Hadron-quark phase transition in neutron stars*

YANG Fang(杨芳)[1)] SHEN Hong(申虹)[2)]

(Department of Physics, Nankai University, Tianjin 300071, China)

Abstract We study the hadron-quark phase transition in the interior of neutron stars. The relativistic mean field (RMF) theory is adopted to describe the hadronic matter phase, while the Nambu-Jona-Lasinio (NJL) model is used for the quark matter phase. We investigate the influence of the hadronic equation of state on the phase transition and neutron star properties. It is found that a neutron star possesses a large population of hyperons, but it is not dense enough to possess a pure quark core. Whether or not the mixed phase of hadronic and quark matter appears in the center of neutron stars depends on the RMF parameters used in the calculation.

Key words hadron-quark phase transition, equation of state, neutron stars

PACS 26.60.+c, 24.10.Jv, 24.85.+p

1 Introduction

It is expected that nuclear matter might undergo a deconfinement phase transition at very high density/temperature. Such phase transition has received much attention in neutron star physics[1—7]. Hyperons may appear around twice normal nuclear matter density through the weak interaction[8], which usually occur earlier than the hadron-quark phase transition. It has been pointed out by Glendenning[9] that the hadron-quark phase transition in neutron stars may proceed through a mixed phase of hadronic and quark matter over a finite range of pressures and densities according to the Gibbs criteria for phase equilibrium. Unfortunately, there is no single model which can be used to describe both phases and the dynamic process of the phase transition. In this work, we adopt the relativistic mean field (RMF) theory to describe the hadronic matter phase, while the Nambu-Jona-Lasinio (NJL) model is used for the quark matter phase, and then perform the Glendenning construction for the charge-neutral mixed phase where the hadronic and quark phases coexist. The choice of the NJL model is motivated by the fact that this model can successfully describe many aspects of quantum chromodynamics such as the non-perturbative vacuum structure and dynamical breaking of chiral symmetry[10—12].

The RMF theory has been successfully and widely used for the description of nuclear matter and finite nuclei[13—17]. It has also been applied to predict the equation of state (EOS) of dense matter for the use in supernovae and neutron stars[18, 19]. In the RMF approach, baryons interact through the exchange of scalar and vector mesons. The meson-nucleon coupling constants are generally determined by reproducing some nuclear matter properties or ground-state properties of finite nuclei. In the present work, we employ two successful parameter sets of the RMF model, NL3[20] and TM1[21]. However, there are large uncertainties in the meson-hyperon couplings due to limited experimental data. We use the vector meson-hyperon coupling constants derived from the quark model, and the scalar meson-hyperon coupling con-

Received 8 July 2008

* Supported by National Natural Science Foundation of China (10675064)

1) E-mail: yangfang022@mail.nankai.edu.cn

2) E-mail: songtc@nankai.edu.cn

stants constrained by reasonable hyperon potentials. Two additional strange mesons, σ^* and ϕ, were originally introduced in order to obtain the strong attractive hyperon-hyperon (YY) interaction deduced from the earlier measurement[22]. A recent observation of the double-Λ hypernucleus ${}^{6}_{\Lambda\Lambda}$He, called the Nagara event[23], suggests that the effective $\Lambda\Lambda$ interaction should be considerably weaker ($\triangle B_{\Lambda\Lambda}\sim 1\,\mathrm{MeV}$) than that deduced from the earlier measurement ($\triangle B_{\Lambda\Lambda}\sim 5\,\mathrm{MeV}$). For each parameter set of the nucleonic sector, we consider two cases of hyperon-hyperon interactions, the weak and strong YY interactions. By comparing the results with different parametrizations in the RMF model, we evaluate how sensitive the hadron-quark phase transition and neutron star properties are to the hadronic EOS used in the calculation.

2 Models

We adopt the relativistic mean field (RMF) theory to describe the hadronic matter phase. The effective Lagrangian is given by

$$\begin{aligned}\mathcal{L}_{RMF} = &\sum_{\mathrm{B}}\bar{\psi}_{\mathrm{B}}\left[\mathrm{i}\gamma_\mu\partial^\mu - m_{\mathrm{B}} - g_{\sigma\mathrm{B}}\sigma - g_{\sigma^*\mathrm{B}}\sigma^* - \right.\\ &\left. g_{\omega\mathrm{B}}\gamma_\mu\omega^\mu - g_{\phi\mathrm{B}}\gamma_\mu\phi^\mu - g_{\rho\mathrm{B}}\gamma_\mu\tau_i\rho_i^\mu\right]\psi_{\mathrm{B}} + \\ &\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - \\ &\frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3\left(\omega_\mu\omega^\mu\right)^2 - \\ &\frac{1}{4}R_{i\mu\nu}R_i^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_{i\mu}\rho_i^\mu + \frac{1}{2}\partial_\mu\sigma^*\partial^\mu\sigma^* - \\ &\frac{1}{2}m_{\sigma^*}^2\sigma^{*2} - \frac{1}{4}S_{\mu\nu}S^{\mu\nu} + \frac{1}{2}m_\phi^2\phi_\mu\phi^\mu + \\ &\sum_{\mathrm{l}}\bar{\psi}_{\mathrm{l}}\left[\mathrm{i}\gamma_\mu\partial^\mu - m_{\mathrm{l}}\right]\psi_{\mathrm{l}},\end{aligned} \quad (1)$$

where the sum on B runs over the baryon octet (p, n, Λ, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-), and the sum on l is over electrons and muons (e^- and μ^-). In the mean-field approximation, the meson field equations in uniform matter are written as

$$m_\sigma^2\sigma + g_2\sigma^2 + g_3\sigma^3 = -\sum_B \frac{g_{\sigma\mathrm{B}}}{\pi^2}\int_0^{k_{\mathrm{F}}^{\mathrm{B}}}\frac{m_{\mathrm{B}}^* k^2}{\sqrt{k^2+m_{\mathrm{B}}^{*2}}}\mathrm{d}k, \quad (2)$$

$$m_\omega^2\omega + c_3\omega^3 = \sum_{\mathrm{B}}\frac{g_{\omega\mathrm{B}}\left(k_{\mathrm{F}}^{\mathrm{B}}\right)^3}{3\pi^2}, \quad (3)$$

$$m_\rho^2\rho = \sum_{\mathrm{B}}\frac{g_{\rho\mathrm{B}}\tau_{3\mathrm{B}}\left(k_{\mathrm{F}}^{\mathrm{B}}\right)^3}{3\pi^2}, \quad (4)$$

$$m_{\sigma^*}^2\sigma^* = -\sum_{\mathrm{B}}\frac{g_{\sigma^*\mathrm{B}}}{\pi^2}\int_0^{k_{\mathrm{F}}^{\mathrm{B}}}\frac{m_{\mathrm{B}}^*}{\sqrt{k^2+m_{\mathrm{B}}^{*2}}}k^2\mathrm{d}k, \quad (5)$$

$$m_\phi^2\phi = \sum_{\mathrm{B}}\frac{g_{\phi\mathrm{B}}\left(k_{\mathrm{F}}^{\mathrm{B}}\right)^3}{3\pi^2}, \quad (6)$$

where $m_{\mathrm{B}}^* = m_{\mathrm{B}} + g_{\sigma\mathrm{B}}\sigma + g_{\sigma^*\mathrm{B}}\sigma^*$ is the effective mass of the baryon species B.

For neutron star matter consisting of a neutral mixture of baryons and leptons, the β equilibrium conditions without trapped neutrinos are given by

$$\mu_{\mathrm{p}} = \mu_{\Sigma^+} = \mu_{\mathrm{n}} - \mu_{\mathrm{e}}, \quad (7)$$

$$\mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_{\mathrm{n}}, \quad (8)$$

$$\mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_{\mathrm{n}} + \mu_{\mathrm{e}}, \quad (9)$$

$$\mu_\mu = \mu_{\mathrm{e}}, \quad (10)$$

and the charge neutrality condition is given by

$$n_{\mathrm{p}} + n_{\Sigma^+} = n_{\Sigma^-} + n_{\Xi^-} + n_{\mathrm{e}} + n_\mu. \quad (11)$$

We can solve the coupled equations self-consistently at a given baryon density $n_{\mathrm{B}} = n_{\mathrm{p}} + n_{\mathrm{n}} + n_\Lambda + n_{\Sigma^+} + n_{\Sigma^0} + n_{\Sigma^-} + n_{\Xi^0} + n_{\Xi^-}$, and then obtain the hadronic matter properties. In the present work, we employ NL3 and TM1 parameter sets, and take the naive quark model values for the vector meson-hyperon coupling constants,

$$\begin{aligned}&\frac{1}{3}g_{\omega\mathrm{N}} = \frac{1}{2}g_{\omega\Lambda} = \frac{1}{2}g_{\omega\Sigma} = g_{\omega\Xi},\\ &g_{\rho\mathrm{N}} = \frac{1}{2}g_{\rho\Sigma} = g_{\rho\Xi}, \quad g_{\rho\Lambda} = 0,\\ &2g_{\phi\Lambda} = 2g_{\phi\Sigma} = g_{\phi\Xi} = -\frac{2\sqrt{2}}{3}g_{\omega\mathrm{N}}, \quad g_{\phi N} = 0.\end{aligned} \quad (12)$$

The scalar coupling constants are chosen to give reasonable hyperon potentials $U_{\mathrm{Y}}^{(\mathrm{N})}(n_0) = g_{\sigma\mathrm{Y}}\sigma(n_0) + g_{\omega\mathrm{Y}}\omega(n_0)$, and we use $U_\Lambda^{(\mathrm{N})} = -28\,\mathrm{MeV}$, $U_\Sigma^{(\mathrm{N})} = +30\,\mathrm{MeV}$, and $U_\Xi^{(\mathrm{N})} = -18\,\mathrm{MeV}$[24—26]. The hyperon couplings to strange meson σ^* are restricted by the relation $U_\Xi^{(\Xi)} \simeq U_\Lambda^{(\Xi)} \simeq 2U_\Xi^{(\Lambda)} \simeq 2U_\Lambda^{(\Lambda)}$[27]. We consider the weak YY interactions $U_\Lambda^{(\Lambda)} \simeq -5\,\mathrm{MeV}$[28—30] and strong YY interactions $U_\Lambda^{(\Lambda)} \simeq -20\,\mathrm{MeV}$[22] for both NL3 and TM1 cases.

We adopt a three-flavor version of the NJL model to describe the deconfined quark phase. The Lagrangian is given by

$$\mathcal{L}_{\rm NJL} = \bar{q}(i\gamma_\mu \partial^\mu - m^0) q + \mathcal{L}_{\rm sym} + \mathcal{L}_{\rm det}, \quad (13)$$

$$\mathcal{L}_{\rm sym} = G \sum_{a=0}^{8} \left[(\bar{q}\lambda_a q)^2 + (\bar{q} i\gamma_5 \lambda_a q)^2 \right], \quad (14)$$

$$\mathcal{L}_{\rm det} = -K \{ \det[\bar{q}(1+\gamma_5) q] + \det[\bar{q}(1-\gamma_5) q] \}, \quad (15)$$

where $\mathcal{L}_{\rm sym}$ and $\mathcal{L}_{\rm det}$ denote the four-point interaction and six-point interaction, respectively. This model has five parameters, namely, the current quark masses m_q^0 and m_s^0, the coupling constants K and G, and the momentum cutoff Λ. In the present calculation, we employ the parameters given in Ref. [31], $m_q^0 = 5.5$ MeV, $m_s^0 = 140.7$ MeV, $\Lambda = 602.3$ MeV, $G\Lambda^2 = 1.835$, and $K\Lambda^5 = 12.36$. In the NJL model, the quark gets constituent quark mass by spontaneous chiral symmetry breaking. The constituent quark mass m_i^* satisfies the following gap equation

$$m_i^* = m_i^0 - 4G\langle \bar{q}_i q_i \rangle + 2K \langle \bar{q}_j q_j \rangle \langle \bar{q}_k q_k \rangle. \quad (16)$$

The quark condensate $C_i = \langle \bar{q}_i q_i \rangle$ is given by

$$C_i = -\frac{3}{\pi^2} \int_{k_F^i}^{\Lambda} \frac{m_i^*}{\sqrt{k^2 + m_i^{*2}}} k^2 dk, \quad (17)$$

where k_F^i denotes the Fermi momentum of the quark flavor i.

For the quark matter consisting of a neutral mixture of quarks (u, d, and s) and leptons (e and μ) in β equilibrium, the charge neutrality condition is expressed as

$$\frac{2}{3} n_u - \frac{1}{3}(n_d + n_s) - n_e - n_\mu = 0. \quad (18)$$

The β equilibrium conditions are given by

$$\mu_s = \mu_d = \mu_u + \mu_e, \quad (19)$$

$$\mu_\mu = \mu_e. \quad (20)$$

The coupled equations can be solved self-consistently at a given baryon density $n_B = (n_u + n_d + n_s)/3$, and then we can obtain the quark matter properties.

3 Hadron-quark phase transition and neutron star properties

It has been discussed extensively in the literature that a mixed phase of hadronic and quark matter could exist over a finite range of pressures and densities according to the Gibbs criteria for phase equilibrium. In the mixed phase, the local charge neutrality condition is replaced by a global one. This means that both hadronic and quark matter are allowed to be separately charged. The condition of global charge neutrality is expressed as

$$\chi n_c^{\rm QP} + (1-\chi) n_c^{\rm HP} = 0, \quad (21)$$

where χ is the volume fraction occupied by quark matter in the mixed phase, which increases from $\chi = 0$ in the pure hadronic phase to $\chi = 1$ in the pure quark phase. $n_c^{\rm HP}$ and $n_c^{\rm QP}$ denote the charge densities of hadronic phase and quark phase, respectively. The Gibbs condition for phase equilibrium at zero temperature is then given by

$$P_{\rm HP}(\mu_n, \mu_e) = P_{\rm QP}(\mu_n, \mu_e) = P_{\rm MP}. \quad (22)$$

In Fig. 1 we plot the full EOS in the form $P = P(\varepsilon)$. The mixed phase part of the EOS is shaded gray, where the pressure varies continuously. It is shown that the onset and width of the mixed phase depend on the RMF parameters used in the calculation. The NL3 model leads to earlier appearance of the mixed phase than the TM1 model, and the weak YY interaction favors earlier onset of the mixed phase than the strong YY interaction. This is mainly because that a harder hadronic EOS prefers an earlier hadron-quark phase transition.

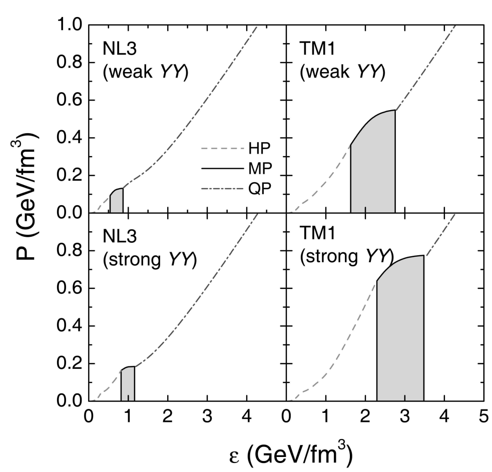

Fig. 1. The full EOS of neutron star matter in the form of pressure P versus energy density ε. The shaded regions correspond to the mixed phase (MP). The dashed and dot-dashed lines show the pressures of hadronic phase (HP) and quark phase (QP), respectively.

In Fig. 2, we present the mass-radius relation of neutron stars by solving the Tolman-Oppenheimer-Volkoff (TOV) equation with the EOS over a wide

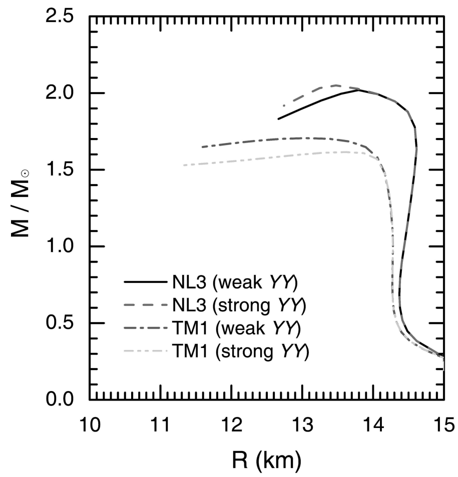

Fig. 2. The mass-radius relation for neutron stars.

density range. Since the pressure and density inside neutron stars decrease from the center to the surface, the most possible region where the deconfined quark phase can exist is the center of the neutron star with maximum mass. When the central density is larger than the critical density of the appearance of quark matter, the hadron-quark phase transition can occur in the core of neutron stars. We find that the neutron star can possess a mixed phase core, but it is not dense enough to possess a pure quark core with the NL3 model. On the other hand, the neutron star is only composed of hadronic matter with the TM1 model. By comparing the results of different cases, we can see the influence of the hadronic EOS on the hadron-quark phase transition and neutron star properties.

4 Summary

We have studied the hadron-quark phase transition which may occur in the core of massive neutron stars. With a definite EOS for the quark phase, we examine the influence of the hadronic EOS on the deconfinement phase transition and neutron star properties. In the present work, we have used NL3 and TM1 parameter sets. For each parameter set of the nucleonic sector, we consider two cases of hyperon-hyperon interactions, the weak and strong YY interactions. With the NL3 model, the mixed phase can exist in the core of massive neutron stars, but no pure quark phase can exist. For the TM1 model, the neutron star is not dense enough to possess the mixed phase, and therefore the hadron-quark phase transition could not occur inside neutron stars in this case. We conclude that whether the mixed phase of hadronic and quark matter exist in the core of neutron stars depends on the RMF parameters used in the calculation.

References

1 Weber F. Prog. Part. Nucl. Phys., 2005, **54**: 193—288
2 Prakash M, Bombaci I, Prakash M et al. Phys. Rep., 1997, **280**: 1—77
3 Heiselberg H, Hjorth-Jensen M. Phys. Rep., 2000, **328**: 237—327
4 Schertler K, Leupold S, Schaffner-Bielich J. Phys. Rev. C, 1999, **60**: 025801
5 Steiner A W, Prakash M, Lattimer J M. Phys. Lett. B, 2000, **486**: 239—248
6 Burgio G F, Baldo M, Sahu P K et al. Phys. Rev. C, 2002, **66**: 025802
7 Shovkovy I, Hanauske M, HUANG M. Phys. Rev. D, 2003, **67**: 103004
8 Schaffner J, Mishustin I N. Phys. Rev. C, 1996, **53**: 1416—1429
9 Glendenning N K. Phys. Rev. D, 1992, **46**: 1274—1287
10 Buballa M. Phys. Rep., 2005, **407**: 205—376
11 Vogl U, Weise W. Prog. Part. Nucl. Phys., 1991, **27**: 195—272
12 Hatsuda T, Kunihiro T. Phys. Rep., 1994, **247**: 221—367
13 Serot B D, Walecka J D. Adv. Nucl. Phys., 1986, **16**: 1—327
14 Gambhir Y K, Ring P, Thimet A. Ann. Phys., 1990, **198**: 132—179
15 Hirata D, Sumiyoshi K, Carlson B V et al. Nucl. Phys. A, 1996, **609**: 131—146
16 REN Z Z, TAI F, CHEN D H. Phys. Rev. C, 2002, **66**: 064306
17 SHEN H, YANG F, Toki H. Prog. Theor. Phys., 2006, **115**: 325—335
18 SHEN H, Toki H, Oyamatsu K et al. Nucl. Phys. A, 1998, **637**: 435—450
19 SHEN H. Phys. Rev. C, 2002, **65**: 035802
20 Lalazissis G A, König J, Ring P. Phys. Rev. C, 1997, **55**: 540—543
21 Sugahara Y, Toki H. Nucl. Phys. A, 1994, **579**: 557—572
22 Schaffner J, Dover C B, Gal A et al. Phys. Rev. Lett., 1993, **71**: 1328—1331
23 Takahashi H et al. Phys. Rev. Lett., 2001, **87**: 212502
24 Millener D J, Dover C B, Gal A. Phys. Rev. C, 1988, **38**: 2700—2708
25 Schaffner-Bielich J, Gal A. Phys. Rev. C, 2000, **62**: 034311
26 Bunta J K, Gmuca S. Phys. Rev. C, 2004, **70**: 054309
27 Schaffner J, Dover C B, Gal A et al. Ann. Phys., 1994, **235**: 35—76
28 SONG H Q, SU R K, LU D H et al. Phys. Rev. C, 2003, **68**: 055201
29 QIAN W L, SU R K, SONG H Q. J. Phys. G: Nucl. Part. Phys., 2004, **30**: 1893—1903
30 Bednarek I, Manka R. J. Phys. G: Nucl. Part. Phys., 2005, **31**: 1009—1023
31 Rehberg P, Klevansky S P, Hüfner J. Phys. Rev. C, 1996, **53**: 410–429