on the nature, composition, and size of the medium from which they are originating. Of particular interest is the extent to which the measured particle yields are showing thermalization<sup>[18, 19]</sup>. In this work we analyzed the thermalization feature of high energy heavy ion collisions at RHIC.

HCFM fails to analyze the charged particle distributions if the collision energies increase beyond 30 GeV. The tail of the distribution of the charged particles at RHIC shows a deviation from the HCFM calculation, increasing with increasing energy. With the increased collision energies also the available phase space for the produced particles increases and thermalization might become more difficult.

On the other hand, the phenomena may suggest that something else happens, including for example the interaction mechanism, such as the onset of de-confinement in the early stage of the reaction with collision energies above 30 GeV, which has been mentioned in Ref. [20]. There, central Pb+Pb collisions were studied in the SPS energy range. At around 30 AGeV the ratio of strangeness to pion production shows a sharp maximum, the rate of increase of the produced pion multiplicity per wounded nucleon increases and the effective temperature of pions and kaons levels off to a constant value. These features are not reproduced by the present hadronic models, however there is a natural explanation in a reaction scenario with the onset of de-confinement in the early

stage of the reaction at SPS energy.

Collective flow in heavy-ion collisions is an unavoidable consequence of thermalization. The extension of the phase space of collective flow can reflect that of the thermalization region. It is found that the TCM can fit the experimental particle production data at the whole AGS, SPS and RHIC energy regions. The percentage of contributions of the particle production from the thermalization region is the largest at AGS, and decreases as collision energies increase at SPS and RHIC, but seems to reach saturation when  $\sqrt{s} = 62.4\text{--}200$  GeV at RHIC. It is also found that the extension of the flow shows a linear with  $\ln\sqrt{s}$ . From that one can predict the thermalization extension for future LHC experimental data.

It is shown from our study that the percentage of particle production from thermalization regions increases with the increase of the centralities at RHIC. The contribution from thermalization region is appreciably larger for the smaller collision system (Cu+Cu) at  $\sqrt{s} = 62.4$  GeV, but independent of the collision system at  $\sqrt{s} = 200$  GeV.

## References

- 1 Jacobs P, WANG X N. Progress in Particle and Nuclear Physics, 2005, **54**(2): 443—534
- 2 Mueller A H. Nucl. Phys. B, 2000, **572**: 227—240
- 3 Schnedermann E, Sollfrank J, Heinz U. Phys. Rev. C, 1993, **48**: 2462
- 4 Schnedermann E, Heinz U. Phys. Rev. C, 1993, **47**:1738
- 5 Blume C. Acta. Phys. Hung. A, 2005, **24**: 31—37
- 6 Back B B et al. Phys. Rev. Lett., 2004, **93**: 082301
- 7 Back B B et al. Phys. Rev. C, 2006, **74**: 021901
- 8 Back B B et al. Phys. Rev. Lett., 2001, **87**: 102303
- 9 Bearden I G et al. Phys. Rev. Lett., 2002, **88**: 202301
- 10 Roland G et al. J. Phys. G, 2004, **30**: S1133—S1138
- 11 Jipa A. Acta Phys. Hung. A, 2005, **22**: 121—137
- 12 Wolschin G. Eur. Phys. J. A, 1999, **5**: 85
- 13 FENG Sheng-Qin. Introduction to Multi-hadron Productions at High Energy. Beijing: Beijing Institute of Technology Press. 2005. 155—164 (in Chinese)
- 14 Bjorken J D. Phys. Rev. D, 1983, **27**: 140
- 15 FENG Sheng-Qin, LIU Feng, LIU Lian-Shou. Phys. Rev. C, 2001, **63**: 014901
- 16 FENG Sheng-Qin, LIU Feng, LIU Lian-Shou. HEP & NP, 2002, **26**: 1277 (in Chinese)
- 17 FENG Sheng-Qin, YUAN Xian-Bao, SHI Ya-Fei. Modern Phys. Lett. A, 2006, **21**(8): 663—673
- 18 YUAN Xian-Bao, FENG Sheng-Qin. HEP & NP, 2005, **29**: 371 (in Chinese)
- 19 Wong C Y. Introduction to High-Energy Heavy-Ion Collisions, Singapore: World Scientific Press, 1994. 24—26
- 20 Afanasiev S V et al. Phys. Rev. C, 2002, **66**: 054902