

# Three dimensional study of harmonic operation at the Shanghai deep ultraviolet free electron laser<sup>\*</sup>

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**Abstract** With the right choice of parameters in the free electron laser (FEL) scheme, the undulator can be primarily operated at high order harmonic modes and the harmonic radiation is expected to be significantly enhanced. Recently, the possibility of proof-of-principle harmonic operation experiments on the basis of the Shanghai deep ultraviolet (SDUV) FEL test facility has been studied. In this paper, the principle of harmonic operation, three dimensional numerical approaches, and detailed performances of proposed harmonic operation at SDUV FEL are presented.

**Key words** harmonic operation, linear harmonic, superradiant

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## 1 Introduction

The availability of high intensity, ever shorter wavelength radiations, especially hard X-ray radiations, is of great interest in the free electron laser (FEL) community [1–4]. However, the available shortest period length of present undulators is limited by the practical difficulty in undulator technology. Thus an FEL scheme operating at fundamental mode requires a high energy electron beam to reach the short-wavelength regime. In order to relax the rigorous requirement for electron beam energy in a short-wavelength FEL, alternative FEL schemes operating at high order harmonic modes have been proposed recently [5–8]. With the right choice of parameters in such “harmonic operation” schemes, the main undulator emits principally at high order harmonics instead of the fundamental frequency in the conventional FEL scheme. A proof-of-principle harmonic operation experiment on the basis of the Shanghai deep ultraviolet (SDUV) FEL test facility [9] has been studied in a one dimensional (1D) way [10]. In this paper, 3D investigations of the proposals of harmonic operation at SDUV FEL are carried out and presented.

## 2 Principle of harmonic operation

In FEL, the fundamental mode grows faster than the harmonic and tends to dominate in the nonlinear region. Thus, the preliminary limitation in harmonic operation is the suppression of the fundamental. Latham proposed an FEL amplifier (see scheme 1, Fig. 1) in which a signal at the harmonic frequency is injected and the fundamental is allowed to grow from shot noise [5]. In such a scheme, harmonic radiation dominates in the undulator. The saturation of harmonic radiation is expected before the fundamental radiation increases. However, the feasibility of the harmonic operation amplifier is absolutely determined by the input signal.

To make harmonic operation schemes more feasible, using the same basic elements as high gain harmonic generation (HGHG) [3] (see scheme 2, Fig. 1), a linear harmonic operation by seeding was proposed [6, 7]. With the chosen parameters of the modulator, the dispersive section, the radiator and the seed laser, the electron beam density modulation entering in the final radiator may be resonant to a higher harmonic of the radiator radiation, instead of the fundamental in standard HGHG FEL. The

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coherent radiation of the  $n^{\text{th}}$  harmonic, which is the  $m^{\text{th}}$  harmonic of the seed laser, is rapidly produced and amplified exponentially until saturation. Meanwhile, the resonant radiation of the radiator starts from spontaneous, incoherent radiation of the electron beam. To generate an interesting wavelength in linear harmonic operation by seeding, a seed laser with a longer wavelength is needed when compared with Latham's amplifier.

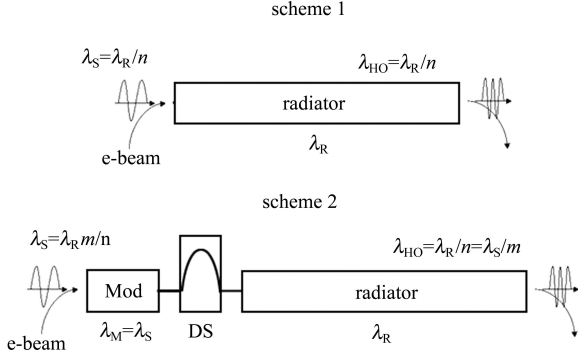


Fig. 1. Layout of harmonic operation schemes.

In the linear harmonic operation scheme, the linear harmonic dominates in the radiator. Also before saturation of harmonic generation, the fundamental radiation is so weak that nonlinear effects can be neglected. Thus the pendulum equation can be written as

$$\theta'' = -\frac{eK}{4k_w mc^2 \gamma^2} [JJ]_n E_n e^{in\theta} + c.c. \quad (1)$$

where  $n$  is the odd harmonic order;  $\theta$  describes the FEL bunching action;  $K$  is the dimensionless undulator magnetic parameter; the  $[JJ]_n$  factor is the difference of the Bessel functions defined as  $[JJ]_n = (-1)^{(n-1)/2} [J_{(n-1)/2}(n\xi) - J_{(n+1)/2}(n\xi)]$ , where  $\xi = K^2/(4+2K^2)$ ;  $k_w$  is the wave number of the undulator magnet;  $\gamma mc^2$  is the energy of the relativistic electron beam and  $E_n$  is the slowly varying envelope of the electric field. If we define  $\phi = n\theta$ , Eq. (1) can be rewritten as

$$\phi'' = -\Omega_n^2 \sin(\phi + \varphi_n), \quad (2)$$

where  $\Omega_n$  is the synchrotron oscillation frequency, which reads

$$\Omega_n^2 = \frac{eK}{2k_w mc^2 \gamma^2} n [JJ]_n E_n. \quad (3)$$

Given the growth rate of linear harmonics [11]

$$\sqrt{3}\rho_n = \sqrt{3}n^{1/3} \left( \frac{[JJ]_n}{[JJ]_1} \right)^{2/3} \rho \quad (4)$$

where  $\rho$  is the FEL pierce parameter [2]. Linear harmonic radiation saturates when the synchrotron os-

cillation rate reaches the linear harmonic growth rate. Thus, the 1D solution for the saturation power yields

$$P_n^{1D} \cong \frac{\rho_n}{n} P_{\text{beam}}. \quad (5)$$

The estimate has been validated in Ref. [10]. However, the 1D model gives the highest possible harmonic gain and can be used just as a reference for a non-ideal electron beam. In order to take the 3D effects into account, we refer to the fundamental saturation power empirically derived by fitting the simulation results [12], and extend it to the linear harmonics,

$$P_n^{3D} \approx 1.6 (L_{gn}^{1D}/L_{gn}^{3D})^2 P_n^{1D}. \quad (6)$$

Furthermore, we validated the analytical estimate by 3D simulation where the dynamics of the electron phase space is only forced by harmonic radiation, and good agreement is observed (see in Fig. 2). The parameters used in simulation are close to SDUV FEL parameters, as will be mentioned below.

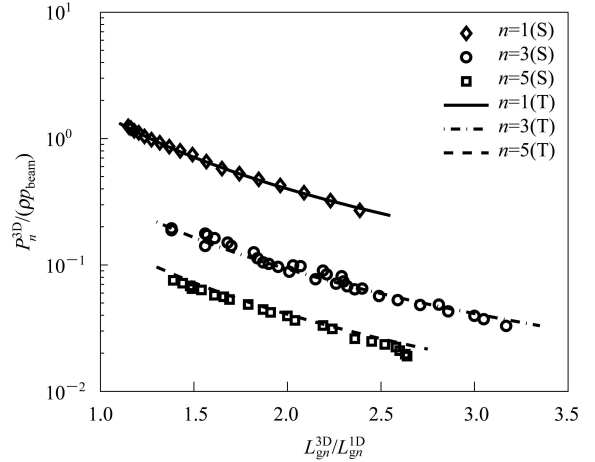


Fig. 2. Saturation power in respect of gain length of linear harmonic radiation. S is the simulation result and T is the empirical estimate.

In general, the saturation power of the 3<sup>rd</sup> and the 5<sup>th</sup> harmonics in the 3<sup>rd</sup> and the 5<sup>th</sup> linear harmonic operation by seeding is 10% and 4% of the fundamental saturation, respectively. This indicates a significant enhancement of the harmonic radiation when compared with nonlinear harmonic generation [13–15]. However, linear harmonic operation by seeding is more sensitive to the energy spread, peak current and emittance of the electron beam than the fundamental.

Another modification of the so-called superradiant harmonic operation by seeding has been proposed [8], where the difference to linear harmonic

operation by seeding is the seed laser. In superradiant harmonic operation by seeding, a high intensity, short pulse seed laser is injected into the modulator to generate strong harmonic bunching. Thus in the radiator, the radiation power of coherent harmonic generation experiences quadratic growth via the superradiant mechanism. The whole process can be analytically optimized by the HGHG theories [16]. Moreover, when operated in the superradiant regime, the energy exchange between the electron beam and the laser field is proportional to the slippage length scaled with the resonant wavelength even if the radiation is amplified at the  $n^{\text{th}}$  harmonic. Thus, superradiant harmonic operation by seeding may be the most feasible and promising harmonic operation scheme.

### 3 Proposals at SDUV FEL

SDUV FEL [9] is a 262 nm HGHG type test facility. Currently, a photocathode gun is under commissioning. In recent tests, a 0.7 nC electron beam with normalized emittance less than 3 mm-mrad is generated by a 50  $\mu\text{J}$ , 262 nm driven laser. This gun is expected to replace the 90 kV grid gun of the existing 100 MeV LINAC whose energy will be increased to 160 MeV. Fabrications of the bunch compressor chicane, the radiator undulator, the seed laser system and beam diagnostics have been achieved. The nominal parameters of the scheme are listed in Table 1.

Table 1. Nominal parameters of SDUV FEL.

parameters	value
seed laser wavelength	1048 nm
seed laser duration	10 ps
electron beam duration	2 ps
electron beam energy	160 MeV
peak current	300 A
normalized emittance	6 mm-mrad
local energy spread	$5 \times 10^{-5}$
modulator period length	50 mm
modulator length	0.80 m
modulator gap	variable
radiator period length	25 mm
radiator length	9.00 m
radiator gap	10 mm
radiator resonant wavelength	262 nm

In SDUV FEL, 1 variable gap modulator segment and 6 fixed gap radiator segments allow configuration of the linear harmonic operation by seeding. For significant power generation in principle verification, we choose  $n=3$  and  $m=4$ . Another advantage of the choice is that the existing 1048 nm Nd: YLF seed

laser system and 262 nm diagnostics system can be directly used. Thus what we need to do is to adjust the gap of the modulator and degrade the electron beam energy to 91 MeV to satisfy the required resonant relationship. Moreover, a two-stage HGHG demonstration has been proposed at SDUV FEL [17], where sub-100 fs, 87 nm FEL radiation will be generated from the 100 fs, 1048 nm seed laser. The 100 fs seed laser shows an opportunity of superradiant harmonic operation by seeding at SDUV FEL.

### 4 Numerical results

In harmonic operation, the fundamental and harmonic radiation grow at different rates. However, whether in linear or superradiant harmonic operation, the harmonics grows much slower than fundamental radiation. Thus, correct modeling of the shot noise for the fundamental and the harmonic at the entrance of the radiator is crucial. The most popular algorithm for harmonic shot noise is given by Fawley [18]. However, in harmonic operation by seeding, it can not straightforwardly set the shot noise consistently in the modulator and the radiator [8]. In order to correctly model the shot noise, in the first run, the initial phase space distribution of the electron beam is loaded in a virtual modulator resonant to 3152 nm which is 3 times the seed laser wavelength, and the particle distribution is dumped without any FEL process. Then the particle is imported to the real modulator where the 1048 nm seed laser's modulation is simulated, and at the end of the real modulator, the particle distribution is re-dumped. Finally, the particle distribution is re-imported to the radiator tuned to be resonant at the  $(4/3)^{\text{th}}$  harmonic of the seed laser, i.e. 786 nm. Because the seed laser and all the harmonic radiations in the radiator are harmonics of the resonant frequency of the virtual modulator, it assures consistent shot noise.

Owing to the introduced virtual modulator, the macro particles are distributed in  $[-n\pi, n\pi]$  in the modulator and  $[-m\pi, m\pi]$  in the radiator. Then the initial electron beam distributions artificially satisfy the correct shot noise statistics in the modulator and radiator. Fig. 3 shows us an example of the phase space at the exit of the modulator and the entrance of the radiator. Therefore, in order to properly model harmonic operation by seeding, modifying the parameter convharm in GENESIS2.0 [19] from integral type to real type is enough. Now we give details of the simulation results.

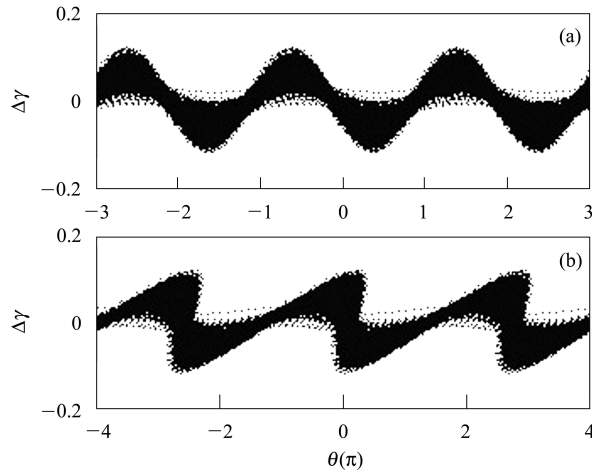


Fig. 3. The phase space at the exit of the modulator (a) and at the entrance of the radiator (b).

#### 4.1 Linear harmonic operation by seeding

As mentioned above, linear harmonic operation is very sensitive to the quality of the electron beam. To avoid bringing an excessively large increase of the energy spread in the electron beam at the entrance of the radiator, the seed laser power cannot be too large in linear harmonic operation. On the other hand, the harmonic efficiency enhancement in linear harmonic

operation is contributed from the limited fundamental effects before the saturation of harmonic radiation. Thus in linear harmonic operation of SDUV FEL, the 1048 nm, 10 ps laser with peak power of 0.5 MW is planned to be injected into the modulator, and the dispersion term  $R_{56}$  of the dispersive section is 0.24 mm.

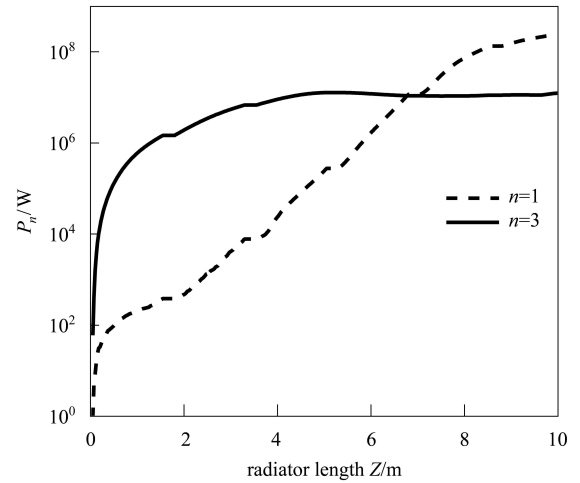


Fig. 4. The peak power growth of the fundamental 786 nm and the 3<sup>rd</sup> harmonic 262 nm in linear harmonic operation by seeding at SDUV FEL.

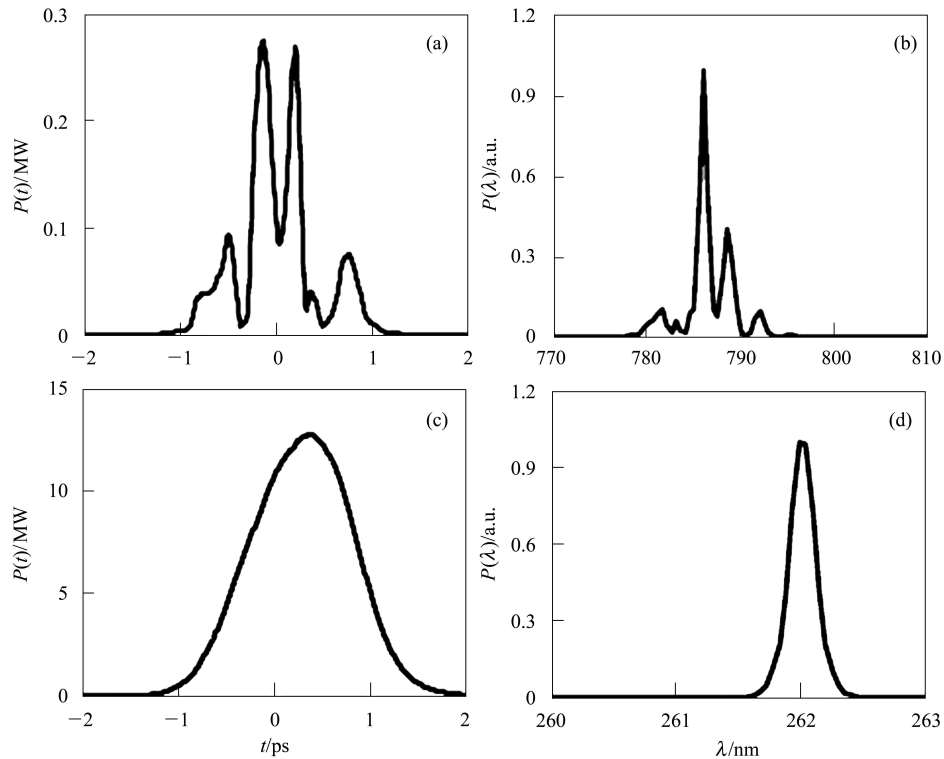


Fig. 5. The output performance after 3 segments of radiator in linear harmonic operation by seeding at SDUV FEL. (a) is the fundamental radiation pulse, (b) is the fundamental radiation spectrum, (c) is the 3<sup>rd</sup> harmonic radiation pulse and (d) is the 3<sup>rd</sup> harmonic radiation spectrum.

According to the simulation results, with the growth of the peak power  $P_n$  shown in Fig. 4, the 3<sup>rd</sup> harmonic 262 nm starts with a strong coherent radiation and dominates in the first 3 segments of radiator. In contrast, the fundamental 786 nm starts from shot noise and evolves as SASE FEL. In detail, 262 nm radiation with a peak power of 12.6 MW is obtained, which is almost at the 10% level of the fundamental saturation power in normal SASE operation.

Figure 5 shows the output power  $P(t)$  and the output spectrum  $P(\lambda)$  after 3 segments of radiator in linear harmonic operation by seeding at SDUV FEL. The output energy of 262 nm radiation is 17  $\mu$ J, which is about 100 times the 786 nm SASE. The pulse length (FWHM) of 262 nm is 1.2 ps. Moreover, the 262 nm radiation has good longitudinal coherence close to the Fourier Transform Limit.

#### 4.2 Superradiant harmonic operation by seeding

Since a strong slippage effect is involved in superradiant harmonic operation by seeding, the harmonic radiation is not very relevant to the modulated part of the electron bunch, but the fresh part of the electron bunch. To obtain a strong harmonic bunching at

the entrance of the radiator and a stable output performance, the future 1048 nm, 100 fs seed laser with peak power of 10 MW is planned to be injected into the modulator in superradiant harmonic operation by seeding at SDUV FEL, and the  $R_{56}$  of the dispersive section is 0.06 mm.

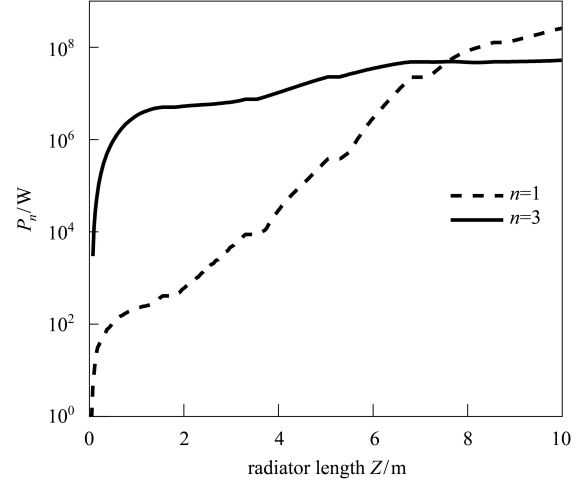


Fig. 6. The peak power growth of the fundamental 786 nm and the 3<sup>rd</sup> harmonic 262 nm in superradiant harmonic operation by seeding at SDUV FEL.

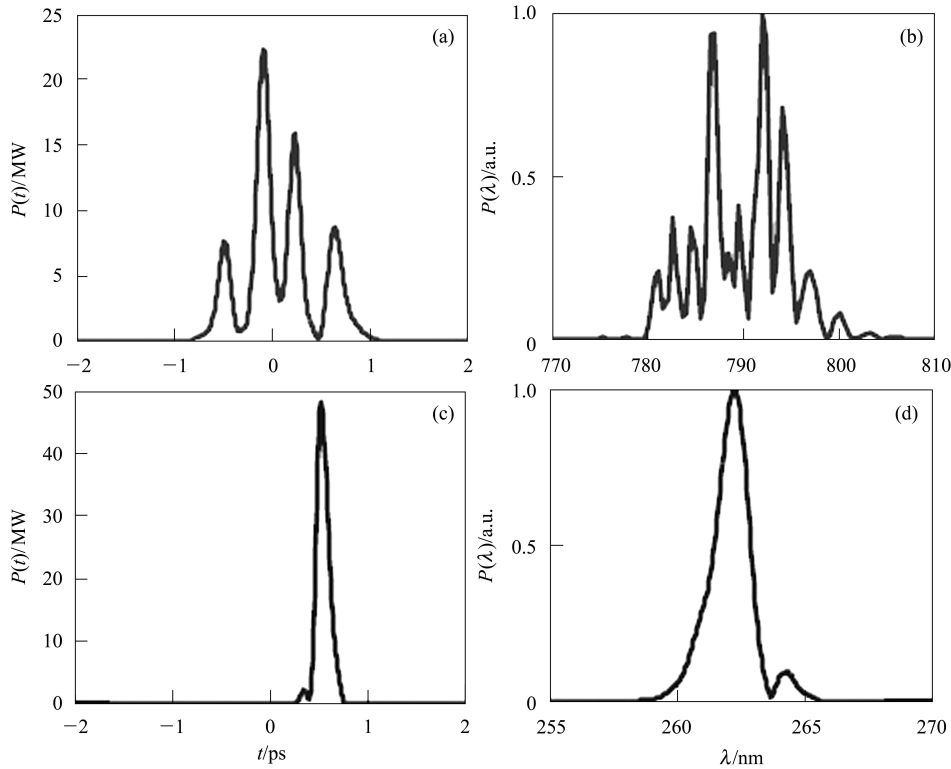


Fig. 7. The output performance after 4 segments of radiator in superradiant harmonic operation by seeding at SDUV FEL. (a) is the fundamental radiation pulse, (b) is the fundamental radiation spectrum, (c) is the 3<sup>rd</sup> harmonic radiation pulse and (d) is the 3<sup>rd</sup> harmonic radiation spectrum.

According to the simulation, the quadratic growth of the 3<sup>rd</sup> harmonic radiation works until the growth of the fundamental stops the superradiant growth by ultimately spoiling the longitudinal phase space of the electrons. Like the growth of the peak power  $P_n$  seen in Fig. 6, the 3<sup>rd</sup> harmonic 262 nm reaches a maximum peak power of 48 MW after 4 segments of radiator, which is 40% of the fundamental saturation power in normal SASE operation.

Figure 7 shows the output power and the FEL spectrum after 4 segments of radiator in superradiant harmonic operation of SDUV FEL. The output energy of 786 nm radiation is 9.6  $\mu$ J, while the 262 nm radiation is 7.3  $\mu$ J. The pulse length (FWHM) of 262 nm is 124 fs, which is larger than that of the seed laser. The lengthening is due to the exponential gain growth associated saturation in the 2<sup>nd</sup> radiator segment [20].

## 5 Conclusions

Compared with the conventional FEL operating at the fundamental frequency, harmonic operation significantly enhances the harmonic efficiency, and generates the short wavelength radiation using a lower energy electron beam. This may be attributed to the miniaturization of the FEL scheme. For the two harmonic operation modes, study shows that the linear harmonic operation has a better spectrum and larger pulse energy, while superradiant mode produces strong peak power, and an ultra-short radiation pulse at the expense of spectral purity.

In this paper, harmonic operation proposals at

SDUV FEL are investigated in a 3D way. We use a compressed electron bunch with a nominal peak current of 300 A and an emittance of 6 mm-mrad to illustrate the principle. An alternative proposal using the uncompressed electron beam has also been studied, where the emittance of the electron bunch is modeled to be 3 mm-mrad, while the peak current is only 100 A. The 3<sup>rd</sup> harmonic 262 nm radiation with peak power of 4.7 MW and 16 MW is calculated in linear and superradiant harmonic operation by seeding, respectively. A good feasibility is shown in harmonic operation proposals at SDUV FEL.

Generally, the harmonic radiation is pretty sensitive to undulator errors. The first measurement of the radiator magnetic field of SDUV FEL shows a random phase error of 4.8° for each segment. Thus, a theoretical estimate of the reduction of the output energy [21] in the proposals can be given. In linear harmonic operation by seeding, the 3<sup>rd</sup> harmonic 262 nm radiation may have an energy reduction of 23% after 3 segments of radiator. In superradiant harmonic operation by seeding, the 3<sup>rd</sup> harmonic 262 nm radiation may degrade by 5% after 4 segments of radiator. This is because, in the superradiant regime, the harmonic energy is proportional to the electron numbers slipped over by the radiation wave, which performs as the fundamental. It should be noted that the numerical model with the measured undulator field and further shimming to the undulator should be carried out before the detailed experiment.

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