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Study of ππ final state interactions in $\psi(2S)\to\pi^+\pi^-J/\psi$ *

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Abstract The $\pi^+\pi^-$ transition of heavy quarkonia in decay $\psi(2S)\to\pi^+\pi^-J/\psi$ is studied. With the BESII data on the decay $\psi(2S)\to\pi^+\pi^-J/\psi$, we update the values of coupling constants (g_i) and chromopolarizability ($\alpha_{\psi(2S)J/\psi}$) in this process.

Key words $\pi^+\pi^-$ transition, $\psi(2S)\to\pi^+\pi^-J/\psi$ decays, coupling constants, chromopolarizability

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1 Introduction

The transition $\psi(2S)\to\pi^+\pi^-J/\psi$ has been a unique laboratory for studying the strong interaction between the heavy quarks and the low energy di-pion system. In recent years, with more and more data accumulation many analyses of this transition have been performed. One commonly used method, the QCD multipole expansion, based on QCD theory for such a study was developed[1]. Recently, the transition of $\psi(2S)\to\pi^+\pi^-J/\psi$ was suggested as a laboratory to extract the value of the chromopolarizability $\alpha_{\psi(2S)J/\psi}$, which appears when describing the interaction of J/ψ with soft gluons in the QCD multipole expansion[2]. This transition was also studied by using phenomenological models, like effective Lagrangian based on the chiral perturbation theory (ChPT)[3]. In the ChPT method, the matrix elements of the decay amplitude are determined by using the mass spectrum of the low energy di-pion system. In recent studies[4, 5], it was found that the final state interaction (FSI) in the di-pion system plays an important role in extracting the value of chromopolarizability $\alpha_{\psi(2S)J/\psi}$ and the ChPT matrix elements. As studied in Ref. [5], it was found that when considering S-wave ππ FSI, the value of chromopolarizability was reduced to about 1/3 of those without ππ FSI. Similarly, in the ChPT model[4], it was also found that the fit to the di-pion mass spectrum is improved when considering the FSI in the di-pion system, and the coupling constant g is reduced to about 1/3 of the ChPT model without FSI.

Recently, the BES collaboration analyzed the transition of $\psi(2S)\to\pi^+\pi^-J/\psi$ with a 14 M $\psi(2S)$ sample and published the results for the $\pi^+\pi^-$ invariant mass spectrum[6]. In this work, we adopt the same amplitude as used in Refs. [4, 5], and update our previous values of coupling constants g_i and chromopolarizability $\alpha_{\psi(2S)J/\psi}$ extracted from fitting to the data and the decay width of the $\psi(2S)\to\pi^+\pi^-J/\psi$.

2 Brief formalism

2.1 Chiral effective Lagriangian

In the $\pi^+\pi^-$ transition process, the Lagrangian in the lowest order based on ChPT constructed by Mannel et al. can be written as[3]

$$\mathscr{L} = \mathscr{L}_0+\mathscr{L}_{\rm S.B.}, \tag{1}$$

$$\begin{aligned}\mathscr{L}_0 = {} & gA_\mu^{(\nu)}B^{(\nu)\mu*}{\rm Tr}\big[(\partial_\nu U)(\partial^\nu U)^\dagger\big]+\\ & g_1A_\mu^{(\nu)}B^{(\nu)\mu*}{\rm Tr}\big[(\upsilon\cdot\partial U)(\upsilon\cdot\partial U)^\dagger\big]+\\ & g_2A_\mu^{(\nu)}B^{(\nu)\mu*}{\rm Tr}\big[(\partial^\mu U)(\partial^\nu U)^\dagger+\\ & (\partial^\mu U)^\dagger(\partial^\nu U)\big]+{\rm h.c.},\end{aligned} \tag{2}$$

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$$\mathscr{L}_{\rm S.B.} = g_3 A_\mu^{(\nu)} B^{(\nu)\mu*} {\rm Tr}\left[\mathscr{M}(U+U^\dagger-2)\right] + {\rm i}g'\varepsilon^{\mu\nu\alpha\beta}\left[\upsilon_\mu A_\nu^{(\upsilon)}\partial_\alpha B_\beta^{(\upsilon)*}\right] - (\partial_\mu A_\nu^{(\upsilon)})\upsilon_\alpha B_\beta^{(\upsilon)*}]{\rm Tr}[\mathscr{M}(U-U^\dagger)] + {\rm h.c.}, \quad (3)$$

where Tr denotes the taking the trace, $\mathscr{L}_0$ is the leading term obeying chiral symmetry, $\mathscr{L}_{\rm S.B.}$ is the leading term caused by chiral symmetry breaking. U is a unitary 3×3 matrix that contains the Goldstone fields, A_μ^ν and B_μ^ν are the fields of the initial and final 1^- states of heavy quarkonia, respectively, and υ is the velocity vector of A. $\mathscr{M}={\rm diag}\{m_{\rm u},\ m_{\rm d},\ m_{\rm s}\}$ is the quark mass matrix, and g_i denotes the coupling constants. Then the tree diagram amplitude for decay $\psi(2S)\to\pi^+\pi^-{\rm J}/\psi$ in the rest frame of $\psi(2S)$ has the form

$$\mathcal{A}(\psi(2S)\to\pi^+\pi^-{\rm J}/\psi) = -\frac{4}{f_\pi^2}\Bigg\{\left[\frac{g}{2}(m_{\pi\pi}^2-2M_\pi^2)+ g_1(\upsilon\cdot p_{\pi^+})(\upsilon\cdot p_{\pi^-})+g_3 M_\pi^2\right]\varepsilon^*_{{\rm J}/\psi}\cdot\varepsilon_{\psi(2S)}+ g_2[p_{\pi^+\mu}p_{\pi^-\nu}+p_{\pi^+\nu}p_{\pi^-\mu}]\varepsilon^{*\mu}_{{\rm J}/\psi}\varepsilon^{\nu}_{\psi(2S)}\Bigg\}, \quad (4)$$

where $m_{\pi\pi}^2=(p_{\pi^+}+p_{\pi^-})^2$, $\varepsilon_{{\rm J}/\psi}$ and $\varepsilon_{\psi(2S)}$ are the polarization vectors of the heavy spin-1 quarkonia, and $f_\pi=93$ MeV is the decay constant of pion. In the rest frame of decaying particle $\psi(2S)$, $\upsilon_\mu=(1,0)$, then $(\upsilon\cdot p_{\pi^+})(\upsilon\cdot p_{\pi^-})=p^0_{\pi^+}p^0_{\pi^-}$, $p^0_{\pi^+}$ and $p^0_{\pi^-}$ are the energy of π^+ and π^- in the lab frame, respectively. It is found that the last term in Eq. (4) is strongly suppressed experimentally in the charmonium decay[3], thus we can set $g_2=0$, and then the amplitude can be written as

$$\mathcal{A}(\psi(2S)\to\pi^+\pi^-{\rm J}/\psi) = -\frac{4}{f_\pi^2}\left[\frac{g}{2}(m_{\pi\pi}^2-2M_\pi^2)+ g_1 p^0_{\pi^+}p^0_{\pi^-}+g_3M_\pi^2\right]\varepsilon^*_{{\rm J}/\psi}\cdot\varepsilon_{\psi(2S)}\ . \quad (5)$$

It is easy to see that the g_1-term contains a D wave component, under Lorentz transformation, $p^0_{\pi^+}p^0_{\pi^-}$ can be expressed as[7]

$$p^0_{\pi^+}p^0_{\pi^-} = A(q^2)P_0(\cos\theta^*_\pi)+B(q^2)P_2(\cos\theta^*_\pi), \quad (6)$$

with

$$A(q^2) = \frac{1}{4}q^2+\frac{1}{6}|\boldsymbol{p}_{{\rm J}/\psi}|^2\left(1+\frac{2m_\pi^2}{q^2}\right),$$

$$B(q^2) = -\frac{1}{2}|\boldsymbol{p}_{{\rm J}/\psi}|^2\left(1-\frac{4m_\pi^2}{q^2}\right),$$

where q is the total four-momentum of the di-pion system. $P_0(\cos\theta^*_\pi)=1$, $P_2(\cos\theta^*_\pi)=\frac{1}{2}\left(\cos^2\theta^*_\pi-\frac{1}{3}\right)$ are the Legendre functions.

On the other hand, the S-wave $\pi\pi$ FSI plays an important role in $\pi^+\pi^-$ transition, and should be included in the calculation. Because ChPT amplitudes in the $O(p^2)$ order are adopted as the kernel of the coupled-channel Bethe-Salpeter equations (BSE), the D-wave $\pi\pi$ FSI cannot be included[4].

Considering the S-wave $\pi\pi$ FSI in the ChUT approach, the S-wave $\pi\pi$ phase shift data can be described with one parameter which is a 3-momentum cut off in the di-pion loop integral $q_{\max}=1.03$ GeV[8]. The full amplitude of the progress $\pi\pi\to\pi^+\pi^-$ can be expressed as

$$\langle\pi^+\pi^-|T|\pi^+\pi^-+\pi^-\pi^++\pi^0\pi^0\rangle = 2T^{I=0}_{\pi\pi,\pi\pi}, \quad (7)$$

where $T^{I=0}_{\pi\pi,\pi\pi}$ is the full S wave $\pi\pi\to\pi\pi$ coupled-channel amplitude for $I=0$. Then the decay amplitude of $\psi(2S)\to\pi^+\pi^-{\rm J}/\psi$ is modified to

$$\mathcal{A}(\psi(2S)\to\pi^+\pi^-{\rm J}/\psi) = \mathcal{A}_0+\mathcal{A}_{0s}\cdot G_\pi(q^2)\cdot 2T^{I=0}_{\pi\pi,\pi\pi}(q^2), \quad (8)$$

where $\mathcal{A}_0$ is the amplitude of the tree diagram, $\mathcal{A}_{0s}$ is the S wave component of $\mathcal{A}_0$. $G_\pi(q^2)$ is the di-pion loop integral

$$G_\pi(q^2) = {\rm i}\int\frac{{\rm d}^4q'}{(2\pi)^4}\frac{1}{q'^2-m_\pi^2+{\rm i}\varepsilon}\frac{1}{(p'-p-q')^2-m_\pi^2+{\rm i}\varepsilon},$$

where $q=p'-p$, p' and p denote the momenta of the initial and the final states, respectively. The loop integral can be calculated by introducing the cutoff $q_{\max}=1.03$ GeV.

The differential width for the $\rm A\to B\pi^+\pi^-$ decay reads

$$d\varGamma = \frac{1}{(2\pi)^3}\frac{1}{8M_{\rm A}^2}\overline{\sum}\sum|\mathcal{A}|^2|\boldsymbol{p}^*_\pi||\boldsymbol{p}_{\rm B}|{\rm d}m_{\pi\pi}{\rm d}\cos\theta^*_\pi, \quad (9)$$

where $\overline{\sum}\sum$ represent the average over the polarizations of A and sum over the polarizations of B. $\boldsymbol{p}^*_\pi$ is the momentum of pion in the rest frame of the $\pi\pi$ system, and $\boldsymbol{p}_{\rm B}$ is the momentum of B in the rest frame of the decaying particle.

$$|\boldsymbol{p}^*_\pi| = \sqrt{\frac{m_{\pi\pi}^2}{4}-m_\pi^2}\,,$$

$$|\boldsymbol{p}_{\rm B}| = \frac{1}{2M_{\rm A}}\times \sqrt{(M_{\rm A}^2-(m_{\pi\pi}+M_{\rm B})^2)(M_{\rm A}^2-(m_{\pi\pi}-M_{\rm B})^2)}.$$

Then the differential decay width for $\psi(2S)\to\pi^+\pi^-{\rm J}/\psi$ can be written as:

$$\mathrm{d}\Gamma(\psi(2S)\to\pi^+\pi^-J/\psi)=\begin{cases}\frac{1}{(2\pi)^3}\frac{2}{M^2_{\psi(2S)}f^4_\pi}\Big|\frac{g}{2}(m^2_{\pi\pi}-2m^2_\pi)+g_1(A(q^2)P_0(\cos\theta^*_\pi)+B(q^2)P_2(\cos\theta^*_\pi))+\\ g_3m^2_\pi\Big|^2|\boldsymbol{p}^*_\pi||\boldsymbol{p}_{J/\psi}|\mathrm{d}m_{\pi\pi}\mathrm{d}\cos\theta^*_\pi, \qquad \text{without } \pi\pi \text{ FSI}\\ \frac{1}{(2\pi)^3}\frac{2}{M^2_{\psi(2S)}f^4_\pi}\Big|\left(\frac{g}{2}(m^2_{\pi\pi}-2m^2_\pi)+g_1A(q^2)P_0(\cos\theta^*_\pi)+g_3m^2_\pi\right)\times\\ (1+2G_\pi(q^2)\cdot T^{I=0}_{\pi\pi,\pi\pi}(q^2))+\\ g_1B(q^2)P_2(\cos\theta^*_\pi)\Big|^2|\boldsymbol{p}^*_\pi||\boldsymbol{p}_{J/\psi}|\mathrm{d}m_{\pi\pi}\mathrm{d}\cos\theta^*_\pi. \qquad \text{with } \pi\pi \text{ FSI.}\end{cases} \tag{10}$$

2.2 Chromopolarizability

By using the QCD multipole expansion method, we can measure the chromopolarizability which is the diagonal amplitude of the E1-E1 chromo electric interaction with soft gluon fields. The amplitude for the $\pi^+\pi^-$ transition can be written as[2]

$$\mathcal{A}(\psi(2S)\to\pi^+\pi^-J/\psi)=2\sqrt{M_{\psi(2S)}M_{J/\psi}}\times \alpha_{\psi(2S)J/\psi}\left\langle\pi^+\pi^-\left|\frac{1}{2}\boldsymbol{E}^a\cdot\boldsymbol{E}^a\right|0\right\rangle\approx \sqrt{M_{\psi(2S)}M_{J/\psi}}\alpha_{\psi(2S)J/\psi}\frac{8\pi^2}{b}(q^2-C), \tag{11}$$

where $\alpha_{\psi(2S)J/\psi}$ is the chromopolarizability we want to measure, the factor $2\sqrt{M_{\psi(2S)}M_{J/\psi}}$ is the normalization constant of the decay amplitude. $\frac{8\pi^2}{b}(q^2-C)$ is the dominant part of the di-pion production amplitude by the gluonic operate $\boldsymbol{E}^a\cdot\boldsymbol{E}^a$ determined by the trace anomaly and chiral algebra, where b is the first coefficient in the QCD β function with three light flavors, and C is a constant which denotes approximately the contributions of subleading terms and depends on the total energy of the pion pair[5].

When the S-wave $\pi\pi$ FSI is taken into account, the amplitude is modified to

$$\mathcal{A}(\psi(2S)\to\pi^+\pi^-J/\psi)=\frac{8\pi^2}{b}\sqrt{M_{\psi(2S)}M_{J/\psi}}\times \alpha_{\psi(2S)J/\psi}(q^2-C)\times(1+2G_\pi(q^2)T^{I=0}_{\pi\pi,\pi\pi}(q^2)), \tag{12}$$

where $G_\pi(q^2)$ and $T^{I=0}_{\pi\pi,\pi\pi}(q^2)$ are the same as in Eq. (8). Then in terms of Eqs. (9), (11) and (12), the differential width for $\psi(2S)\to\pi^+\pi^-J/\psi$ can be written as:

$$\mathrm{d}\Gamma(\psi(2S)\to\pi^+\pi^-J/\psi)=\begin{cases}\frac{2\pi M_{J/\psi}}{b^2M_{\psi(2S)}}|\alpha_{\psi(2S)J/\psi}|^2(q^2-C)^2|\boldsymbol{p}^*_\pi||\boldsymbol{p}_{J/\psi}|\mathrm{d}m_{\pi\pi}, \qquad \text{without } \pi\pi \text{ FSI}\\ \frac{2\pi M_{J/\psi}}{b^2M_{\psi(2S)}}|\alpha_{\psi(2S)J/\psi}|^2(q^2-C)^2\times|1+2G_\pi(q^2)T^{I=0}_{\pi\pi,\pi\pi}(q^2)|^2|\boldsymbol{p}^*_\pi||\boldsymbol{p}_{J/\psi}|\mathrm{d}m_{\pi\pi}.\\ \text{with } \pi\pi \text{ FSI}\end{cases} \tag{13}$$

The parameters in the amplitudes, g, g_1, g_3 and $\alpha_{\psi(2S)J/\psi}$, will be determined by fitting the amplitudes to the BESII data[6], together with the decay width of $\Gamma(\psi(2S)\to\pi^+\pi^-J/\psi)=107.166\pm6.156$ keV from PDG[9].

3 Results and discussion

The fit method is similar to the one used in Ref. [6]: the detection efficiency is considered in the definition of likelihood function, and the free parameters are optimized by using MINUIT[10] in the CERN library, and the χ^2 definition used to check the goodness of fit has the same definition as in Ref. [6].

The fitted $\pi^+\pi^-$ invariant mass spectra are shown in Fig. 1, and the resultant parameters through the best fitting are listed in Table 1 and Table 2, respectively. If we ignore g_3 in fitting and without inclusion of $\pi\pi$ FSI as suggested in Ref. [3], the value of g_1/g will be almost the same one as in Ref. [3], while the value of g is reduced to about 2/3 of the value given in Ref. [3]. In the case of with $\pi\pi$ FSI, the fitting result shows that the the value of g_3 is so small that we can take $g_3=0$, which is consistent with the result in Ref. [4]. The value of g_1/g is almost the same as in Ref. [3]. It should be noted that the unit of the value of g in Ref. [3] is GeV, so we cannot compare the value we get with the value in Ref. [3] directly, however, this value is consistent with the one given in the Doctoral dissertation[11] of the Ref. [3] author. The values of $\alpha_{\psi(2S)J/\psi}$ and C are slightly different

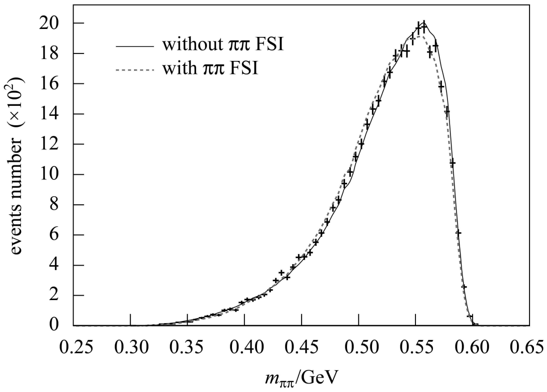

Fig. 1. The fitting result of the $\pi^+\pi^-$ invariant mass spectrum for the $\psi(2S)\to\pi^+\pi^-J/\psi$ decay using the decay amplitude Eqs. (5), (8). The solid and dashed curves are fitted without and with S-wave $\pi\pi$ FSI, respectively. The points with error bar are data [6]. The fit using amplitudes Eqs. (11), (12) have similar distribution.

from the values given in Ref. [5]. The χ^2 value/ number of degrees of freedom (ndf) is almost the same in the two sets of the fit with or without $\pi^+\pi^-$ FSI. It should be mentioned that the errors in Table 1 and Table 2 not only include the statistical errors, but also include the experimental uncertainties of the total decay width. That is why the errors are larger than the ones given in Ref. [5].

Table 1. Resultant coupling constants g_i for $\psi(2S)\to\pi^+\pi^-J/\psi$ decay without and with $\pi\pi$ S-wave FSI.

FSI	g	g_1/g	g_3	χ^2/ndf
without $\pi\pi$ FSI	0.219 ± 0.007	-0.336 ± 0.005	0	1.2
with $\pi\pi$ FSI	0.081 ± 0.003	-0.327 ± 0.006	0	1.3

Table 2. Resultant parameters $\alpha_{\psi(2S)J/\psi}$ and C for $\psi(2S)\to\pi^+\pi^-J/\psi$ decay without and with $\pi\pi$ S-wave FSI.

FSI	$\alpha_{\psi(2S)J/\psi}$/GeV^{-3}	$C(m^2_{\pi\pi})$	χ^2/ndf
without $\pi\pi$ FSI	2.53 ± 0.09	4.49 ± 0.04	1.2
with $\pi\pi$ FSI	0.92 ± 0.03	4.36 ± 0.05	1.2

4 Summary

With the published BESII data on the $\psi(2S)\to\pi^+\pi^-J/\psi$, we re-analyze the matrix elements of the decay amplitudes, and the coupling constants g_i and chromopolarizability $\alpha_{\psi(2S)J/\psi}$ are obtained. From the fitting results we can see that the $\pi\pi$ FSI does not modify the value of g_1/g and C significantly, but causes about $\frac{2}{3}$ decrease of the value of g and $\alpha_{\psi(2S)J/\psi}$. This indicates that the S-wave $\pi\pi$ FSI plays an important role in extracting the value of chromopolarizability $\alpha_{\psi(2S)J/\psi}$ and the ChPT matrix elements.

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