

Study of various charged ρ -meson masses in asymmetric nuclear matter^{*}

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Abstract We study the effective masses of ρ -mesons for different charged states in asymmetric nuclear matter (ANM) using the Quantum Hadrodynamics II model. The closed form analytical results are presented for the effective masses of ρ -mesons. We have shown that the different charged ρ -mesons have mass splitting similar to various charged pions. The effect of the Dirac sea is also examined, and it is found that this effect is very important and leads to a reduction of the different charged ρ -meson masses in ANM.

Key words asymmetric nuclear matter, ρ -meson, effective mass, Dirac sea

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1 Introduction

As is well known, pions and ρ -mesons are both iso-vector mesons. The two introduced meson fields have an obvious contribution to ANM. Only by considering them can one explain the electromagnetic characteristic of finite nuclei. In recent years, pion and ρ -meson properties have attracted great interest. The experimental data of CERN^[1, 2] and TAGX^[3] seem to support the reduction of the ρ -meson mass in a dense medium; the latter ones in particular also give a quantitative result when the density of the nucleon medium equals $0.7\rho_0$, ρ_0 is the saturation density, and the mass of a neutral ρ -meson reduces to 610 MeV. Subsequently, more efforts have been devoted to studying the effective mass of the ρ -meson in a dense medium. There have been many published papers^[4–8] which employ different models and different methods for giving a reasonable explanation of experiments. Refs. [4–7] also pointed out that the Dirac vacuum gave an important correction to the self-energy of the ρ -meson and led to a reduction of the ρ -meson mass in dense matter. Recently in Ref. [9], it was found that the masses split for the various charged states of the pion in ANM. Such mode splitting is, in fact, a generic feature of all the isovector mesons. Therefore in this paper we study different

charged ρ -meson mass splitting cases in ANM and examine the effects of the Dirac sea on different charged ρ -meson masses.

2 Theory

The Lagrangian density of the QHD-II model is given^[10]

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\gamma_\mu\partial^\mu - M)\psi - \frac{1}{2}g_\rho\bar{\psi}\gamma_\mu(\boldsymbol{\tau}\cdot\boldsymbol{\Phi}_\rho^\mu)\psi + g_s\bar{\psi}\Phi_s\psi - \\ & g_\omega\bar{\psi}\gamma_\mu\Phi_\omega^\mu\psi - ig_\pi\bar{\psi}\gamma_5(\boldsymbol{\tau}\cdot\boldsymbol{\Phi}_\pi)\psi + \\ & \frac{1}{2}(\partial_\mu\Phi_s\partial^\mu\Phi_s - m_s^2\Phi_s^2) + \frac{1}{2}(\partial_\mu\boldsymbol{\Phi}_\pi - g_\rho\boldsymbol{\Phi}_{\rho\mu} \times \boldsymbol{\Phi}_\pi) \cdot \\ & (\partial^\mu\boldsymbol{\Phi}_\pi - g_\rho\boldsymbol{\Phi}_\rho^\mu \times \boldsymbol{\Phi}_\pi) - \frac{1}{2}m_\pi^2\boldsymbol{\Phi}_\pi^2 + \frac{1}{2}g_{\Phi\pi}m_s\Phi_s\boldsymbol{\Phi}_\pi^2 - \\ & \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}\mathbf{B}_{\mu\nu}\cdot\mathbf{B}^{\mu\nu} + \frac{1}{2}m_\omega^2\Phi_\omega\Phi_\omega^\mu + \\ & \frac{1}{2}m_\rho^2\boldsymbol{\Phi}_{\rho\mu}\cdot\boldsymbol{\Phi}_\rho^\mu, \end{aligned} \quad (1)$$

where

$$G_{\mu\nu} = \partial_\mu\Phi_{\omega\nu} - \partial_\nu\Phi_{\omega\mu}, \quad (2)$$

$$\mathbf{B}_{\mu\nu} = \partial_\mu\boldsymbol{\Phi}_{\rho\nu} - \partial_\nu\boldsymbol{\Phi}_{\rho\mu} - g_\rho\boldsymbol{\Phi}_{\rho\mu}\cdot\boldsymbol{\Phi}_{\rho\nu}. \quad (3)$$

Here, ψ , $\boldsymbol{\Phi}_\pi$, Φ_s , $\boldsymbol{\Phi}_\rho$, and Φ_ω represent the nucleon, π , σ , ρ , and ω fields, respectively, and their masses are

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denoted by M , m_π , m_s , m_ρ , and m_ω . This model is a successful relativistic model for describing the properties of both nuclear matter and finite nuclei^[11].

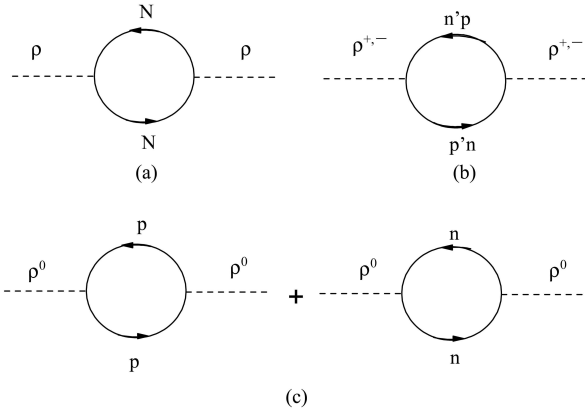


Fig. 1. One-loop self-energy diagrams for the ρ -mesons.

In this model, the ρ -N dynamics is described by

$$\mathcal{L}_{\rho N} = -\frac{1}{2}g_\rho \bar{\psi} \gamma_\mu (\boldsymbol{\tau} \cdot \boldsymbol{\Phi}_\rho^\mu) \psi. \quad (4)$$

where g_ρ is the ρ -N coupling constant. At the self-energy level, Eq. (4) will generate the exchange diagram, as shown in Fig. 1(a), and that will involve various combinations of neutron (n) and proton (p) for the various charged states of ρ -mesons, as shown in Fig. 1(b, c). According to the Feynman rules, the self-energy of the ρ -mesons reads

$$\Pi^{*\mu\nu}(q) = -i \int \frac{d^4k}{2\pi^4} \text{Tr} [i\Gamma^\mu iG_i(k+q) i\Gamma^\nu iG_j(k)], \quad (5)$$

where the subscripts i and j denote either p or n. $\Gamma^\mu = -\frac{1}{2}g_\rho \gamma_\mu$ is the interaction vertex.

The propagator of a nucleon is

$$G_i(k) = G_i^F(k) + G_i^D(k), \quad (6)$$

where

$$G_i^F(k) = \frac{\not{k} + M_i^*}{k^2 - M_i^{*2} + i\epsilon}, \quad (7)$$

$$G_i^D(k) = \frac{i\pi(\not{k} + M_i^*)}{E_i^*} \delta(k_0 - E_i^*) \theta(k_i^F - |k|). \quad (8)$$

Here, $G_i^F(k)$ and $G_i^D(k)$ represent the free and density dependent parts of the propagator. In Eq. (8) k_i^F denotes either the proton or neutron Fermi momenta, M_i^* is the effective nucleon mass, its numerical result has been given in Ref. [12] under the one-loop approximation and mean field. The two approximate results are very close in lower density, while as the nuclear density increases, the result of the one-loop approximation will be a little lower than that of the mean field. In this paper, for the purpose of revealing

the Dirac sea effect qualitatively and preliminarily, we adopt the result under the mean field^[11]:

$$M_i^* = M_i - \frac{g_s^2}{m_s^2} (\rho_p^s + \rho_n^s), \quad (9)$$

$$\rho_i^s = \frac{M_i^*}{2\pi^2} \left[E_i^* k_i - M_i^{*2} \ln \left(\frac{E_i^* + k_i}{M_i^*} \right) \right]. \quad (10)$$

In Eq. (9), it is easily seen that the modification of nucleon masses does not distinguish between p and n. Therefore, for the moment, we can neglect the explicit symmetry breaking (p, n mass difference), i.e., $M_p^* = M_n^* = M^*$.

Because of the self-energy correction, the propagator of the ρ -meson in a dense medium reads

$$D_\rho^{\mu\nu}(q) = -\frac{P_L^{\mu\nu}}{q^2 - m_\rho^2 - \Pi_L^\rho} - \frac{P_T^{\mu\nu}}{q^2 - m_\rho^2 - \Pi_T^\rho}, \quad (11)$$

where $P_L^{\mu\nu}$ and $P_T^{\mu\nu}$ are the projection tensors. Π_T^ρ and Π_L^ρ are the longitudinal and the transverse components of the ρ -meson self-energy, respectively. In the limit $\mathbf{q} \rightarrow 0$, it can be proven that^[6, 13]

$$\Pi_L^\rho(q_0, \mathbf{q} \rightarrow 0) = \Pi_T^\rho(q_0, \mathbf{q} \rightarrow 0) = -\frac{1}{3} \Pi_{\rho\mu}^\mu(q_0, \mathbf{q} \rightarrow 0). \quad (12)$$

The effective mass of the ρ meson is defined as the pole of the propagator $D_\rho^{\mu\nu}(q)$ in the limit $\mathbf{q} \rightarrow 0$. Now let us calculate $\Pi_{\rho\mu}^\mu(q)$ for the purpose of this paper. Substituting Eq. (6) into Eq. (5), the expression for ρ -meson self-energy takes the form

$$\begin{aligned} \Pi_{\rho\mu}^{*\mu}(q) &= -i \frac{g_\rho^2}{4} \int \frac{d^4k}{2\pi^4} T^{FF} + T^{FD+DF} + T^{DD} = \\ &= \Pi^{*FF}(q) + \Pi^{*DF+FD}(q) + \Pi^{*DD}(q). \end{aligned} \quad (13)$$

For ρ^\pm the coupling constant g_ρ is replaced by $\sqrt{2}g_\rho$. T^{**} is the trace factor, whose expressions will be discussed later. As described in Ref. [8], the term T^{DD} contains the product of two δ functions $[\Gamma G^D(k+q) \Gamma G^D(k)]$. This means that the ρ -meson can decay into a nucleon-antinucleon pair which happens only in the high momentum limit. Therefore, T^{DD} is neglected in the present calculation. Thus, Eq. (13) can now be written as

$$\Pi_{\rho\mu}^{*\mu}(q) = \Pi^{*FF}(q) + \Pi^{*DF+FD}(q). \quad (14)$$

The first term of Eq. (13) is the same for the different charged states of ρ -mesons.

$$\begin{aligned} T^{FF} &= 2\text{Tr} [\gamma^\mu G^F(k+q) \gamma_\mu G^F(k)] = \\ &= 16 \left[\frac{2M^{*2} - (k+q) \cdot k}{(k^2 - M^{*2})((k+q)^2 - M^{*2})} \right]. \end{aligned} \quad (15)$$

where the factor 2 follows from isospin symmetry for $M_n^* = M_p^*$. Substituting Eq. (15) into Eq. (13), it

is observed that $\Pi^{*FF}(q)$ is quadratically divergent. To eliminate these divergences, we need to renormalize $\Pi^{*FF}(q)$. Here we adopt Feynman's parametrization and dimensional regularization technique^[14–16] to regularize $\Pi^{*FF}(q)$ with the following results (the details are discussed in the Appendix).

$$\begin{aligned} \Pi^{*R}(q) = & \frac{g_\rho^2}{2\pi^2} \left[\frac{1}{2}(M^2 - M^{*2}) - \right. \\ & \left. \frac{3}{2} \int_0^1 dx q^2 x(1-x) \ln \frac{M^{*2} - q^2 x(1-x)}{M^2 - m_\rho^2 x(1-x)} + \right. \\ & \left. (q^2 - m_\rho^2) \int_0^1 dx \frac{3m_\rho^2 x(1-x)}{2[m_\rho^2 x(1-x) - M^2]} \right]. \quad (16) \end{aligned}$$

If we consider that $(M^* - M)$ is small enough, then Eq. (16) can be approximatively written as

$$\Pi^{*R}(q) \simeq -A + Bq^2, \quad (17)$$

where

$$\begin{aligned} A = & \frac{g_\rho^2}{4\pi^2} \left[(M^{*2} - M^2) + \int_0^1 dx \frac{3m_\rho^4 x(1-x)}{m_\rho^2 x(1-x) - M^2} \right], \\ B = & \frac{g_\rho^2}{4\pi^2} \frac{3m_\rho^2 x(1-x)}{m_\rho^2 x(1-x) - M^2}. \quad (18) \end{aligned}$$

The $\Pi^{*(FD+DF)}(q)$ part of self-energy Eq. (14) is different for the different charged states of ρ -mesons. For a ρ^0 -meson (see Fig. 1(c))

$$\begin{aligned} T^{0(FD+DF)} = & \text{Tr} [\gamma^\mu G_\rho^F(k+q) \gamma_\mu G_\rho^D(k) + \\ & \gamma^\mu G_\rho^D(k+q) \gamma_\mu G_\rho^F(k) + \text{p} \rightarrow \text{n}]. \quad (19) \end{aligned}$$

Substituting Eq. (19) into Eq. (13), we can get

$$\Pi^{*0(FD+DF)}(q) = 2g_\rho^2 \int \frac{d^3k}{(2\pi)^3 E^*} \mathbf{S}. \quad (20)$$

where

$$\mathbf{S} = \left[\frac{M^{*2} q^2 + 2(k \cdot q)^2}{q^4 - 4(k \cdot q)^2} \right] (\theta_p + \theta_n). \quad (21)$$

In the long wavelength limit, we neglect the term q^4 compared with the term $4(k \cdot q)^2$ from the denominator of \mathbf{S} in Eq. (21). After a straightforward calculation, we get

$$\begin{aligned} \Pi^{*0(FD+DF)}(q) = & -\frac{g_\rho^2}{4\pi^2} \left[k_p E_p^* - \frac{M^{*2} c_0}{2} \ln \left| \frac{c_0 + v_p}{c_0 - v_p} \right| + \right. \\ & \left. k_n E_n^* - \frac{M^{*2} c_0}{2} \ln \left| \frac{c_0 + v_n}{c_0 - v_n} \right| \right], \quad (22) \end{aligned}$$

where $v_{p,n} = k_{p,n}^F/E_{p,n}^*$, $E_{p,n}^* = \sqrt{M^{*2} + k_{p,n}^{F2}}$, and $c_0 = q_0/|q|$. By using the power series expansion method, the approximate results of Eq. (22) are given

below:

$$\Pi^{*0(FD+DF)}(q) \simeq C \frac{q^2}{q_0^2} + D, \quad (23)$$

where

$$\begin{aligned} C = & -\frac{g_\rho^2 M^{*2}}{12\pi^2} \left(\frac{k_p^3}{E_p^{*3}} + \frac{k_n^3}{E_n^{*3}} \right), \\ D = & -\frac{g_\rho^2}{4\pi^2} \left(k_p E_p^* - \frac{k_p}{E_p^*} M^{*2} - \frac{k_p^3}{3E_p^{*3}} M^{*2} + \right. \\ & \left. k_n E_n^* - \frac{k_n}{E_n^*} M^{*2} - \frac{k_n^3}{3E_n^{*3}} M^{*2} \right). \quad (24) \end{aligned}$$

For a ρ^+ -meson (see Fig. 1(b))

$$\begin{aligned} T^{+(FD+DF)} = & \text{Tr} [\gamma^\mu G_\rho^F(k+q) \gamma_\mu G_n^D(k) + \\ & \gamma^\mu G_p^D(k+q) \gamma_\mu G_n^F(k)]. \quad (25) \end{aligned}$$

Substituting Eq. (25) into Eq. (13), we can get

$$\begin{aligned} \Pi^{*+(FD+DF)}(q) = & 2g_\rho^2 \int \frac{d^3k}{(2\pi)^3 E^*} [\mathbf{S} + \mathbf{dS}] = \\ & \Pi^{*0(FD+DF)}(q) + \delta \Pi^{*(FD+DF)}(q), \quad (26) \end{aligned}$$

where

$$\mathbf{dS} = \left[\frac{(2M^{*2} + q^2)(k \cdot q)}{q^4 - 4(k \cdot q)^2} \right] (\theta_p - \theta_n). \quad (27)$$

With the same consideration as Eq. (20), we get

$$\begin{aligned} \delta \Pi^{*(FD+DF)}(q) = & -\frac{g_\rho^2}{4\pi^2} \frac{(q^2 + 2M^{*2})}{|q|} \times \\ & \left[\frac{1}{2} E_p^* \ln \left| \frac{c_0 + v_p}{c_0 - v_p} \right| - \frac{M^*}{\sqrt{c_0^2 - 1}} \arctan \left(\frac{k_p \sqrt{c_0^2 - 1}}{c_0 M^*} \right) \right] \simeq \\ & E \frac{q^2}{q_0} + F \frac{1}{q_0}, \quad (28) \end{aligned}$$

where

$$\begin{aligned} E = & -\frac{g_\rho^2}{12\pi^2} \frac{k_p^3 - k_n^3}{M^{*2}}, \\ F = & -\frac{g_\rho^2}{6\pi^2} (k_p^3 - k_n^3). \quad (29) \end{aligned}$$

For a ρ^- -meson (see Fig. 1(b))

$$\begin{aligned} T^{-(FD+DF)} = & \text{Tr} [\gamma^\mu G_n^F(k+q) \gamma_\mu G_p^D(k) + \\ & \gamma^\mu G_n^D(k+q) \gamma_\mu G_p^F(k)], \quad (30) \end{aligned}$$

$$\begin{aligned} \Pi^{*-(FD+DF)}(q) = & 2g_\rho^2 \int \frac{d^3k}{(2\pi)^2 E^*} [\mathbf{S} - \mathbf{dS}] = \\ & \Pi^{*0(FD+DF)}(q) - \delta \Pi^{*(FD+DF)}(q). \quad (31) \end{aligned}$$

According to the definition of effective mass^[6] and Eq. (14), we can get the effective masses of $\rho^{0,\pm}$ with-

out the Dirac sea

$$m_{\rho^0}^{*2} \simeq m_{\rho^0}^2 - \frac{C+D}{3}, \quad (32)$$

$$m_{\rho^\pm}^{*2} \simeq \frac{m_{\rho^\pm}^2 - \frac{C+D \pm F/m_{\rho^\pm}}{3}}{1 \pm \frac{E}{3m_{\rho^\pm}}}, \quad (33)$$

and the effective masses with the Dirac sea

$$m_{\rho^0}^{*2} \simeq \frac{m_{\rho^0}^2 - \frac{C+D-A}{3}}{1 + \frac{B}{3}}, \quad (34)$$

$$m_{\rho^\pm}^{*2} \simeq \frac{m_{\rho^\pm}^2 - \frac{C+D-A \pm F/m_{\rho^\pm}}{3}}{1 + \frac{B}{3} \pm \frac{E}{3m_{\rho^\pm}}}. \quad (35)$$

From Eq. (29), it can easily be shown that E and F are vanishing in symmetric nuclear matter, but not in ANM. They are responsible for the masses splitting for the different charged states of ρ -mesons.

3 Results and discussion

Typical values of the ρ -meson mass shifts at normal nuclear density ($\rho_0 = 0.17 \text{ fm}^{-3}$) for Pb-like nuclei ($\alpha = 0.2$) are $\Delta m_{\rho^0} = m_{\rho^0}^* - m_{\rho^0} = -0.968$, $\Delta m_{\rho^+} = -0.974$, and $\Delta m_{\rho^-} = -0.962 \text{ fm}^{-1}$ with Dirac vacuum correction, and the corresponding values are 0.062, 0.055, and 0.069 fm^{-1} without Dirac vacuum correction.

The numerical results of the effective masses which depend on the nuclear density ρ and asymmetry parameter α for the various charged states of ρ -mesons are shown in Fig. 2 and Fig. 3, where we choose the parameters as $M = 4.7585 \text{ fm}^{-1}$, $m_\rho = 3.902 \text{ fm}^{-1}$, $m_s = 2.7872 \text{ fm}^{-1}$, $g_\rho^2 = 36.8195$, and $g_s^2 = 91.6088$. We see from Fig. 2 that the various charged states of ρ -mesons have mass splitting phenomena. Ref. [9] notes that the π^- meson mass increases in nuclear matter, while the π^+ meson mass decreases at higher density, the π^- meson mass is obviously higher than the π^+ and π^0 meson mass (for PV coupling). Compared with pions, all of the ρ -meson masses grow with nuclear density without Dirac vacuum correction. When taking the Dirac sea effect into account, the effective masses of ρ -mesons for the various charged states decrease with nuclear density, which agrees with the experimental data; moreover the mass splitting phenomena become obscure. Fig. 3 depicts the variation relation of ρ^- , ρ^0 , ρ^+ meson masses along with the asymmetry parameter α at normal nuclear density. Obviously,

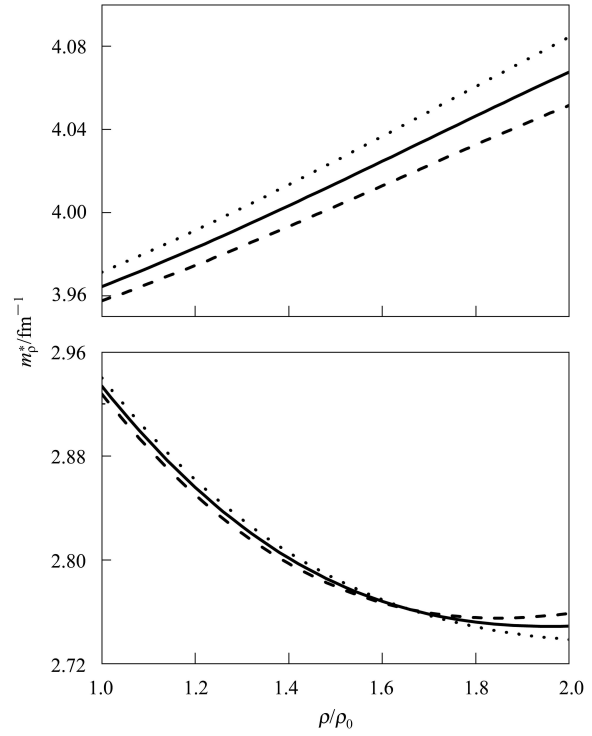


Fig. 2. Nuclear density (ρ) dependent effective ρ -meson masses at $\alpha=0.2$. The dotted, dashed, and solid curves, respectively, represent ρ^- , ρ^+ , and ρ^0 without (upper panel) and with (lower panel) the Dirac sea effect.

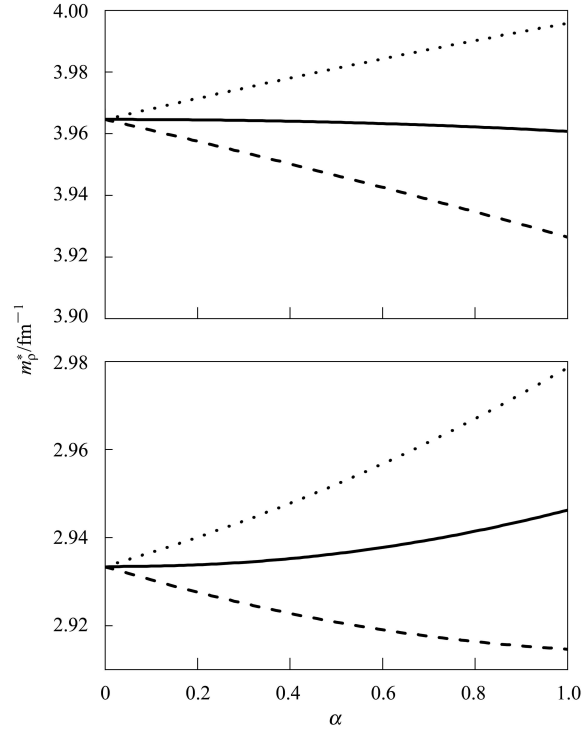


Fig. 3. Asymmetry parameter (α) dependent effective ρ -meson masses at $\rho = \rho_0$. The dotted, dashed, and solid curves, respectively, represent ρ^- , ρ^+ , and ρ^0 without (upper panel) and with (lower panel) the Dirac sea effect.

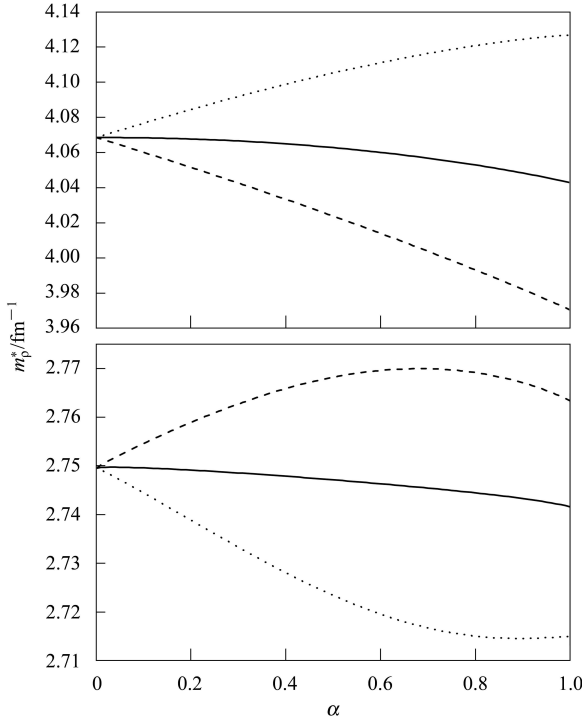


Fig. 4. Asymmetry parameter (α) dependent effective ρ -meson masses at $\rho = 2\rho_0$. The dotted, dashed, and solid curves, respectively, represent ρ^- , ρ^+ , and ρ^0 without (upper panel) and with (lower panel) the Dirac sea effect.

the ρ -mass splitting which depends on the asymmetry parameter is clear. Here, the ρ^- meson mass increases with α , the ρ^0 meson mass almost remains invariable, and the ρ^+ meson mass decreases with α . These phenomena are very similar to π^- , π^0 , π^+ mesons (for PV coupling)^[9]. In Fig. 4, when the nuclear density is twice the normal density, the trend of ρ^- and ρ^+ meson mass along with the asymmetry parameter will change obviously taking the Dirac sea effect into account.

In summary, using the QHD-II model, we have studied the effective masses for the various charged states of ρ -mesons in ANM. It is found that various charged states of ρ -mesons have mass splitting similar to pion^[9]. This is a generic feature of all the isovector mesons. We also have examined the effects of the Dirac sea on the effective masses of ρ -mesons. We have shown that the contributions of the Dirac sea to

the masses of the ρ -mesons are very important and lead to a reduction of the different charged ρ -meson masses in ANM.

4 Appendix

Using Feynman's parametrization and dimensional regularization technique, the $\Pi^{*FF}(q)$ can be written as

$$\begin{aligned} \Pi^{*FF} &= \int \frac{d^N k}{(2\pi)^N} \frac{-i4g_\rho^2[2M^{*2} - (k+q)\cdot k]}{[(k+q)^2 - M^{*2}](k^2 - M^{*2})} = \\ &= \int \frac{d^N k}{(2\pi)^N} \int_0^1 \frac{-i4g_\rho^2[2M^{*2} - (k+q)\cdot k]}{[(k+qx)^2 + q^2x(1-x) - M^{*2}]^2} = \\ &= \frac{g_\rho^2}{2\pi^2} \int_0^1 \left\{ K^2 \left[\frac{2}{\varepsilon} - \ln(\pi K^2) + \frac{1}{2} - \gamma_E \right] + \right. \\ &\quad \left. \left(\frac{q^2x(1-x)}{2} + M^{*2} \right) \left[\frac{2}{\varepsilon} - \ln(\pi K^2) - \gamma_E \right] \right\}, \end{aligned} \quad (36)$$

where $N = 4 - \varepsilon$, $\varepsilon \rightarrow 0$, $K^2 = q^2x(1-x) - M^{*2}$ and γ_E is the Euler-Mascheroni constant.

The diverging part of Eq. (36) is

$$D(q) = \frac{g_\rho^2}{2\pi^2} \int_0^1 \left[K^2 + \frac{q^2x(1-x)}{2} + M^{*2} \right] \frac{2}{\varepsilon}. \quad (37)$$

To remove the divergence, we need to add the counterterms^[17] in the original Lagrangian interaction.

$$\mathcal{L}_{CT} = -\frac{1}{2}\beta_1\Phi_\rho\cdot(\partial^2 + m_\rho^2)\cdot\Phi_\rho + \frac{1}{2}\beta_2\Phi_\rho^2. \quad (38)$$

The values of the counterterms β_1 and β_2 are determined by imposing the appropriate renormalization conditions, that is,

$$\beta_1 = \left. \frac{\partial \Pi^{*FF}(q)}{\partial q^2} \right|_{q^2=m_\rho^2, M^* \rightarrow M}, \quad (39)$$

$$\beta_2 = \Pi^{*FF}(q)|_{q^2=m_\rho^2, M^* \rightarrow M}. \quad (40)$$

Here β_1 and β_2 are the wave function and ρ -meson mass renormalization counterterms, respectively. Then we get

$$\begin{aligned} \beta_1 &= \frac{3g_\rho^2}{2\pi^2} \int_0^1 dx \left\{ \frac{x(1-x)}{2} \left[\frac{2}{\varepsilon} - \ln[\pi(m_\rho^2x(1-x) - M^2)] - \gamma_E \right] + \frac{x(1-x)}{6} - \frac{m_\rho^2x(1-x)}{2[m_\rho^2x(1-x) - M^2]} \right\}, \\ \beta_2 &= \frac{3g_\rho^2}{2\pi^2} \int_0^1 dx \left\{ \frac{x(1-x)}{2} \left[\frac{2}{\varepsilon} - \ln[\pi(m_\rho^2x(1-x) - M^2)] - \gamma_E \right] + \frac{m_\rho^2x(1-x) - M^2}{6} \right\}. \end{aligned} \quad (41)$$

The renormalized $\Pi^{*FF}(q)$ is

$$\Pi^{*R}(q) = \Pi^{*FF}(q) - \beta_1 (q^2 - m_\rho^2) - \beta_2 = \frac{-3g_\rho^2}{2\pi^2} \left\{ \int_0^1 dx \left[\frac{q^2 x(1-x)}{2} \ln \left[\frac{q^2 x(1-x) - M^{*2}}{m_\rho^2 x(1-x) - M^2} \right] - \frac{(q^2 - m_\rho^2)m_\rho^2 x(1-x)}{2[m_\rho^2 x(1-x) - M^2]} - \frac{M^2 - M^{*2}}{6} \right] \right\}. \quad (42)$$

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