Particle-number-conserving analysis of multiquasiparticle bands in ^{177}Lu*

ZHANG Zhen-Hua(张振华)[1] QI Shou-Tao(齐寿涛)[1,2] SUN Bao-Xi(孙宝玺)[1,2]
LEI Yi-An(雷奕安)[1;1)] ZENG Jin-Yan(曾谨言)[1]

[1] State Key Lab of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China
[2] Institute of Theoretical Physics, College of Applied Sciences, Beijing University of Technology, Beijing 100124, China

Abstract The experimental one-, three-, and five-quasiparticle bands in ^{177}Lu are analyzed by the particle-number conserving (PNC) method for treating the cranked shell model with pairing interaction, in which the blocking effects are taken into account exactly. The experimental moments of inertia are reproduced very well by PNC calculations with us free parameter.

Key words multiquasiparticle band, blocking effect, moment of inertia, particle-number-conserving method

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1 Introduction

It has long been established that the reduction of the experimental nuclear moment of inertia (MOI) of the ground-state band (gsb) of an even-even well-deformed nucleus compared with the rigid-body value is a consequence of nuclear pairing interaction [1]. Moreover, the experimental MOIs of one-quasiparticle (qp) bands in odd-A nuclei are systematically larger than those of the gsb's of adjacent even-even nuclei [2]. Usually, the odd-even differences in MOIs are attributed to the presence of an odd particle (Pauli blocking effect), which leads to a reduction of the pairing gap parameter Δ, and hence the rotational parameter ($A = \hbar^2/2J$). As a rough estimation, a reduction of Δ^2 by half will cause a 15% increase in MOI [2]. However, the experimental odd-even differences in MOIs show large fluctuations [2, 3]. Assuming Δ_0 is the gap parameter for the gsb of an even-even nucleus, and Δ_ν is that for a 1-qp band of an odd-A neighboring nucleus with the Nilsson level ν being blocked by an unpaired nucleon, it is shown that the pairing reduction of gap parameter, $\delta\Delta_\nu = \Delta_0 - \Delta_\nu$, depends sensitively on the single-particle level spacing near the Fermi surface and the distance of the blocked Nilsson orbital ν to the Fermi surface [4]. However, as Rowe emphasized [5], while the blocking effects are straightforward, it is very difficult to treat them in the usual BCS (Bardeen-Cooper-Schriffer) formalism, because it introduces different quasiparticle bases for different blocked levels.

In recent years, a lot of low-lying excited rotational bands with intrinsic multi-quasiparticle states have been observed [6–9]. For the well-deformed rare-earth nuclei with $A \sim 180$ ($Z \sim 71$ – 74, $N \sim 98$ – 108), the single-particle spectrum is dominated by high-Ω (projection of angular momentum along the symmetry axis) orbitals near the Fermi surface (see Fig. 1), e.g., the proton orbitals **7/2$^-$[523]** (boldface denotes the high-j intruder orbitals), 7/2$^+$[404], **9/2$^-$[514]**, 5/2$^+$[402], and the neutron orbitals **5/2$^+$[642]**, 5/2$^-$[523], **7/2$^+$[633]**, 5/2$^-$[512], 7/2$^-$[514], **9/2$^+$[624]**, etc. This special situation gives rise to low-lying excited high-K multi-quasiparticle bands.

It is well known that, besides the space reflection symmetry, the well-deformed rare-earth nuclei also exhibit axial symmetry (rotation around the symmetry z axis) and symmetry of rotation of π around the x axis, $R_x(\pi) = \mathrm{e}^{-\mathrm{i}\pi J_x}$ [2]. Thus, the projection of nuclear total angular momentum along the symmetry

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1) E-mail: lei@phy.pku.edu.cn
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axis, $K=\sum_i \Omega_i$, is a good quantum number. For an even-even nucleus, $R_x(\pi)^2=1$, the eigenvalue of $R_x(\pi)$, signature $r=\mathrm{e}^{-\mathrm{i}\pi\alpha}=\pm 1$ or equivalently, the signature exponent $\alpha=0,1$. For an odd-A nucleus, $R_x(\pi)^2=-1$, $r=\pm i$ and $\alpha=\pm 1/2$. For the ground-state band of an even-even nucleus, all the nucleons are paired up due to strong pairing interaction, parity $\pi=+$, $\alpha=0$, $I=0,\ 2,\ 4,\ \cdots$. For an odd-A nucleus, where the Nilsson orbital 1 is blocked by an unpaired nucleon, we have two sequences of rotational levels with $\pi=\pi_1$, $K=\Omega_1$, $I\geqslant K$, $\alpha=\pm 1/2$. For a 3-quasiparticle band in an odd-A nucleus, when the Nilsson orbitals 1, 2 and 3 are blocked by unpaired nucleons, we have eight sequences with $\pi=\pi_1\pi_2\pi_3$, $K=|\Omega_1\pm\Omega_2\pm\Omega_3|$, $I\geqslant K$, $\alpha=\pm 1/2$. For the high-K 3-qp band, $K=|\Omega_1+\Omega_2+\Omega_3|$, $\alpha=\pm 1/2$.

In this article the experimental 1-qp, 3-qp and 5-qp bands in ^{177}Lu [7–9] are analyzed using the particle number conserving (PNC) method for treating the cranked shell model (CSM) with pairing interaction, in which the blocking effects are exactly taken into account. Details of the PNC formalism for treating the eigenvalue problem of the CSM Hamiltonian are given in [10, 11]. For convenience, the main formulation used in the present calculation is presented in Sect. 2. The PNC analysis for the low-lying multi-quasiparticle bands in ^{177}Lu, including the MOIs, bandhead energies and occupation probabilities of each cranked Nilsson orbital, are given in Sect. 3. Sect. 4 gives a brief summary.

2 A brief review of the PNC method for the CSM with pairing interaction

The CSM Hamiltonian of an axially symmetric nucleus in the rotating frame is [10, 11]

$$\begin{aligned} H_{\mathrm{CSM}} &= H_0+H_P, \\ H_0 &= H_{\mathrm{Nil}}-\omega J_x, \end{aligned} \tag{1}$$

$H_0=H_{\mathrm{Nil}}-\omega J_x$ is the one-body part of H_{CSM}, where H_{Nil} is the Nilsson Hamiltonian, $-\omega J_x$ is the Coriolis interaction with cranking frequency ω about the x axis (perpendicular to the nuclear symmetry z axis). H_{P} is the pairing interaction

$$H_{\mathrm{P}}=-G\sum_{\xi\eta} a^+_\xi a^+_{\bar\xi} a_{\bar\eta} a_\eta, \tag{2}$$

where $\bar\xi$ ($\bar\eta$) labels the time-reversal state of the Nilsson state ξ (η), and G is the effective strength of pairing interaction, which is determined by the experimental odd-even difference in binding energies, and it is not a free parameter.

The key point of the PNC method is that a cranked many-particle configuration (CMPC) truncation (Fock space truncation) is used instead of the single-particle level (SPL) truncation in common shell-model calculations. This is crucial to make the PNC calculations for low-lying excited states both workable and sufficiently accurate [12, 13]. The stability of the calculations using the Fock space truncation has been investigated in detail by the Dudek group [13].

Assume that an eigenstate of H_{CSM} is

$$|\psi\rangle=\sum_i C_i|i\rangle \qquad (C_i \text{ real}), \tag{3}$$

where $|i\rangle$ is a CMPC (an eigenstate of the one-body operator H_0). In the following calculations, the CSM Hamiltonian (1) is diagonized in a sufficiently large CMPC space to obtain the solutions to low-lying excited eigenstates of H_{CSM}. The dimension of the CMPC space is about 700 for proton and 800 for neutron. As we are only interested in the yrast and low-lying excited states, the number of important CMPC's involved (weight $>1\%$) is very limited (usually <20) and almost all the CMPC's with weight $>0.1\%$ are taken into account, so the solutions to the low-lying excited states are accurate enough.

The angular momentum alignment of $|\psi\rangle$ is

$$\langle\psi|J_x|\psi\rangle=\sum_i C_i^2\langle i|J_x|i\rangle+2\sum_{i<j} C_iC_j\langle i|J_x|j\rangle, \tag{4}$$

and the kinematic moment of inertia for state $|\psi\rangle$ is

$$J^{(1)}=\frac{1}{\omega}\langle\psi|J_x|\psi\rangle. \tag{5}$$

The occupation probability n_μ of the cranked orbital μ is $n_\mu=\sum_i |C_i|^2P_{i\mu}$, where $P_{i\mu}=1$ if μ is occupied in $|i\rangle$, and $P_{i\mu}=0$ otherwise.

Two points should be noted:

(a) Besides the parity π, the eigenvalue of $R_x(\pi)$, i.e., the signature $r=\mathrm{e}^{-\mathrm{i}\pi\alpha}$, or the signature exponent α is believed to be a good quantum number. Since $R_x(\pi)=\mathrm{e}^{-\mathrm{i}\pi J_x}$, $[J_x,J_z]\neq 0$, the signature scheme breaks the validity of K quantum number. Considering $[J_x,J_z^2]=0$, we can construct the simultaneous eigenstates of $(R_x(\pi),J_z^2)$. In fact, in the PNC calculations, each $|i\rangle$ in (3) is chosen as a simultaneous eigenstate of (H_0,J_z^2). Walker and Draculis [6] pointed out that some forms of K-mixing must exist to enable the K-forbidden transition observed in a lot of low-lying rotational bands of axially symmetric nuclei. However, by convention, K is still used as a convenient quantum number to describe rotational bands of deformed spheroidal nuclei, but it should

be kept in mind that the K structure of a rotational band may change with ω.

(b) In the PNC treatment for the CSM with Hamiltonian (1), the seniority (number of unpaired particles) is not a good quantum number either, due to the appearance of the antipairing Coriolis interaction. Moreover, though the total number of particles n ($=\sum_\mu n_\mu$) stays exactly constant, the occupation probability n_μ for each orbital μ may change with increasing rotational frequency ω.

3 Analysis of low-lying rotational bands in ^{177}Lu

3.1 The cranked Nilsson orbitals near the Fermi surface of ^{177}Lu

The Lund systematics of the Nilsson level scheme [14, 15] has been very successful for predicting the ground-state spins of well-deformed odd-A nuclei, particularly for the well deformed rare-earth ($150 < A < 190$) and actinide ($A > 225$) nuclei. However, deviation of the experimental bandhead energies of 1-qp bands in some rare-earth nuclei from the Lund systematics was found, e.g., see Ref. [16]. To give a reliable calculation for multiquasiparticle bands, it is necessary to adopt a more realistic single particle scheme. The Nilsson level scheme (Lund systematics) for protons and neutrons is adjusted to reproduce the bandhead of energies of the low-lying 1-qp bands in ^{177}Lu and ^{177}Hf respectively. The slightly adjusted Nilsson level scheme for ^{177}Lu is shown in Fig. 1. The deformation $\varepsilon_2 = 0.257$, $\varepsilon_4 = 0.057$ are taken from the Lund systematics [14] and no change is made, but a slight change in Nilsson parameters κ and μ is made (see the caption of Fig. 1).

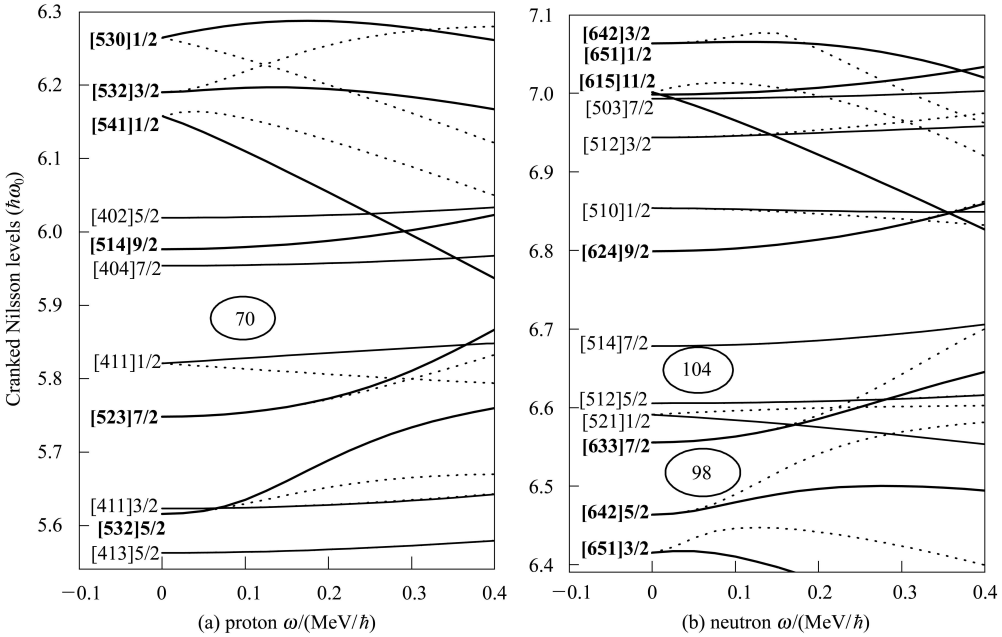

Fig. 1. The cranked Nilsson orbitals near the Fermi surface of ^{177}Lu. The deformation parameters (ε_2,ε_4) are taken from the Lund systematics [14], (ε_2,ε_4)=(0.257, 0.057). The Nilsson parameters κ and μ (Lund systematics [15]) are slightly adjusted to reproduce the bandhead energies of the low-lying 1-qp bands of ^{177}Lu (see Fig. 2). (a) For proton, $\kappa_4 = 0.061$ ($N = 4$ major shell), $\kappa_5 = 0.060$ ($N = 5$ major shell), $\mu_4 = 0.609$, $\mu_5 = 0.609$. In addition, the Nilsson level $1/2^+[411]$ is slightly shifted upward by $0.023\hbar\omega_0$, $7/2^+[404]$ is slightly shifted upward by $0.015\hbar\omega_0$. (b) For neutron, $\kappa_5 = 0.0677$, $\kappa_6 = 0.0636$, $\mu_5 = 0.432$, $\mu_6 = 0.370$, and $1/2^-[510]$ is shifted downward by $0.070\hbar\omega_0$, $\mathbf{7/2^+[633]}$ is shifted downward by $0.010\hbar\omega_0$, $\mathbf{9/2^+[624]}$ is shifted upward by $0.060\hbar\omega_0$.

Using Fig. 1(a), the bandhead energies and MOIs of the low-lying excited 1-quasiproton bands in ^{177}Lu [8] are well reproduced (see Figs. 2). Similarly, the experimental bandhead energies of low-lying 1-quasineutron bands in ^{177}Hf [17] are reproduced by Fig. 1(b). The simple shell model calculation (pairing interaction neglected) for the bandhead energies of four low-lying excited 1-qp bands of ^{177}Lu by the modified proton Nilsson level scheme (Fig. 1(a)) is shown in Fig. 2(a). The PNC calculated bandhead

energies with the pairing interaction involved are shown in Fig. 2(b), and are very close to the experimental results (Fig. 2(c)). The effective pairing interaction strength is determined by the experimental odd-even differences, $G_{\rm p}=0.32$ MeV, $G_{\rm n}=0.30$ MeV.

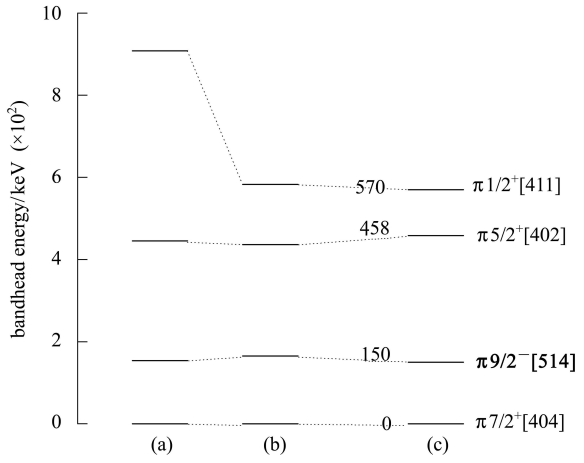

Fig. 2. The bandhead energies of the low-lying 1-quasiproton bands of ^{177}Lu. (a) The shell model calculation of the bandhead energies using the Nilsson level scheme given in Fig. 1. (b) The PNC calculation for the bandhead energies. The proton effective pairing interaction strength $G_{\rm p}$ is determined by the experimental odd-even difference in binding energies, $G_{\rm p}=0.32$ MeV. (c) The experimental results.

3.2 The MOIs of 1-qp bands in ^{177}Lu

The experimental $J^{(1)}$'s and the occupation probabilities of low-lying 1-qp bands of ^{177}Lu [8] are shown in Fig. 3, by ■ ($\alpha=1/2$) and □ ($\alpha=-1/2$) respectively. The experimental $J^{(1)}$ of the reference band ($K^\pi=0^+$, gsb of ^{176}Yb, denoted by •) is also presented. The PNC calculated $J^{(1)}$'s for all the low-lying 1-quasiproton bands are shown by solid ($\alpha=1/2$) and dotted ($\alpha=-1/2$) lines, respectively. The experimental $J^{(1)}(\omega)$'s are reproduced very well PNC calculations with no free parameter, which in turn confirms the assigned configurations for 1-qp bands in ^{177}Lu [8, 9].

Discussions:

(a) Signature splitting.

For the former three bands, $K^\pi=7/2^+$ (the gsb), $K^\pi=9/2^-$ at 150.9 keV, and $K^\pi=5/2^+$ at 457.9 keV, no signature splitting is observed, which is in accordance with the behavior of the cranked Nilsson levels $\pi7/2^+[404]$, $\pi\mathbf{9/2^-[514]}$ and $\pi5/2^+[402]$ (Fig. 1(a)). For the $\pi1/2^+[411]$ at 569.7 keV and $\pi\mathbf{1/2^-[541]}$ band, the observed signature splitting is well reproduced by the PNC calculations, which is understandable from the behavior of the cranked Nilsson orbital $\pi1/2^+[411]$ and $\pi\mathbf{1/2^-[541]}$(Fig. 1(a)). As for the $K^\pi=1/2^-$ band ($\pi\mathbf{1/2^-[541]}$), a slightly larger deformation parameter $\varepsilon_2=0.271$ is adopted according to Ref. [9].

(b) The so-called "identical bands".

From Fig. 3 it is seen that the experimental $J^{(1)}$'s for two low-lying excited bands of ^{177}Lu [8], $K^\pi=7/2^+$ (gsb) and $K^\pi=5/2^+$ band at 457.9 keV, are "identical" to the reference band ($K^\pi=0^+$, gsb of ^{176}Yb) [18], i.e., the odd-even difference in MOIs vanishes, $\delta J_\lambda\approx 0$. This seemingly strange behavior is reproduced very well by the PNC calculations with no free parameter. This can be easily understood, because for the two 1-qp bands, the blocked Nilsson levels are deformation aligned normal orbitals ($\pi7/2^+[404]$, $\pi5/2^+[402]$) with a negligibly small Coriolis response. As for the 1-qp band $K^\pi=9/2^-$ at 150.9 keV, both the experimental and calculated $J^{(1)}$ ($\pi\mathbf{9/2^-[514]}$) are a little larger than the reference band. This can also be understood, because even $\pi\mathbf{9/2^-[514]}$ is a high-j intruder orbital, but it has a high-Ω value (i.e., deformation aligned). The MOI of the $\pi\mathbf{1/2^-[541]}$ band ($\alpha=1/2$) is much larger than the reference band at low ω; this is also understandable, because the blocked orbital $\pi\mathbf{1/2^-[541]}$ ($h_{9/2}$) is of a high-j low-Ω, and has a very strong Coriolis response.

3.3 The 3- and 5-qp bands in ^{177}Lu

The experimental $J^{(1)}$'s of two low-lying excited high-K 3-qp and one 5-qp bands of ^{177}Lu [9] are shown in Fig. 4 by ■ ($\alpha=1/2$) and □ ($\alpha=-1/2$), and the calculated $J^{(1)}$'s are shown by solid ($\alpha=1/2$) and dotted ($\alpha=-1/2$) lines, respectively. No signature splitting is found. The experimental $J^{(1)}$ of the reference band ($K^\pi=0^+$, $\alpha=0$, gsb of ^{176}Yb, denoted by •) is also presented. The occupation probabilities for these bands are given in the right side of Fig. 4. The experimental $J^{(1)}(\omega)$'s of these bands are reproduced very well by the PNC calculations, in which no free parameters are involved. The difference between two high-K 3-qp bands is that the blocked normal proton orbital $\pi7/2^+[404]$ in the $K^\pi=23/2^-$ band is replaced by the high-j, but high-Ω (deformation aligned) orbital $\pi\mathbf{9/2^-[514]}$ in the $K^\pi=25/2^+$ band. This can also be seen from the proton occupation probabilities n_μ of each Nilsson level. Thus we can understand why $J^{(1)}(K^\pi=25/2^+)$ is a little larger than $J^{(1)}(K^\pi=23/2^-)$, and in turn $J^{(1)}(K^\pi=23/2^-)$

is a little larger than $J^{(1)}(K^\pi = 0^+,\ \alpha = 0$, gsb of ^{176}Yb). For the two high-K 3-qp bands of ^{177}Lu, the neutron occupation probabilities of each Nilsson level are the same.

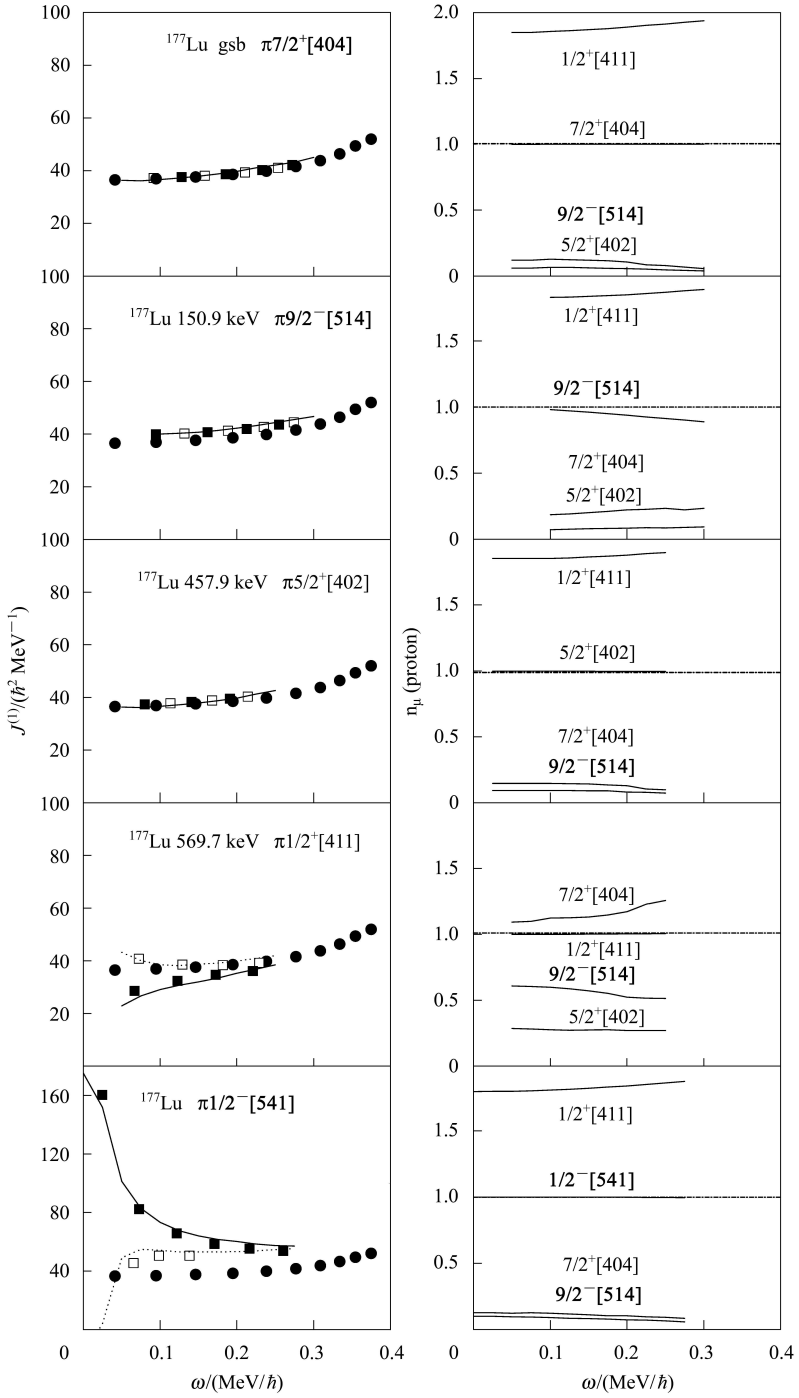

Fig. 3. The MOIs and the occupation probabilities of the low-lying 1-qp bands in ^{177}Lu. The experimental MOIs are denoted by ■ ($\alpha = 1/2$) and □ ($\alpha = -1/2$), respectively. The calculated MOIs by the PNC method are denoted by solid lines ($\alpha = 1/2$) and dotted lines ($\alpha = -1/2$), respectively. The experimental MOIs of the reference band ($K^\pi = 0^+, \alpha = 0$, ground state band of ^{176}Yb) are denoted by solid circles •.

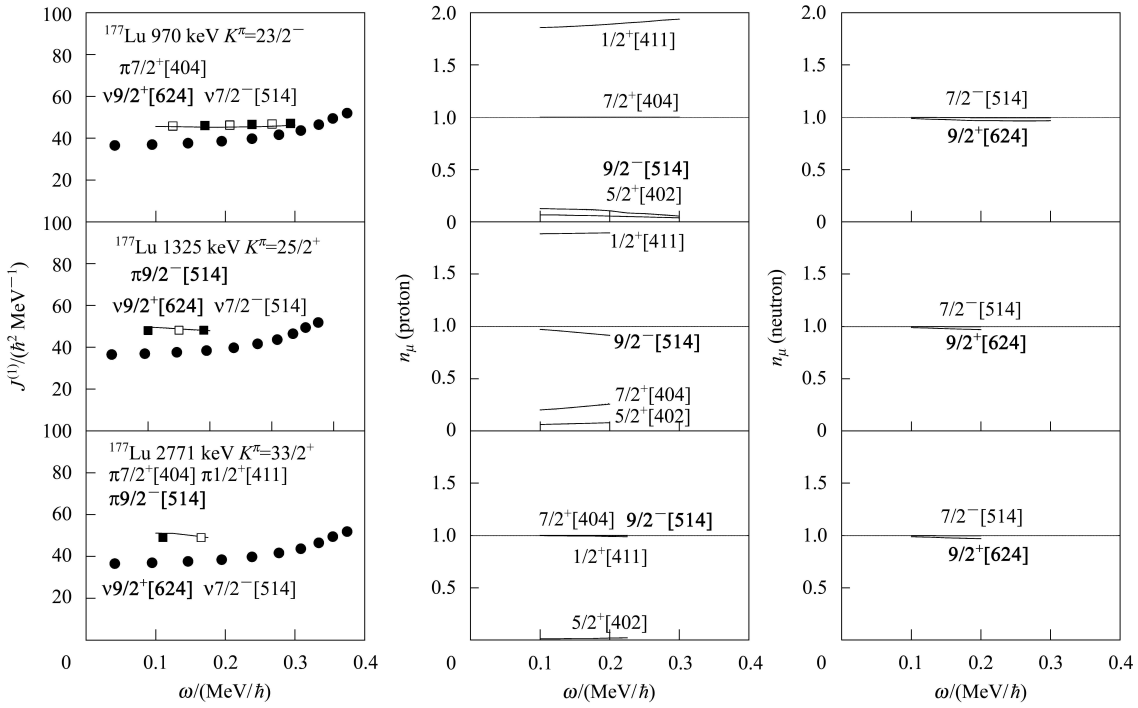

Fig. 4. The same as Fig. 3, but for the low-lying 3-qp and 5-qp bands of ^{177}Lu.

4 Summary

The experimental 1-qp bands and low-lying high-K 3-qp and 5-qp bands are analyzed by the PNC method for treating the CSM with pairing interaction, in which the blocking effects are taken into account exactly. The Nilsson level scheme (Lund systematics) is slightly adjusted to reproduce the bandhead energies of the 1-qp bands in the PNC calculation, the effective pairing interaction strength is determined by the experimental odd-even difference in binding energies, so no free parameters are involved. The experimental $J^{(1)}$'s of the five 1-qp bands are reproduced very well by PNC calculations, including the ω variation of MOIs and signature splitting. The so-called "identical band" is due to the negligibly small Coriolis response of the blocked orbitals (e.g. $\pi 7/2^+[404]$, $\pi 5/2^+[402]$, etc.). To our knowledge, no such satisfactory calculations were reported. The experimental MOIs of two high-K 3-qp bands and one 5-qp band are also reproduced very well by the PNC calculation without additional free parameters, which in turn confirms the configurations assigned to these multiquasiparticle bands in previous articles[8, 9].

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