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# Multi-modal analysis for low energy neutron-induced fission of $^{235}$U*

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**Abstract** Low energy neutron induced fission of $^{235}$U is studied in the framework of the multi-modal fission model. The fission fragment properties, such as the yields, the average total kinetic energy distribution and the average neutron separation energy, are investigated for incident neutron energies from thermal to 6.0 MeV. The multi-modal fission approach is also used to evaluate the prompt fission neutron multiplicity and spectra for the neutron-induced fission of $^{235}$U with an improved version of the Los Alamos model for incident neutrons below the (n, nf) threshold. The three most dominant fission modes are taken into account. The model parameters are determined on the basis of experimental data. The calculated results are in good agreement with the experimental data.

**Key words** $^{235}$U (n, f), multi-model, prompt neutron, fragment property

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## 1 Introduction

The fission fragment yield and kinetic energy distributions, and the prompt fission neutron multiplicity and spectrum of actinide nuclei are crucially important nuclear data for nuclear application and theory, and new calculations of these quantities with higher accuracy are required [1].

For low energy fission of the actinide nuclei, a detailed analysis of the experimental data of the fragment mass distributions seems to show that multi-modal fission is the most promising model [2–6]. Along the lines of the multi-modal model, Knitter et al [4] have presented a formula for the fission fragment (FF) yield as a function of fragment mass and total kinetic energy, which is in good agreement with the fragment mass, total kinetic energy and variance distributions of the thermal neutron induced fission of $^{235}$U. Brosa [2] et al have calculated the potential energy surfaces for several nuclides, and their calculations have shown that there are indeed channels on the potential energy surface of the fissioning system and the fragment properties can be predicted with the random neck rupture model.

In the prompt fission neutron energy spectrum and multiplicity of actinide calculation, several new models instead of the Maxwellian and Watt spectra [7] have been developed in the recent two decades. The Dresden model [8] and the Hauser-Feshbach calculations [9] require too many input data though they probably yield more accurate results. The widely-used Los Alamos (LA) model [10] accounts for some physical effects such as the variety of the FFs with various excitation energies and has good predictive power with using less input parameters.

With the development of the multi-modal random neck-rupture model, the different fission modes can now generate the different energy partitions and the nuclear properties of the FFs before neutron emission. Thus the prompt neutron spectrum and the multiplicity are dependent on the fission mode. For the neutron induced fission of actinides, the multi-

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modal analysis of the prompt neutron spectrum and the multiplicity is necessary and has been applied to some calculations of the prompt neutron spectrum of actinide isotopes by Ohsawa [11–13], Hambsch [14–16], Vladuca [17] and Zheng [18] et al. The basic discrepancy among recent studies arises from the fact that different analysis methods for the fission fragment distributions were used.

In the present paper a detailed analysis of the new experimental data of the fragment mass distributions for $^{235}$U (n, f) at the Institute for Reference Materials and Measurements (IRMM) is given. The three main fission modes, standard 1 (S1), standard 2 (S2) and super long (SL), are taken into account to determine the input model parameters and to produce the FF yields and total kinetic energy distributions. The prompt fission neutron spectrum and the multiplicity are calculated by using the multi-modal approach. The partial spectrum and multiplicity for each mode are calculated separately by using the improved LA model, and the total neutron spectrum and multiplicity are synthesized and compared with the experimental data.

## 2 Methods and results

### 2.1 Multi-modal mass distribution of the FF

According to the Moriyama-Ohnish systematics [19], the FF mass distribution can be written as

$$Y(A)=\sum_{m} Y_m(A), \tag{1}$$

where the FF mass distribution of mode $m$ is expressed in the following form

$$Y_m(A)=\frac{\omega_m}{\sigma_{Am}\sqrt{2\pi}}\exp\left(-\frac{(A-\overline{A}_m^2)}{2\sigma_{Am}^2}\right), \tag{2}$$

where $\omega_m$ is the channel probability of channel $m$, $\overline{A}_m$ is the most probable mass number of channel $m$, $\sigma_{Am}$ is the width of channel $m$. The sum over all of them must reproduce the experimental data.

The present work useds the Gauss-Newton nonlinear least square method to analyze the FF mass distribution. The experimental FF mass ranges $A_{\rm L}\in[76,118]$ and $A_{\rm H}\in[118,160]$ are taken into account. For each FF mass pair, four isobars per mass are taken into account with values of the nuclear charge $Z$ which are the nearest integer values above and below the most probable charge. Comparisons between the calculations and the experimental data for $^{235}$U (n, f) at $E_{\rm n}=1.0$, 2.0, 3.0, 4.0, 5.0, 6.0 MeV are shown in Fig. 1.

The behavior of the modal branching ratio versus the incident neutron energy is shown in Fig. 2. From Figs. 1, 2, it is found that the channel probability of SL grows obviously with the incident neutron energy.

Compared with Fan's [5,6] work, the parameters of the three modes in the present work show some differences, and our calculations can reproduce the experimental data much better. A comparison between the present calculation and Fan's work for $^{235}$U (n, f) at $E_{\rm n}=2.5$ MeV is shown in Fig. 3.

### 2.2 Multi-modal average total kinetic energy of the FF

The average total kinetic energy of the FF for fission mode $m$ is calculated as

$$\langle TKE\rangle_m=\frac{\sum_i Y_{m,i}\overline{TKE_i}}{\sum_i Y_{m,i}}, \tag{3}$$

where $\overline{TKE}_i$ is the average total kinetic energy of the $i^{\rm th}$ FF pair which can be written as

$$\overline{TKE}_i=E_{{\rm Cou},i}+E_{{\rm nuc},i}+K_{{\rm s},i}, \tag{4}$$

with $E_{{\rm Cou},i}$ and $E_{{\rm nuc},i}$ being the Coulomb repulsion and nuclear energy between the newborn fragments, and $K_{{\rm s},i}$ is the prescission kinetic energy.

In the present work, the formulae given by Brosa et al are used to calculate $E_{{\rm Cou},i}$ and $E_{{\rm nuc},i}$. There are experimental indications that the values of $K_{{\rm s},i}$ are small and an upper limit of 8 MeV [20] is used in this work. The calculated values of the average total kinetic energy $\overline{TKE}(A)$ of the FFs from $^{235}$U (n, f) at $E_{\rm n}$ = 5.5 MeV are plotted versus the heavy fragment mass in Fig. 4(a). The average total kinetic energies of the fragments for the S1, S2 and SL modes each as a function of the incident neutron energy obtained from our calculations are shown in Fig. 4(b) as obtained by our calculations.

It is evident from Fig. 4 that the present calculated FF kinetic energies agree well with the experimental data and there is no evident dependence on incident neutron energy. This conclusion is the same as in Fan's early work [5,6], but there are several MeV difference between the two works, and the present work can reproduce the experiment much better.

### 2.3 Average energy released from each mode

The average energy released from fission mode $m$ is calculated by the following relation:

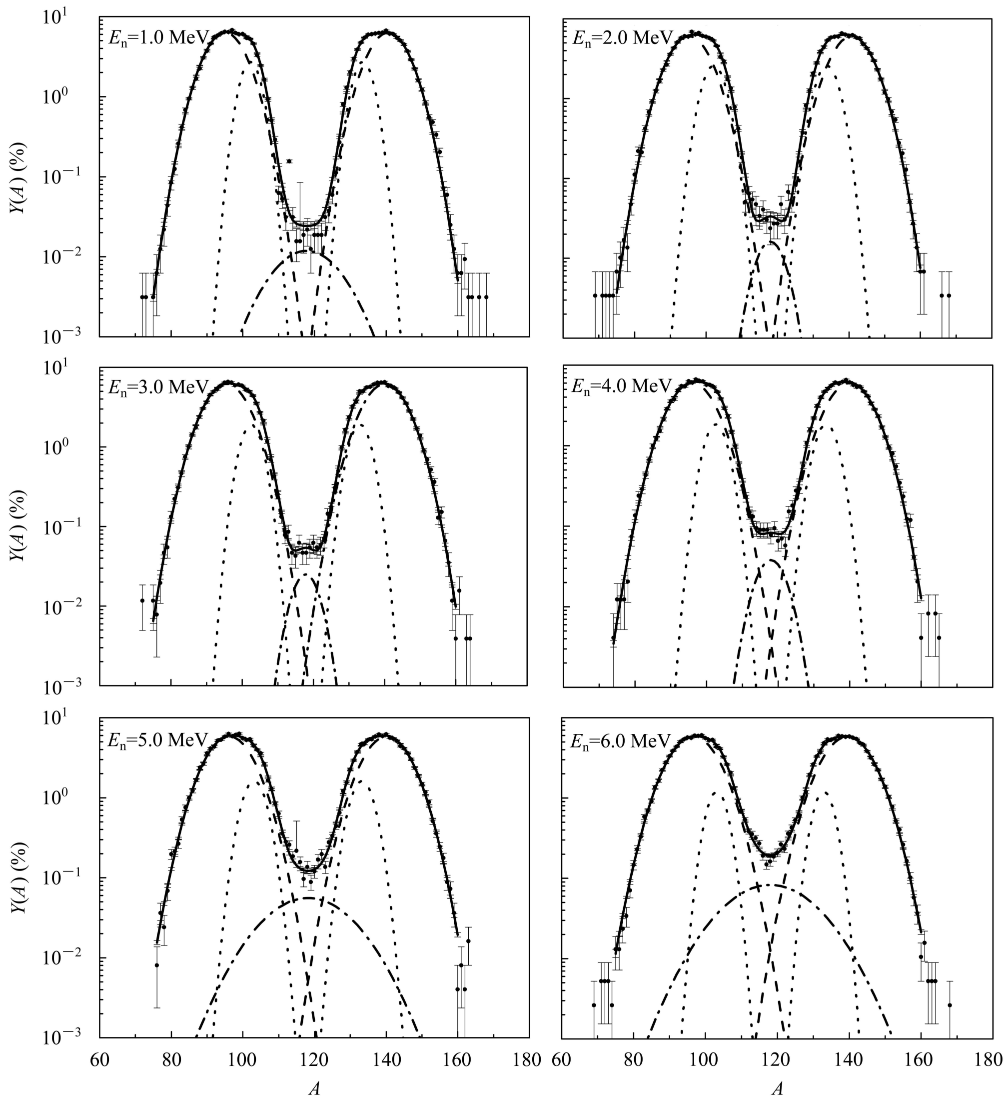


Fig. 1. Mass distributions of fission fragments for $^{235}$U(n, f) for incident neutron energies from 1.0 to 6.0 MeV. The contributions of fission modes are shown with dotted (for S1 mode), dashed (for S2 mode), and dash-dotted (for SL mode) lines separately. The solid curves show the superposition results of the three modes. The experimental data are shown with solid circles.

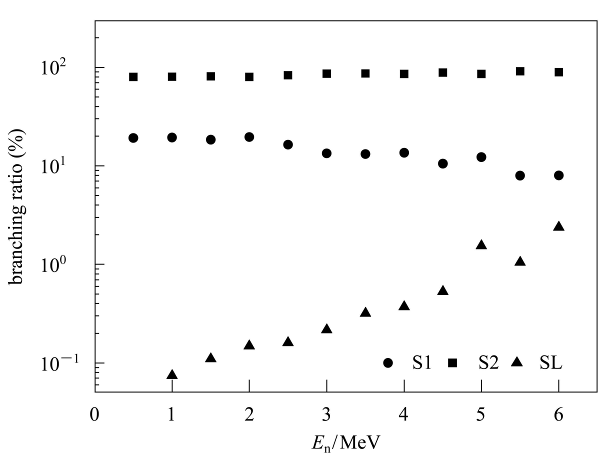


Fig. 2. Multi-modal fission branching ratios as a function of the incident neutron energy for $^{235}$U (n, f).

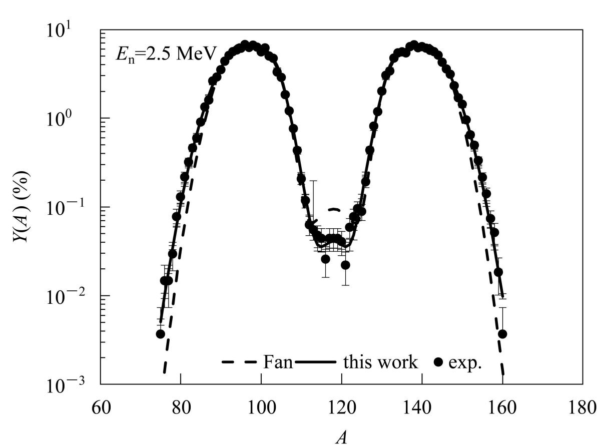


Fig. 3. Comparison of the calculated and experimental mass distributions of the FFs for $^{235}$U(n, f) at $E_n = 2.5$ MeV.

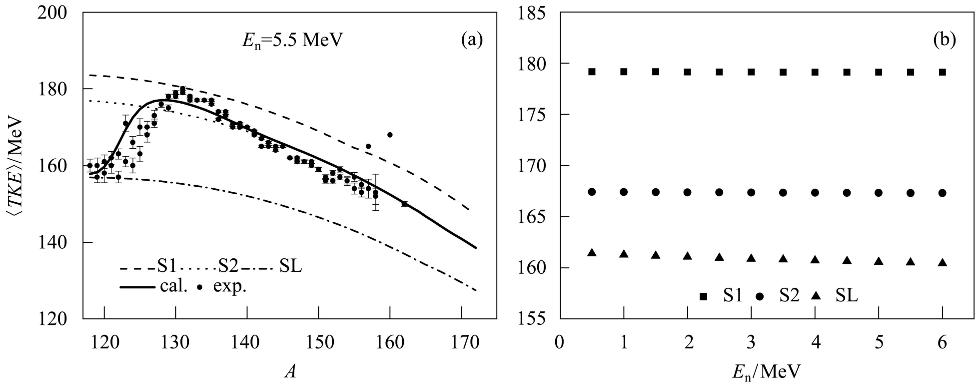


Fig. 4. (a) Comparison of experimental data [21] with the corresponding fit of the $\overline{TKE}(A)$ for $^{235}$U(n, f) at $E_n = 5.5$ MeV, where the dashed lines represent the contributions from the single modes and the solid line denotes the superposition result of the three modes; (b) Calculated values of the average total kinetic energy $\overline{TKE}(A)$ of the fragments of $^{235}$U(n, f) for the S1, S2 and SL modes as a function of the incident neutron energy.

$$\langle E_r\rangle_m = \frac{\sum_i Y_{mi} \sum_{j=z_p-2}^{j=z_p+2} pz_{i,j} E_{ri,j}}{\sum_i Y_{mi} \sum_{j=z_p-2}^{j=z_p+2} pz_{i,j}}, \tag{5}$$

where $m$ is the index of the mode (S1, S2, SL), $i$ is the range of FF pairs, $Y_{mi}$ is the FF mass yield for mode $m$ and FF pair $i$, and $pz_i$ is the charge distribution corresponding to the FF pair $i$, approximated by the Gaussian function [22]. The dependence of $\langle E_r\rangle_m$ on the incident neutron energy $E_n$ is shown in Fig. 5(a).

### 2.4 Multi-modal average neutron separation energy of the FF

The average neutron separation energy of the FF for fission mode $m$ is calculated by the following expression:

$$\langle S_n\rangle_m = \frac{\sum_i Y_{mi} \sum_{j=z_p-2}^{j=z_p+2} pz_{i,j} \overline{S_{n,i,j}}}{\sum_i Y_{mi} \sum_{j=z_p-2}^{j=z_p+2} pz_{i,j} \overline{S_{n,i,j}}}, \tag{6}$$

where the average neutron separation energy of FF pair $i$ is given by:

$$\overline{S_{n,i}} = \frac{1}{2}(\overline{S^L_{n,i}} + \overline{S^H_{n,i}}) \quad \text{and}$$

$$\overline{S^{L,H}_{n,i}} = \frac{1}{2}\left(S^{L,H}_{n,i} + \frac{S^{L,H}_{2n,i}}{2}\right), \tag{7}$$

where $S_n$ and $S_{2n}$ are calculated from the mass excess by using the Reference Input Parameter Library, and $S_{2n}$ is used to remove pairing effects [15].

The dependence of $\langle S_n\rangle_m$ on the incident neutron energy $E_n$ is plotted in Fig. 5(b).

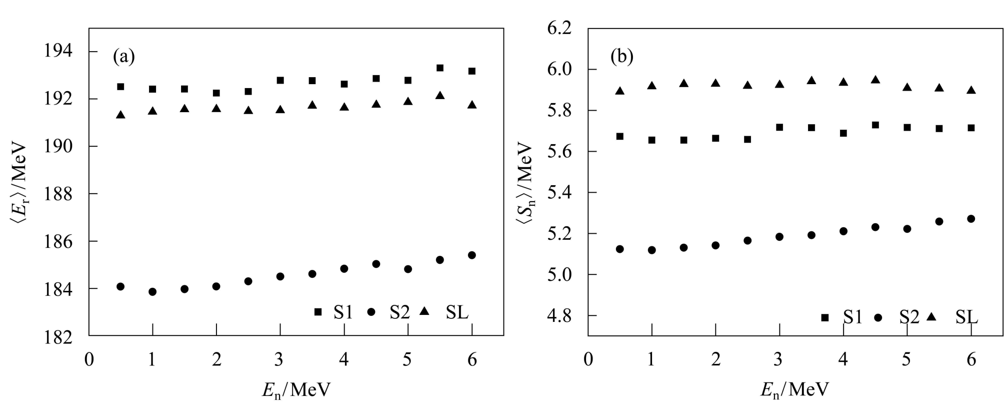


Fig. 5. Average FF neutron separation energies (a) and average energies released from fission (b) as a function of the incident neutron energy for S1, S2 and SL for $^{235}$U(n, f).

### 2.5 Multi-modal calculation of the prompt neutron spectrum and multiplicity

In the frame of the multi-modal fission approach, the prompt neutron spectrum $N_{\rm tot}(E)$ and multiplicity $\langle\upsilon\rangle_{\rm tot}$ at incident energy are calculated as a superposition of the prompt neutron spectra and the multiplicities associated with a particular fission mode from the following expressions

$$N_{\rm tot}(E)=\frac{\sum_m \omega_m\langle\upsilon\rangle_m N_m(E)}{\sum_m \omega_m\langle\upsilon\rangle_m}, \tag{8}$$

and

$$\langle\upsilon\rangle_{\rm tot}=\frac{\sum_m \omega_m\langle\upsilon\rangle_m}{\sum_m \omega_m}, \tag{9}$$

where $\omega_m$ is the branching ration of mode $m$, $N_m(E)$ and $\langle\upsilon\rangle_m$ are, respectively, the prompt fission neutron spectrum and the multiplicity for the mode $m$:

$$N_m(E)=\frac{1}{2}(N_m^{\rm L}(E,E_{\rm f}^{\rm L},\sigma_{\rm c,L})+N_m^{\rm H}(E,E_{\rm f}^{\rm H},\sigma_{\rm c,H})). \tag{10}$$

Here $N_m^{\rm L}(E,E_{\rm f}^{\rm L},\sigma_{\rm c,L})$ and $N_m^{\rm H}(E,E_{\rm f}^{\rm H},\sigma_{\rm c,H})$ are the spectra of the individual light fragment (LF) and heavy fragment (HF) for mode $m$. The present work assumes the numbers of neutrons emitted from the LF and the HF to be identical. For each fission mode the prompt fission neutron spectrum $N^{\rm j}(E,E_{\rm f}^{\rm j},\sigma_{\rm c,j})$ of the individual FF (j = L or H) is calculated by using the improved LA model [23].

The average prompt neutron multiplicity of mode $m$ is obtained from energy conservation and can be calculated by using the following relationship:

$$\langle\upsilon\rangle_m=\frac{\langle E_{\rm r}\rangle_m+E_{\rm n}+B_{\rm n}-\langle TKE\rangle_m-\langle E_\gamma\rangle_m}{\langle S_{\rm n}\rangle_m+\langle\varepsilon\rangle_m}, \tag{11}$$

where $E_{\rm n}$ and $B_{\rm n}$ are the incident neutron energy and the neutron binding energy in the fissioning nucleus, $\langle E_{\rm r}\rangle_m$ and $\langle S_{\rm n}\rangle_m$ are the average energy released from fission and the average neutron separation energy from the FFs for mode $m$, and $\langle E_\gamma\rangle_m$ is the average prompt gamma ray energy [24], respectively. $\langle\varepsilon\rangle_m$ is the first-order moment of the center-of-mass system spectrum of mode $m$.

In the present work the neutron spectra of $^{235}$U(n, f) for incident energies below 6.0 MeV are calculated. The total prompt neutron spectra for $E_{\rm n}=0.0253$ eV and $E_{\rm n}$ = 0.5, 1.5, 2.0, 2.9 MeV are shown in Fig. 6 together with experiments [25–32]. From these figures it can be seen that the multi-modal calculations agree quite well with the experiments. From the calculations we find that the S1-spectrum is the softest one. The partial spectra for the S2 and SL modes are

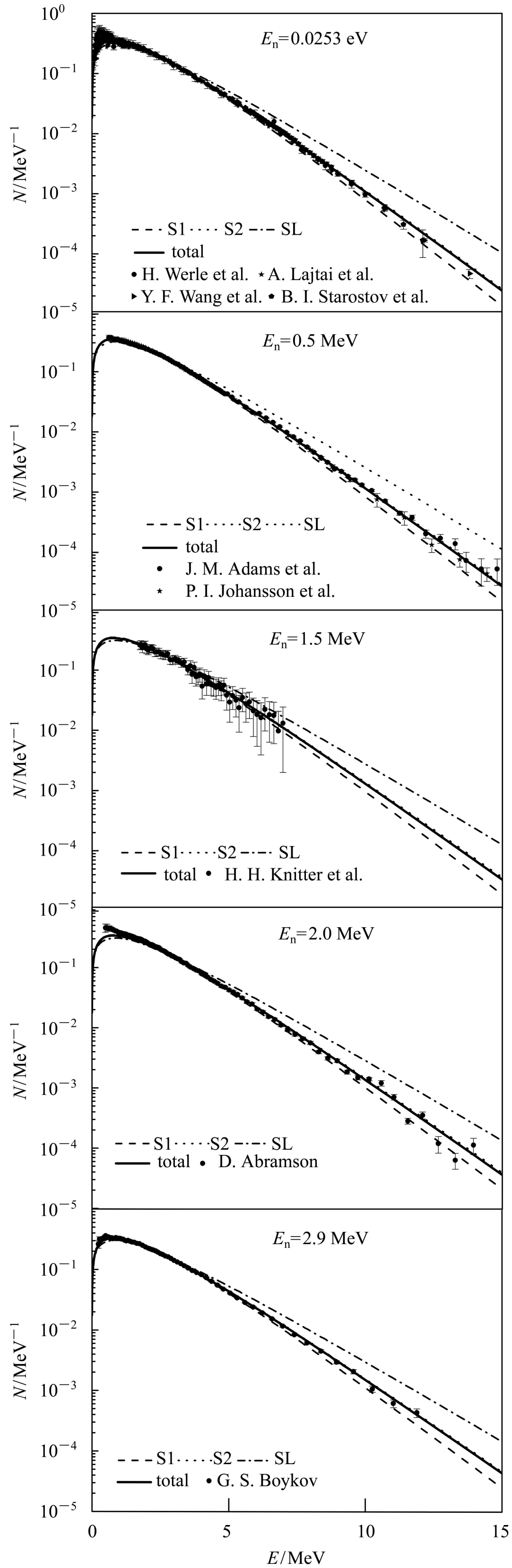


Fig. 6. Comparison of the partial spectra for the S1, S2, SL modes and the calculated total neutron spectra with the experimental data [25–32] for $E_{\rm n}$ = 0.0253 eV and $E_{\rm n}$ = 0.5, 1.5, 2.0, 2.9 MeV for $^{235}$U(n, f).

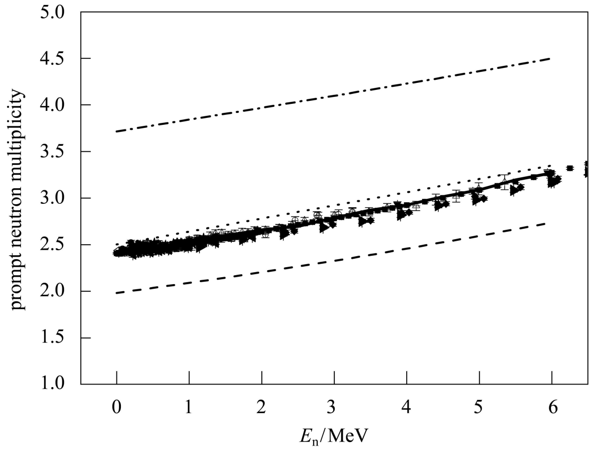


Fig. 7. Average prompt fission neutron multiplicity (solid line) as a function of incident neutron energy compared with the experimental data [33–43] for $^{235}$U(n, f), where the contributions of the three fission modes are also shown.

harder, i.e. the larger the deformation of the compound nucleus is, the harder its corresponding partial spectrum will be. These results agree with the experimental observation.

The total average prompt neutron multiplicity calculated with expression (11) agrees well with the experimental data [33–43], as shown in Fig. 7.

## 3 Conclusions

Low energy neutron induced fission of $^{235}$U is systematically studied in the framework of the multi-modal fission approach. The three most dominant fission modes (S1, S2, SL) are taken into account. The multi-modal fission branching ratios as a function of incident neutron energy for these three modes are presented. The fission fragment yield and average total kinetic energy distributions are investigated for incident neutron energies from thermal to 6.0 MeV. The prompt neutron spectra and multiplicities are evaluated by using improved version of the Los Alamos model within the multi-modal fission approach. The calculated results for the fission fragment and emission neutron properties are in good agreement with the experimental data. This work shows that the multi-modal fission model can be used to understand the systematic change of the post-scission phenomena for the reaction $^{235}$U(n, f) in the incident neutron energy range below the second chance fission threshold.

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