

# Properties of deformed $\Lambda$ hypernuclei<sup>\*</sup>

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**Abstract** The properties of Be and B isotopes and the corresponding  $\Lambda$  hypernuclei are studied by using a deformed Skyrme Hartree-Fock approach with realistic nucleonic Skyrme forces, pairing correlations, and a microscopically determined lambda-nucleon interaction based on Brueckner-Hartree-Fock calculations of hypernuclear matter. The results suggest that the core nuclei and the corresponding hypernuclei have similar deformations with the same sign.

**Key words** deformed  $\Lambda$  hypernuclei, Skyrme Hartree-Fock

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## 1 Introduction

The study of hypernuclei is crucial to provide the information about hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions. Currently there are many experimental data for various single- $\Lambda$  hypernuclei over almost the whole mass table<sup>[1]</sup> and a few double- $\Lambda$  hypernuclei. Many theoretical calculations of hypernuclei were based on spherical symmetry, except some attempts of deformed Hartree-Fock (HF) calculations with nonrealistic interactions<sup>[2]</sup> and the Nilsson model study of  $p$ -shell nuclei in Ref. [3]. However, it is well known that many  $p$ -shell and  $d$ -shell nuclei are deformed in the ground state. For example, according to experiment,  $^{10}\text{B}$  and  $^{11}\text{C}$  have large quadrupole moments<sup>[4]</sup>. So far there is no study of a self-consistent model treating the core and the hypernuclei with realistic effective interactions for both nucleon-nucleon and  $\Lambda\text{N}$  channels.

The aim of this paper is to investigate how much the observables of hypernuclei depend on the deformation in a self-consistent deformed SHF (DSHF) model including the hyperon degree of freedom (hereafter we call this model the extended DSHF model). As an example, we study first the well-deformed Be and B isotopes and the corresponding  $\Lambda$  hypernuclei. The DSHF method has been used to de-

scribe the properties of light and medium-heavy normal nuclei in Ref. [5] and is extended in this paper to the study of hypernuclei. For this purpose we generalize the microscopic  $\Lambda\text{N}$  force developed in Refs. [6, 7] for nearly symmetric nuclei to isospin asymmetric nuclei. This effective  $\Lambda\text{N}$  interaction is derived from Brueckner-Hartree-Fock (BHF) calculations of isospin-asymmetric hypernuclear matter<sup>[8]</sup> with the Nijmegen soft-core hyperon-nucleon potential NSC89 and the Argonne  $V_{18}$  nucleon-nucleon interaction, including explicitly the coupling of the lambda-nucleon to the sigma-nucleon states.

This paper is organized as follows. In Section 2 we briefly introduce the extended DSHF model including an effective hyperon-nucleon interaction derived from microscopic BHF calculations of asymmetric nuclear matter. The calculated results of DSHF for core nuclei and extended DSHF for hypernuclei with one or two  $\Lambda$  are given in Section 3. Finally, a summary is given in Section 4.

## 2 Extended deformed Skyrme Hartree-Fock

We extend the self-consistent DSHF method solved in coordinate space with axially symmetric shape<sup>[9]</sup>, including the  $\Lambda\text{N}$  interaction. Namely, we

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solve the SHF Schrödinger equation

$$\left[ -\nabla \cdot \frac{1}{2m_q^*(\mathbf{r})} \nabla + V_q(\mathbf{r}) - i\nabla W_q(\mathbf{r}) \cdot (\nabla \times \boldsymbol{\sigma}) \right] \times \phi_q^i(\mathbf{r}) = e_q^i \phi_q^i(\mathbf{r}), \quad (1)$$

where  $V_q$  is the Skyrme mean field of nucleons or hyperon(s) and  $W_q$  the nucleonic spin-orbit mean field. In Eq. (1), the extended SHF mean fields are given by

$$\begin{aligned} V_q &= V_q^{\text{SHF}} + \frac{\partial \epsilon_{N\Lambda}}{\partial \rho_q} + \frac{\partial}{\partial \rho_q} \left( \frac{m_\Lambda}{m_\Lambda^*} \right) \frac{\tau_\Lambda - C \rho_\Lambda^{5/3}}{2m_\Lambda}, \\ V_\Lambda &= \frac{\partial \epsilon_{N\Lambda}}{\partial \rho_\Lambda} + \frac{\partial}{\partial \rho_\Lambda} \left( \frac{m_\Lambda}{m_\Lambda^*} \right) \frac{\tau_\Lambda - C \rho_\Lambda^{5/3}}{2m_\Lambda} - \\ &\quad \left( \frac{m_\Lambda}{m_\Lambda^*} - 1 \right) \frac{5}{3} \frac{C \rho_\Lambda^{2/3}}{2m_\Lambda}, \end{aligned} \quad (2)$$

where  $V_q^{\text{SHF}}$  ( $q = n, p$ ) is the nucleonic Skyrme mean field without hyperons and

$$\begin{aligned} \epsilon_{N\Lambda} &\approx - \left[ 368 - (1717 + 268\alpha - 920\alpha^2) \rho_N - \right. \\ &\quad \left. (2932 - 776\alpha + 2483\alpha^2) \rho_N^2 \right] \rho_N \rho_\Lambda + \\ &\quad \left[ 449 - 2470\rho_N + 5834\rho_N^2 \right] \rho_N \rho_\Lambda^{5/3}, \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{m_\Lambda^*}{m_\Lambda} &\approx 1 - \\ &\quad \left[ 1.58 + 0.12\alpha - 0.12\alpha^2 + 0.54y - 0.14y^2 \right] \rho_N + \\ &\quad \left[ 4.11 + 2.11\alpha + 2.88\alpha^2 + 0.35y + 1.17y^2 \right] \rho_N^2 - \\ &\quad \left[ 4.03 + 7.08\alpha + 5.18\alpha^2 - 0.93y + 3.27y^2 \right] \rho_N^3, \end{aligned} \quad (4)$$

with  $\rho_N = \rho_n + \rho_p$ ,  $\alpha = (\rho_n - \rho_p)/\rho_N$ , and  $y = \rho_\Lambda/\rho_N$ . For more details about the above equations, please refer to Ref. [10].

We take into account pairing interactions of nucleons with a BCS approximation which is taken to be a density-dependent delta-force<sup>[11]</sup>,

$$V_q(\mathbf{r}_1, \mathbf{r}_2) = V_q' \left( 1 - \frac{\rho_N(r)}{\rho_0} \right) \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad (5)$$

where  $\rho_N(r)$  is the nucleonic HF density at  $\mathbf{r} = (\mathbf{r}_1 + \mathbf{r}_2)/2$  and  $\rho_0 = 0.16 \text{ fm}^{-3}$ . We use the pairing strength  $V_q' = -410 \text{ MeV} \cdot \text{fm}^3$  for both neutrons and protons of light nuclei<sup>[12]</sup> and  $V_p' = -1146 \text{ MeV} \cdot \text{fm}^3$ ,  $V_n' = -999 \text{ MeV} \cdot \text{fm}^3$  for medium-mass and heavy nuclei. A smooth energy cutoff is employed in the BCS calculations<sup>[13]</sup>. In the case of an odd number of nucleons, the orbit occupied by the odd nucleon is blocked in the BCS calculations, as described in Ref. [14].

### 3 Results

The nucleus  ${}^8\text{Be}$  is known to be strongly deformed due to its double- $\alpha$  structure. As a first step, we study the well-deformed Be and B isotopes and the corresponding  $\Lambda$  hypernuclei by performing the DSHF+BCS and extended DSHF+BCS calculations with SkI4 Skyrme interaction and the microscopic  $\Lambda N$  force.

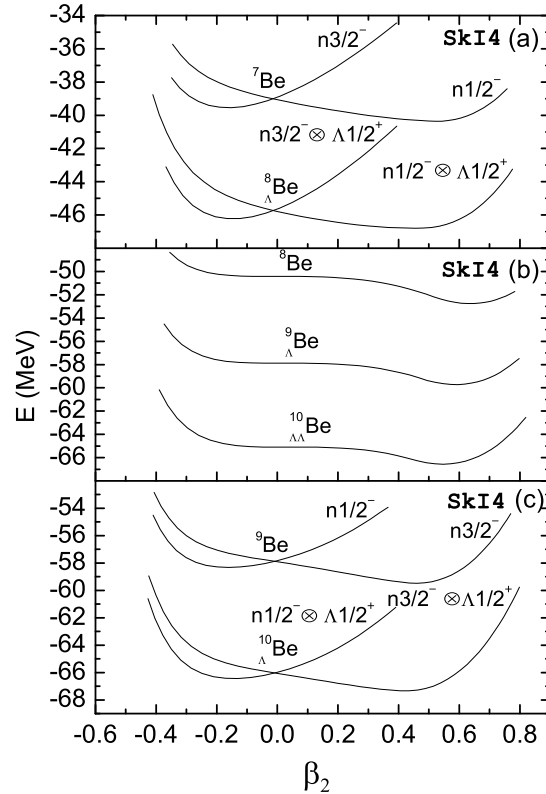


Fig. 1. Self-consistent DSHF calculations with the SkI4 interaction for (a)  ${}^7\text{Be}$  and  ${}^8_\Lambda\text{Be}$ , (b)  ${}^8\text{Be}$ ,  ${}^9_\Lambda\text{Be}$ , and  ${}^{10}_{\Lambda\Lambda}\text{Be}$ , and (c)  ${}^9\text{Be}$  and  ${}^{10}_\Lambda\text{Be}$ . The abbreviation “n” stands for the neutron configuration.

Figures. 1, 2 show the binding energy surfaces for the core nuclei  ${}^{7,8,9}\text{Be}$ ,  ${}^{8,9,10}\text{B}$  and the corresponding hypernuclei with the SkI4 force. For  ${}^8\text{Be}$  we got the  $\Lambda$  binding energy,

$$B_\Lambda = E({}^{A-1}Z) - E({}^A_\Lambda Z), \quad (6)$$

and the  $\Lambda\Lambda$  bond energy,

$$\Delta B_{\Lambda\Lambda} = 2E({}^{A-1}_\Lambda Z) - E({}^{A-2}Z) - E({}^A_{\Lambda\Lambda} Z), \quad (7)$$

with  $B_\Lambda = 6.96 \text{ MeV}$  and  $\Delta B_{\Lambda\Lambda} = -0.12 \text{ MeV}$ , respectively, compared with the experimental values of  $B_\Lambda = 6.71 \pm 0.04 \text{ MeV}$ <sup>[15]</sup> or  $5.99 \pm 0.07 \text{ MeV}$ <sup>[1]</sup>,

and  $\Delta B_{\Lambda\Lambda} = 4.3 \pm 0.4$  MeV<sup>[16]</sup> or  $-4.9 \pm 0.7$  MeV<sup>[17]</sup>. The relativistic mean-field model of Ref. [18] predicts  $\Delta B_{\Lambda\Lambda} \approx 0.3$  MeV for this nucleus. Although there are two experimental reports about the double- $\Lambda$  hypernucleus  ${}^{10}_{\Lambda\Lambda}\text{Be}$ , more experimental events are desperately needed to confirm the data with better statistics, as discussed in Ref. [19]. In fact a recent measurement of  ${}_{\Lambda\Lambda}^6\text{He}$  shows a weakly attractive

$\Delta B_{\Lambda\Lambda} \approx +1$  MeV for this nucleus<sup>[20]</sup>. We remark that the slightly repulsive bond energy is obtained because of no  $\Lambda\Lambda$  interaction in our model, since the underlying NSC89 potential does not provide it. Results obtained with the NSC97 potentials including hyperon-hyperon interactions yield similarly small numbers, however<sup>[7, 21]</sup>.

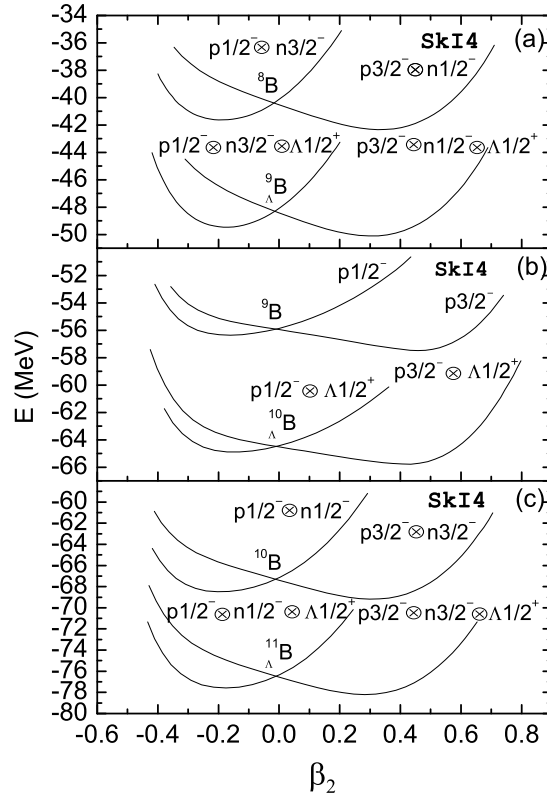


Fig. 2. Self-consistent DSHF calculations with the SkI4 interaction for (a)  ${}^8\text{B}$  and  ${}^9_{\Lambda}\text{B}$ , (b)  ${}^9\text{B}$  and  ${}^{10}_{\Lambda}\text{B}$ , and (c)  ${}^{10}\text{B}$  and  ${}^{11}_{\Lambda}\text{B}$ . The abbreviation “p” stands for the proton configuration.

The predicted deformations of  ${}^8\text{Be}$ ,  ${}^9_{\Lambda}\text{Be}$  and  ${}^{10}_{\Lambda\Lambda}\text{Be}$  are very similar, namely  $\beta_2 = 0.63$ ,  $\beta_2 = 0.59$  and  $\beta_2 = 0.55$ , respectively. The calculations suggest that the core nucleus  ${}^8\text{Be}$  and the  ${}^9_{\Lambda}\text{Be}$ ,  ${}^{10}_{\Lambda\Lambda}\text{Be}$  hypernuclei have similar deformation parameters with the same sign, which agree with the results of Ref. [22]. These results also justify the assumption of the same deformations in the core and the hypernuclei made in the Nilsson model potential<sup>[3]</sup>.

One notes in Fig. 1 that the ground state of  ${}^7\text{Be}$  has the quantum number  $K^{\pi} = n\frac{1}{2}^{-}$ , while  $K^{\pi} = n\frac{3}{2}^{-}$  for  ${}^9\text{Be}$ . All the core nuclei  ${}^{7,8,9}\text{Be}$  have large prolate deformations, especially  ${}^8\text{Be}$ . The corresponding hypernuclei have similar shapes for the ground states. We can see the same phenomena in Fig. 2: the ground

states of  ${}^{8,9,10}\text{B}$  and the corresponding hypernuclei are prolate with little difference of the deformation parameter  $\beta_2$ . Our calculations predict that the ground states of  ${}^9\text{Be}$  and  ${}^{10}\text{B}$  are prolate, which is consistent with the experimental data of the  $Q$ -moments of these nuclei<sup>[4]</sup>.

## 4 Summary

In summary, we studied deformations of core and hypernuclei of Be and B isotopes using an extended DSHF formalism. To this purpose we introduced the microscopic  $\Lambda N$  interaction of Refs. [6, 7] extended to isospin-asymmetric matter, together with nuclear pairing correlations and SkI4 Skyrme forces.

We found that the calculated large prolate deformations of the  $p$ -shell nuclei  ${}^9\text{Be}$  and  ${}^{10}\text{B}$  are confirmed by the experimental data of  $Q$  moments. The calculated core nuclei and the corresponding hypernuclei have similar deformations with the same sign, which agrees with the calculations of the  $\alpha$ -cluster model in Ref. [22]. The obtained  $\Lambda$  binding energies

$B_\Lambda$  confront satisfactorily with the experimental values. A proper treatment of deformations is important in the future study of hypernuclei regarding not only the binding energies of hypernuclei  $B_\Lambda$  and  $B_{\Lambda\Lambda}$ , but also the other properties like the fine structure and nonmesonic decays of hypernuclei.

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