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Non-thermal Hawking radiation from the Kerr black hole*

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Abstract We present a short and direct derivation of Hawking radiation by using the Damour-Ruffini method, as taking into account the self-gravitational interaction from the Kerr black hole. It is found that the radiation is not exactly thermal, and because the derivation obeys conservation laws, the non-thermal Hawking radiation can carry information from the black hole. So it can be used to explain the black hole information paradox, and the process satisfies unitary.

Key words black hole, Hawking radiation, Damour-Ruffini method, quantum theory

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1 Introduction

In recent years the thermal radiation of a black hole has become a hot spot in theoretical physics[1, 2]. Many valuable models, such as the tunneling method, the Hamilton Jacobi method and the gravitational anomaly method, try to explain the dynamical origin of a black hole's thermal radiation. Furthermore, Damour and Ruffini suggested[3] that a massive charged particle could tunnel out over the horizon by a wave function which gives rise to the creation of a pair: one particle will go out and one antiparticle will fall back towards the singularity. This way they obtained the spectrum of the Hawking radiation. Recently, a semi-classical method of modeling the Hawking radiation as a tunneling effect has been developed[4—6]. It's main point is that the Hawking radiation of a black hole has not a pure thermal spectrum if the self-gravitation taken into account. A key insight is to find a coordinate system well behaved at the event horizon to calculate the emission rate. Tunneling provides not only a useful verification of thermodynamic properties of black holes but also an alternate conceptual means for understanding the underlying physical process of black hole radiation. It has been shown to be very robust, having been successfully applied to a wide variety of exotic space-times[7—13]. However, there is a complicated calculating process of the imaginary part of the action for the outgoing particle. Lastly, the authors of Ref. [14] have based on the Damour-Ruffini method and considering the self-gravitation interaction and energy conservation, derived the Hawking radiation from a static spherically symmetric black hole. Their result shows that the radiation is not exactly thermal and this non-thermal Hawking radiation can carry information from the black hole. This can be used to explain the black hole information paradox and in addition the process satisfies unitary. In this paper, we attempt to extend this method to stationary axisymmetric Kerr black holes. A new method to calculate the corrected non-thermal Hawking radiation of the stationary black hole is given, which is more accurate and more general.

2 Review of the Damour-Ruffini method

As usual we consider the Kerr metric of the form[15]

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$$\mathrm{d}s^2 = -\left(1-\frac{2Mr}{\rho^2}\right)\mathrm{d}t^2+\frac{\rho^2}{\Delta}\mathrm{d}r^2+\rho^2\mathrm{d}\theta^2+ \left[(r^2+a^2)\sin^2\theta+\frac{2Mra^2\sin^4\theta}{\rho^2}\right]\mathrm{d}\varphi^2- \frac{4Mra^2\sin^2\theta}{\rho^2}\mathrm{d}t\mathrm{d}\varphi \ , \tag{1}$$

where $\rho = r^2+a^2\cos^2\theta$, and $\Delta = r^2+a^2-2Mr = (r-r_+)(r-r_-)$. So we can obtain

$$r_\pm = M\pm\sqrt{M^2-a^2} \ . \tag{2}$$

In this expression, M is the total mass of the black hole, and a is the angular momentum per unit mass of the black hole, respectively. The surface gravity of the event horizon is

$$\kappa=\frac{r_+-r_-}{2(r_+^2+a^2)} \ . \tag{3}$$

There is a coordinate singularity in the metric (1) at the radius of the event horizon. To extend Damour-Ruffini's work to Kerr space-time, we should first find a coordinate system from which we expect that it be well behaved at the event horizon, and its coordinate clock synchronization can be transmitted from one place to another. We first investigate the dragging coordinate system. Let there be

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t}=-\frac{g_{03}}{g_{33}}=\Omega \ . \tag{4}$$

The space-time metric corresponding to the Kerr black hole can be rewritten as

$$\mathrm{d}s^2 = -\frac{\rho^2\Delta}{(r^2+a^2)^2-\Delta a^2\sin^2\theta}\mathrm{d}t^2+\frac{\rho^2}{\Delta}\mathrm{d}r^2+\rho^2\mathrm{d}\theta^2 = \hat{g}_{00}\mathrm{d}t^2+g_{11}\mathrm{d}r^2+g_{22}\mathrm{d}\theta^2 \ . \tag{5}$$

In fact, the line element (5) represents a three-dimensional hyper-surface in four-dimensional Kerr space-time. We can get the pure thermal spectrum of the Hawking radiation by using the Damour-Ruffini method. It means that we can also discuss the non-thermal Hawking radiation in the dragging coordinate system. From Eq. (5), we have

$$\begin{aligned}\hat{g}^{00} &= -\frac{(r^2+a^2)^2-\Delta a^2\sin^2\theta}{\rho^2\Delta},\\ g^{11} &= \frac{\Delta}{\rho^2},\quad g^{22}=\frac{1}{\rho^2},\\ \sqrt{-g} &= \frac{\rho^2\sqrt{\rho^2}}{\sqrt{(r^2+a^2)^2-\Delta a^2\sin^2\theta}} \ .\end{aligned} \tag{6}$$

In curved space-time the Klein-Gordon equation is

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi)-m_0^2\phi=0 \ . \tag{7}$$

With Eq. (5), the Klein-Gordon equation can be reduced to

$$g^{11}\frac{\mathrm{d}^2R(r)}{\mathrm{d}r^2}+\frac{1}{\sqrt{-g}}\frac{\partial}{\partial r}(\sqrt{-g}g^{11})\frac{\mathrm{d}R(r)}{\mathrm{d}r}+ \frac{R(r)}{\psi(\theta)}F(r,\theta)=\left[m_0^2+\left(\omega+\frac{g_{03}}{g_{33}}\right)^2\right]R(r) \ , \tag{8}$$

where

$$F(r,\theta)=g^{22}\frac{\mathrm{d}^2\psi(\theta)}{\mathrm{d}\theta^2}+\frac{1}{\sqrt{-g}}\frac{\partial\phi}{\partial\theta}\frac{\partial}{\partial\theta}(\sqrt{-g}g^{22})\frac{\mathrm{d}\psi(\theta)}{\mathrm{d}\theta} \ , \tag{9}$$

and the wave function ϕ has been separated as

$$\phi=\mathrm{e}^{-\mathrm{i}\omega t}R(r)\psi(\theta)\mathrm{e}^{\mathrm{i}m\varphi} \ . \tag{10}$$

Introduce the Tortoise coordinates

$$r_*=\frac{1}{2\kappa}\ln[(r-r_+)/r_+], \tag{11}$$

thus

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}r} &= \frac{1}{2\kappa(r-r_+)}\frac{\mathrm{d}}{\mathrm{d}r_*} \ ,\\ \frac{\mathrm{d}^2}{\mathrm{d}r^2} &= \frac{1}{4\kappa^2(r-r_+)^2}\frac{\mathrm{d}^2}{\mathrm{d}r_*^2}-\frac{1}{2\kappa(r-r_+)^2}\frac{\mathrm{d}}{\mathrm{d}r_*} \ .\end{aligned} \tag{12}$$

Substituting Eq. (12) into Eq. (8), we obtain

$$\frac{\mathrm{d}^2R(r)}{\mathrm{d}r_*^2}-2\kappa\frac{\mathrm{d}R(r)}{\mathrm{d}r_*}+ 2\kappa(r-r_+)\left(\frac{1}{\sqrt{-g}}\frac{\partial}{\partial r}\sqrt{-g}+\frac{1}{g^{11}}\frac{\partial g^{11}}{\partial r}\right)\frac{\mathrm{d}R(r)}{\mathrm{d}r_*}+ \frac{4\kappa^2(r-r_+)^2}{g^{11}}\frac{R(r)}{\psi(\theta)}F(r,\theta)= \frac{4\kappa^2(r-r_+)^2}{g^{11}}\left[m_0^2+\left(\omega+m\frac{g_{03}}{g_{33}}\right)^2g^{00}\right]R(r) \ . \tag{13}$$

Obviously, for $r\to r_+$, we get

$$\begin{aligned}&2\kappa(r-r_+)\left(\frac{1}{\sqrt{-g}}\frac{\partial}{\partial r}\sqrt{-g}\right)=0,\\ &2\kappa(r-r_+)\left(\frac{1}{g^{11}}\frac{\partial g^{11}}{\partial r}\right)=2\kappa,\\ &\frac{4\kappa^2(r-r_+)^2}{g^{11}}\left[m_0^2+\left(\omega+m\frac{g_{03}}{g_{33}}\right)^2g^{00}\right]R(r)= -(\omega-m\Omega)^2R(r) \ .\end{aligned} \tag{14}$$

Thus, we get the standard wave equation near the event horizon

$$\frac{\mathrm{d}^2R(r)}{\mathrm{d}r_*^2}+(\omega-\omega_0)^2R(r)=0 \ , \tag{15}$$

where $\omega_0=m\Omega$. By a simple calculation we get the radial wave solutions of Eq. (15) as

$$R_1(r)=\mathrm{e}^{-\mathrm{i}(\omega-\omega_0)r_*} \ , \quad R_2(r)=\mathrm{e}^{\mathrm{i}(\omega-\omega_0)r_*} \ . \tag{16}$$

From $R(t,r)=\mathrm{e}^{-\mathrm{i}\omega t}R(r)$, we obtain the ingoing-wave and outgoing-wave solutions

$$R^{\mathrm{in}}=R(t,r)R_1(r)=\mathrm{e}^{-\mathrm{i}\omega\upsilon}\ , \tag{17}$$

$$R^{\mathrm{out}}=R(t,r)R_2(r)=\mathrm{e}^{2\mathrm{i}(\omega-\omega_0)r_*}\mathrm{e}^{-\mathrm{i}\omega\upsilon}\ . \tag{18}$$

$\upsilon=t+[(\omega-\omega_0)/\omega]r_*$ is the advanced Eddington-Finkelstein coordinates.

3 Analytical continuations and the self-gravitation interaction

In the following discussion, we first rewrite R^{out}as follows

$$R^{\mathrm{out}}=\mathrm{e}^{-\mathrm{i}\omega\upsilon}\left(\frac{r-r_+}{r_+}\right)^{\frac{\mathrm{i}(\omega-\omega_0)}{2\kappa_+}}\ , \tag{19}$$

which shows that the outgoing wave is not analytic on the event horizon r_+. Therefore we have to extend it by analytical continuation to inside the horizon through the lower-half complex r-plane

$$(r\rightarrow r_+)\rightarrow|r-r_+|\,\mathrm{e}^{-\mathrm{i}\pi}=(r_+-r)\mathrm{e}^{-\mathrm{i}\pi}\ , \tag{20}$$

then near the event horizon r_+, R^{out} can be rewritten as

$$R^{\mathrm{out}}=\mathrm{e}^{-\mathrm{i}\omega\upsilon}\mathrm{e}^{\frac{\pi(\omega-\omega_0)}{\kappa_+}}\mathrm{e}^{2\mathrm{i}(\omega-\omega_0)r_*}\ . \tag{21}$$

The scattering probability of the outgoing wave at the event horizon is

$$\Gamma=\left|\frac{R^{\mathrm{out}}(r>r_+)}{R^{\mathrm{out}}(r<r_-)}\right|^2=\mathrm{e}^{-\frac{2\pi(\omega-\omega_0)}{\kappa_+}}\ . \tag{22}$$

If a particle with energy radiates from the black hole and back-reaction of the particle to the space-time is considered, M should be replaced by $(M-\omega_i)$, and the emission probability will be

$$\Gamma_i=\exp\left[-\frac{2\pi(\omega_i-\omega_0)}{\kappa_i}\right]=\exp\left(-\frac{2\pi[(M-\omega_i+\omega_0)(\omega_i-\omega_0)]}{\kappa_i}\right). \tag{23}$$

For many particles, assuming that they radiate one by one, we have

$$\Gamma=\prod_i\Gamma_i=\exp\left(\sum_i-\frac{2\pi[(M-\omega_i+\omega_0)(\omega_i-\omega_0)]}{\kappa_i}\right). \tag{24}$$

If the emission is regarded as a continuous process and considering $\omega_0=m\Omega$ as a constant, the sum in (24) should be substituted by an integration. The emission probability will be

$$\Gamma_i=\exp\left(-\int_0^{\omega}\frac{2\pi}{\kappa_+}\mathrm{d}\omega\right)=\exp\left(-\int_0^{\omega'}\frac{2\pi(M-\omega')}{\kappa_+}\mathrm{d}\omega'\right). \tag{25}$$

According to the first law of thermodynamics, we have

$$T\mathrm{d}S'=-\mathrm{d}\omega'\ . \tag{26}$$

Therefore, Eq. (25) can be expressed as

$$\Gamma=\mathrm{e}^{-\int_0^{\omega}\frac{1}{T}\mathrm{d}\omega'}=\mathrm{e}^{\Delta S}\ , \tag{27}$$

where

$$\Delta S=\frac{1}{4}\times4\pi\times\left[(M-\omega)+\sqrt{(M-\omega)^2-a^2}\,\right]^2-\frac{1}{4}\times4\pi\times(M+\sqrt{M^2-a^2})^2\approx -2\pi\left[\frac{\omega}{\kappa_+}-\left(1+\frac{3M}{2\sqrt{M^2-a^2}}+\frac{M^3}{4(M^2-a^2)^{3/2}}\right)\omega^2+\cdots\right]. \tag{28}$$

ΔS is the entropy change of the black hole between before and after the emission. This result is obviously consists with an underlying unitary theory.

In fact, Quantum theory shows that the transmission of the outgoing-wave can be expressed as

$$\Gamma(\mathrm{i}\rightarrow\mathrm{f})=|M_{\mathrm{fi}}|^2A\ , \tag{29}$$

where A is a phase factor, and $|M_{\mathrm{fi}}|^2$ is the square of the probability amplitude in the transition process, and A can be obtained by summing over all final states and averaging over all initial states. The number of final states can be expressed in exponential form through the final states entropy $\mathrm{e}^{\Delta S}$, and similarly, the number of the initial states can be expressed in exponential form through the final states entropy $\mathrm{e}^{\Delta S}$. We obtain

$$\Gamma=\frac{\exp(S_{\mathrm{final}})}{\exp(S_{\mathrm{initial}})}=\mathrm{e}^{\Delta S}\ . \tag{30}$$

Eq. (25) is as same as Eq. (30).

4 Conclusion

In this Letter, we obtained the result that the permeation ratio of the outgoing-wave revises the thermal radiation spectrum from the Kerr black hole, while taking into account the self-gravitational interaction of the radiant particle energy to space-time background. This derived result is in contrast with previous analysis of the same subject by using the tunneling method, and satisfies unitary. However, we use a quite different way that is more simple, more direct, and clearer in its physical meaning. Moreover, the calculation is easy, and we don't have to bother whether a radiant particle has restmass. It is obvious that if we consider the self-gravitational interaction, the transmission ratio of outgoing-wave at the event area appears to deviate from the thermal radiation spectrum of the black hole, which might contain the related information about the material that makes up the black hole. This result may lead to resolve the problem of information loss. In fact, if we calculate the Hawking radiation of the Kerr black hole by using the Damour-Ruffini method without considering the self-gravitational interaction of the radiation energy to time-space background, we get a precise radiation spectrum of the tunneling process through the event area. The outgoing-wave has a potential barrier in Eq. (13), which lies between the event area and infinite distance. Therefore, in the opinion of an observer looking at the event area from an infinite distance the black hole radiation spectrum, dispersed by the shape, a gray spectrum.

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