Casas-Ibarra parametrization and leptogenesis*

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Abstract The Casas-Ibarra parametrization is a description of the Dirac neutrino mass matrix $M_{\rm D}$ in terms of the neutrino mixing matrix V, an orthogonal matrix O and the diagonal mass matrices of light and heavy Majorana neutrinos in the type-I seesaw mechanism. Because $M^\dagger_{\rm D} M_{\rm D}$ is apparently independent of V but dependent on O in this parametrization, a number of authors have claimed that unflavored leptogenesis has nothing to do with CP violation at low energies. Here we question this logic by clarifying the physical meaning of O. We establish a clear relationship between O and the observable quantities, and find that O does depend on V. We show that both unflavored leptogenesis and flavored leptogenesis have no direct connection with low-energy CP violation.

Key words Casas-Ibarra parametrization, type-I seesaw, leptogenesis

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1 Introduction

Very compelling evidence for finite neutrino masses and large neutrino mixing angles has been achieved from solar, atmospheric, reactor and accelerator neutrino oscillation experiments. This exciting breakthrough opens a new window to physics beyond the standard electroweak model, because the standard model itself only contains three massless neutrinos whose flavor states ν_α (for $\alpha = {\rm e}, \mu, \tau$) and mass states ν_i (for $i = 1, 2, 3$) are identical. A very natural and elegant way of generating non-zero but tiny masses m_i for ν_i is to extend the standard model by introducing three right-handed neutrinos and allowing lepton number violation. In this case, the $SU(2)_{\rm L} \times U(1)_{\rm Y}$ gauge-invariant neutrino mass terms are given by

$$-\mathcal{L}_{\rm mass} = \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2} \overline{N^{\rm c}_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}\,, \quad (1)$$

where $\tilde{H} \equiv {\rm i}\sigma_2 H^*$, $l_{\rm L}$ denotes the left-handed lepton doublet, and $M_{\rm R}$ is the mass matrix of right-handed neutrinos. After spontaneous gauge symmetry breaking, we are left with the Dirac neutrino mass matrix $M_{\rm D} = Y_\nu v$, where $v \approx 174$ GeV is the vacuum expectation value of the neutral component of the Higgs doublet H. The scale of $M_{\rm R}$ can be much higher than v, as right-handed neutrinos belong to the $SU(2)_{\rm L}$ singlet and are not subject to electroweak symmetry breaking. It is therefore natural to obtain the effective mass matrix for three light neutrinos [1]:

$$M_\nu \approx -M_{\rm D} M^{-1}_{\rm R} M^{\rm T}_{\rm D}\,. \quad (2)$$

Such a relation is commonly referred to as the type-I seesaw mechanism. Let us denote the mass states of three right-handed neutrinos and their corresponding masses as $\rm N_i$ and M_i (for $i = 1, 2, 3$), respectively. Then Eq. (2) implies $m_i \sim v^2/M_i$ as a naive result, which explains why m_i is small but non-vanishing. Note that both light and heavy neutrinos are Majorana particles in this seesaw picture. Without loss of generality, one often chooses the flavor basis with both the charged-lepton mass matrix and $M_{\rm R}$ being diagonal, real and positive (i.e., the mass eigenstates of three charged leptons are identified with their flavor eigenstates, and $M_{\rm R} = \widehat{M}_{\rm N} \equiv {\rm Diag}\{M_1, M_2, M_3\}$). In this basis, Casas and Ibarra (CI) proposed an interesting parametrization of $M_{\rm D}$ [2]:

$$M_{\rm D} \approx {\rm i} V \sqrt{\widehat{M}_\nu}\, O \sqrt{\widehat{M}_{\rm N}}\,, \quad (3)$$

where V is the 3×3 neutrino mixing matrix which can be obtained from the diagonalization of M_ν (i.e.,

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$V^\dagger M_\nu V^* = \widehat{M}_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\}$)[1)], and O is a complex orthogonal matrix.

Associated with the above seesaw mechanism, the leptogenesis mechanism [3] may naturally work to account for the cosmological matter-antimatter asymmetry via the CP-violating and out-of-equilibrium decays of $\rm N_i$ and the $\rm (B-L)$-conserving sphaleron processes [4]. The CP-violating asymmetry between $\rm N_\mathit{i} \to l + H^c$ and $\rm N_\mathit{i} \to l + H$ decays, denoted as ε_i (for $i = 1, 2, 3$), has been calculated in the single flavor approximation (i.e., the final-state lepton flavors are not distinguished and are simply summed) [5]:

$$\varepsilon_i = \frac{\displaystyle\sum_{j \neq i} \left\{ \mathcal{F}(x_{ij}) \ {\rm Im}\left[(M_{\rm D}^\dagger M_{\rm D})_{ij} \right]^2 \right\}}{8\pi v^2 (M_{\rm D}^\dagger M_{\rm D})_{ii}} \ , \qquad (4)$$

where

$$\mathcal{F}(x_{ij}) = \sqrt{x_{ij}} \{(2-x_{ij})/(1-x_{ij}) + (1+x_{ij}) \ln[x_{ij}/(1+x_{ij})]\}$$

with $x_{ij} \equiv M_j^2/M_i^2$ is the loop function of self-energy and vertex corrections. In this unflavored leptogenesis scenario, a non-vanishing ε_i depends on the imaginary part of $M_{\rm D}^\dagger M_{\rm D}$. Given the CI parametrization in Eq. (3), it is straightforward to obtain

$$M_{\rm D}^\dagger M_{\rm D} \approx \sqrt{\widehat{M}_{\rm N}} \ O^\dagger \widehat{M}_\nu O \sqrt{\widehat{M}_{\rm N}} \ , \qquad (5)$$

which is apparently independent of V but dependent on O. Hence a number of authors have taken it for granted that unflavored leptogenesis has nothing to do with CP violation at low energies (see, e.g., Refs. [6–11]). We find that this logic is questionable, because the physical meaning of O has never been clarified in the literature.

The main purpose of this note is to clarify the physical meaning of O in the CI parametrization by establishing a relationship between O and the observable quantities in a generic type-I seesaw model without any special assumptions. Contrary to the naive observation, we find that O depends not only on the neutrino mixing matrix V but also on the matrix responsible for the charged-current interactions of heavy neutrinos $\rm N_\mathit{i}$. The latter, which has clear physical meaning and is denoted as R, governs the strength of CP violation in V and that in leptogenesis. After a detailed analysis of the correlation between R and V, we draw a general conclusion that both unflavored leptogenesis and flavored leptogenesis have no direct connection with low-energy CP violation.

2 Physical meaning of *O*

After spontaneous $SU(2)_{\rm L} \times U(1)_{\rm Y} \to U(1)_{\rm em}$ symmetry breaking, the mass terms in Eq. (1) turn out to be

$$-\mathcal{L}'_{\rm mass} = \frac{1}{2} \overline{(\nu_{\rm L} \, N_{\rm R}^c)} \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^{\rm T} & M_{\rm R} \end{pmatrix} \begin{pmatrix} \nu_{\rm L}^c \\ N_{\rm R} \end{pmatrix} + {\rm h.c.} \ , \qquad (6)$$

where $\nu_{\rm L}$ represents the column vector of $(\nu_e, \nu_\mu, \nu_\tau)_{\rm L}$. The overall 6×6 neutrino mass matrix in Eq. (6) can be diagonalized by a unitary transformation:

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^{\rm T} & M_{\rm R} \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_{\rm N} \end{pmatrix} , \qquad (7)$$

where $\widehat{M}_\nu$ and $\widehat{M}_{\rm N}$ have been defined before. After this diagonalization, the flavor states of light neutrinos (ν_α for $\alpha = e, \mu, \tau$) can be expressed in terms of the mass states of light and heavy neutrinos (ν_i and $\rm N_\mathit{i}$ for $i = 1, 2, 3$), and thus the standard charged-current interactions between ν_α and α (for $\alpha = e, \mu, \tau$) can be written as

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e \ \mu \ \tau)_{\rm L}} \ \gamma^\mu \left[V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} \right] W^-_\mu + {\rm h.c.} \qquad (8)$$

in the basis of mass states. So V is just the neutrino mixing matrix responsible for neutrino oscillations, while R describes the strength of charged-current interactions between (e, μ, τ) and $(\rm N_1, N_2, N_3)$. V and R are correlated with each other through the normalization condition $VV^\dagger + RR^\dagger = \mathbf{1}$. Hence V itself is not exactly unitary in the type-I seesaw mechanism and its deviation from unitarity is simply characterized by non-vanishing R.

Because both V and R are well-defined in Eq. (8), they can be used to understand the physical meaning of O in the CI parametrization. To do so, we first derive the seesaw relation from Eq. (7). The latter yields

$$V \widehat{M}_\nu V^{\rm T} + R \widehat{M}_{\rm N} R^{\rm T} = \mathbf{0} \, , \qquad (9)$$

and

$$S \widehat{M}_\nu S^{\rm T} + U \widehat{M}_{\rm N} U^{\rm T} = M_{\rm R} \, . \qquad (10)$$

If Eq. (7) is rewritten as

$$\begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^{\rm T} & M_{\rm R} \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} V & R \\ S & U \end{pmatrix} \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_{\rm N} \end{pmatrix} , \qquad (11)$$

1) Note that we have tentatively ignored tiny differences between the eigenvalues of $M_{\rm R}$ (or M_ν) and the physical masses M_i (or m_i). See the next section for a detailed discussion.

we can directly obtain the exact results

$$R = M_{\rm D} U^* \widehat{M}_{\rm N}^{-1} \; , \tag{12}$$

and

$$S^* = M_{\rm D}^{-1} V \widehat{M}_\nu \; . \tag{13}$$

Let us substitute Eqs. (12) and (13) into Eq. (9) and (10), respectively. Then we arrive at

$$V \widehat{M}_\nu V^{\rm T} = - M_{\rm D} \left(U^* \widehat{M}_{\rm N}^{-1} U^\dagger \right) M_{\rm D}^{\rm T} \; , \tag{14}$$

and

$$M_{\rm R} = U \widehat{M}_{\rm N} U^{\rm T} + (M_{\rm D}^{-1})^* V^* \widehat{M}_\nu^3 V^\dagger (M_{\rm D}^{-1})^\dagger \approx U \widehat{M}_{\rm N} U^{\rm T} \; . \tag{15}$$

The excellent approximation made in Eq. (15) implies that U is essentially unitary. Taking U to be unitary and combining Eqs. (14) and (15), we obtain

$$M_\nu \equiv V \widehat{M}_\nu V^{\rm T} \approx - M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^{\rm T} \; , \tag{16}$$

where V is also unitary in this leading-order approximation. Eq. (16) reproduces the seesaw formula given in Eq. (2). It is obvious that $R \sim S \sim \mathcal{O}(M_{\rm D}/M_{\rm R})$ holds, and thus the seesaw relation actually holds up to the accuracy of $\mathcal{O}(R^2)$ [12].

Now we look at the orthogonal matrix O in the CI parametrization. Given the basis where $M_{\rm R}$ is diagonal, real and positive, Eq. (15) implies that $M_{\rm R} \approx \widehat{M}_{\rm N}$ and $U \approx 1$ are very good approximations. In this case, we get $M_{\rm D} \approx R \widehat{M}_{\rm N}$ from Eq. (12). Substituting this relation into Eq. (3), we obtain

$$O \approx -{\rm i} \sqrt{\widehat{M}_\nu^{-1}} \; V^\dagger M_{\rm D} \sqrt{\widehat{M}_{\rm N}^{-1}} \approx -{\rm i} \sqrt{\widehat{M}_\nu^{-1}} \; V^\dagger R \sqrt{\widehat{M}_{\rm N}} \; , \tag{17}$$

which shows that O is definitely dependent on V. It is worth remarking that both V and R, which are respectively associated with the charged-current interactions of light and heavy Majorana neutrinos, have clear physical meaning. Hence it seems improper to draw the conclusion from Eq. (5) that unflavored leptogenesis is independent of low-energy neutrino mixing and CP violation described by V. If Eq. (17) is substituted into Eq. (5), however, we shall arrive at a much simpler expression

$$M_{\rm D}^\dagger M_{\rm D} \approx \widehat{M}_{\rm N} R^\dagger R \widehat{M}_{\rm N} \; . \tag{18}$$

This result is actually straightforward, just because of $M_{\rm D} \approx R \widehat{M}_{\rm N}$. It apparently has nothing to do with V. So the question becomes whether unflavored leptogenesis depends on V through R. We have known that V is correlated with R via the exact seesaw relation in Eq. (9) and the normalization condition $VV^\dagger + RR^\dagger = 1$. To see this correlation more clearly, one has to adopt an explicit and self-consistent parametrization of V and R.

3 Unflavored leptogenesis

To be specific, here we make use of a very instructive and useful parametrization of $V \equiv AV_0$ and R advocated in Ref. [13]:

$$V_0 = \begin{pmatrix} c_{12} c_{13} & \hat{s}_{12}^* c_{13} & \hat{s}_{13}^* \\ -\hat{s}_{12} c_{23} - c_{12} \hat{s}_{13} \hat{s}_{23}^* & c_{12} c_{23} - \hat{s}_{12}^* \hat{s}_{13} \hat{s}_{23}^* & c_{13} \hat{s}_{23}^* \\ \hat{s}_{12} \hat{s}_{23} - c_{12} \hat{s}_{13} c_{23} & -c_{12} \hat{s}_{23} - \hat{s}_{12}^* \hat{s}_{13} c_{23} & c_{13} c_{23} \end{pmatrix} , \tag{19}$$

and

$$A = \begin{pmatrix} c_{14} c_{15} c_{16} & 0 & 0 \\ \begin{matrix} -c_{14} c_{15} \hat{s}_{16} \hat{s}_{26}^* - c_{14} \hat{s}_{15} \hat{s}_{25}^* c_{26} \\ -\hat{s}_{14} \hat{s}_{24}^* c_{25} c_{26} \end{matrix} & c_{24} c_{25} c_{26} & 0 \\ \begin{matrix} -c_{14} c_{15} \hat{s}_{16} c_{26} \hat{s}_{36}^* + c_{14} \hat{s}_{15} \hat{s}_{25}^* \hat{s}_{26} \hat{s}_{36}^* \\ -c_{14} \hat{s}_{15} c_{25} \hat{s}_{35}^* c_{36} + \hat{s}_{14} \hat{s}_{24}^* c_{25} \hat{s}_{26} \hat{s}_{36}^* \\ + \hat{s}_{14} \hat{s}_{24}^* \hat{s}_{25} \hat{s}_{35}^* c_{36} - \hat{s}_{14} c_{24} \hat{s}_{34}^* c_{35} c_{36} \end{matrix} & \begin{matrix} -c_{24} c_{25} \hat{s}_{26} \hat{s}_{36}^* - c_{24} \hat{s}_{25} \hat{s}_{35}^* c_{36} \\ -\hat{s}_{24} \hat{s}_{34}^* c_{35} c_{36} \end{matrix} & c_{34} c_{35} c_{36} \end{pmatrix} ,$$

$$R = \begin{pmatrix} \hat{s}^*_{14} c_{15} c_{16} & \hat{s}^*_{15} c_{16} & \hat{s}^*_{16} \\ \begin{array}{l} -\hat{s}^*_{14} c_{15} \hat{s}_{16} \hat{s}^*_{26} - \hat{s}^*_{14} \hat{s}_{15} \hat{s}^*_{25} c_{26} \\ +c_{14} \hat{s}^*_{24} c_{25} c_{26} \end{array} & -\hat{s}^*_{15} \hat{s}_{16} \hat{s}^*_{26} + c_{15} \hat{s}^*_{25} c_{26} & c_{16} \hat{s}^*_{26} \\ \begin{array}{l} -\hat{s}^*_{14} c_{15} \hat{s}_{16} c_{26} \hat{s}^*_{36} + \hat{s}^*_{14} \hat{s}_{15} \hat{s}^*_{25} \hat{s}_{26} \hat{s}^*_{36} \\ -\hat{s}^*_{14} \hat{s}_{15} c_{25} \hat{s}^*_{35} c_{36} - c_{14} \hat{s}^*_{24} c_{25} \hat{s}_{26} \hat{s}^*_{36} \\ -c_{14} \hat{s}^*_{24} \hat{s}_{25} \hat{s}^*_{35} c_{36} + c_{14} c_{24} \hat{s}^*_{34} c_{35} c_{36} \end{array} & \begin{array}{l} -\hat{s}^*_{15} \hat{s}_{16} c_{26} \hat{s}^*_{36} - c_{15} \hat{s}^*_{25} \hat{s}_{26} \hat{s}^*_{36} \\ +c_{15} c_{25} \hat{s}^*_{35} c_{36} \end{array} & c_{16} c_{26} \hat{s}^*_{36} \end{pmatrix}, \quad (20)$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $\hat{s}_{ij} \equiv e^{i\delta_{ij}} \sin\theta_{ij}$ with θ_{ij} and δ_{ij} (for $1 \leqslant i < j \leqslant 6$) being rotation angles and phase angles, respectively. One can see that V_0 is just the standard parametrization of the unitary neutrino mixing matrix (up to some proper phase rearrangements) [14], and thus non-vanishing A signifies the non-unitarity of V. One can also see that A and R involve the same parameters: nine rotation angles and nine phase angles[1)]. If all of them are switched off, we shall be left with $R = 0$ and $A = 1$.

In view of the fact that the unitarity violation of V must be very small effects (at most at the percent level as constrained by current experimental data on neutrino oscillations, rare lepton-flavor-violating or lepton-number-violating processes and precision electroweak tests [15]), one may treat A as a perturbation to V_0. The smallness of θ_{ij} (for $i = 1,2,3$ and $j = 4,5,6$) allows us to make the following excellent approximations:

$$A = \mathbf{1} - \begin{pmatrix} \frac{1}{2}(s^2_{14} + s^2_{15} + s^2_{16}) & 0 & 0 \\ \hat{s}_{14}\hat{s}^*_{24} + \hat{s}_{15}\hat{s}^*_{25} + \hat{s}_{16}\hat{s}^*_{26} & \frac{1}{2}(s^2_{24} + s^2_{25} + s^2_{26}) & 0 \\ \hat{s}_{14}\hat{s}^*_{34} + \hat{s}_{15}\hat{s}^*_{35} + \hat{s}_{16}\hat{s}^*_{36} & \hat{s}_{24}\hat{s}^*_{34} + \hat{s}_{25}\hat{s}^*_{35} + \hat{s}_{26}\hat{s}^*_{36} & \frac{1}{2}(s^2_{34} + s^2_{35} + s^2_{36}) \end{pmatrix} + \mathcal{O}(s^4_{ij}),$$

$$R = \mathbf{0} + \begin{pmatrix} \hat{s}^*_{14} & \hat{s}^*_{15} & \hat{s}^*_{16} \\ \hat{s}^*_{24} & \hat{s}^*_{25} & \hat{s}^*_{26} \\ \hat{s}^*_{34} & \hat{s}^*_{35} & \hat{s}^*_{36} \end{pmatrix} + \mathcal{O}(s^3_{ij}) \quad (21)$$

with $s_{ij} \equiv \sin\theta_{ij}$ being real. Note that the approximation made in Eq. (15) is equivalent to $A \approx 1$, leading to unitary V and U. One may therefore take $V \approx V_0$ when applying the approximate seesaw relation given in Eqs. (2) or (16) to the phenomenology of neutrino flavor mixing and leptogenesis. In this case, Eq. (9) is simplified to

$$V_0 \widehat{M}_\nu V^{\rm T}_0 \approx -R \widehat{M}_{\rm N} R^{\rm T}. \quad (22)$$

The total number of free parameters in $\widehat{M}_\nu$, $\widehat{M}_{\rm N}$, V (or V_0) and R is thirty (six masses, twelve mixing angles and twelve CP-violating phases). However, either Eq. (9) or Eq. (22) can give twelve real constraint conditions. Hence we are left with eighteen independent parameters in the type-I seesaw mechanism.

Given the expression of R in Eq. (21), it is straightforward to obtain

$$\begin{aligned} {\rm Im}\left(R\widehat{M}_{\rm N}R^{\rm T}\right)_{ij} = & -M_1 s_{i4} s_{j4} \sin(\delta_{i4} + \delta_{j4}) - \\ & M_2 s_{i5} s_{j5} \sin(\delta_{i5} + \delta_{j5}) - \\ & M_3 s_{i6} s_{j6} \sin(\delta_{i6} + \delta_{j6}), \end{aligned} \quad (23)$$

where $1 \leqslant i < j \leqslant 3$. In comparison, Eqs. (4) and (18) tell us that the CP-violating asymmetries ε_i (for $i = 1,2,3$) in unflavored leptogenesis are associated with

$$\begin{aligned} {\rm Im}\left(R^\dagger R\right)_{12} &= \sum^3_{i=1} s_{i4} s_{i5} \sin(\delta_{i4} - \delta_{i5}), \\ {\rm Im}\left(R^\dagger R\right)_{13} &= \sum^3_{i=1} s_{i4} s_{i6} \sin(\delta_{i4} - \delta_{i6}), \\ {\rm Im}\left(R^\dagger R\right)_{23} &= \sum^3_{i=1} s_{i5} s_{i6} \sin(\delta_{i5} - \delta_{i6}). \end{aligned} \quad (24)$$

1) Note that none of the phases of R (or A) can be rotated away by redefining the phases of three charged-lepton fields, because such a phase redefinition will also affect the phases of A (or R), as one can easily see from Eq. (8).

We see that there are in general nine independent phase combinations in Eq. (23), while there are only six independent phase combinations in Eq. (24). It is possible to acquire $\mathrm{Im}(R\widehat{M}_{\rm N}R^{\rm T}) = 0$ by fine-tuning the free parameters in Eq. (23), such that $\mathrm{Im}(V_0\widehat{M}_\nu V_0^{\rm T}) \approx 0$ holds (i.e., the neutrino mixing matrix V_0 is real) as one can see from Eq. (22). In this special case, there is no low-energy CP violation but viable unflavored leptogenesis is likely to take place. To achieve a direct connection between the CP-violating phases of V_0 and the CP-violating asymmetries ε_i, one should switch off as many phases of R as possible. Such a treatment can be realized in some specific type-I seesaw models [16], in which the texture of Y_ν (or $M_{\rm D}$) might get constrained from a certain flavor symmetry in the basis of $M_{\rm R} = \widehat{M}_{\rm N}$. But our general conclusion is that there is only indirect connection between unflavored leptogenesis and low-energy observables.

4 Flavored leptogenesis

The same conclusion as obtained above is true for flavored leptogenesis. When the mass of the lightest heavy Majorana neutrino is lower than about 10^{12} GeV, flavor-dependent effects matter in leptogenesis [17] and have to be carefully handled [18]. In this case, the CP-violating asymmetries $\varepsilon_{i\alpha}$ between $\mathrm{N}_i \to \mathrm{l}_\alpha + \mathrm{H}^{\rm c}$ and $\mathrm{N}_i \to \mathrm{l}^{\rm c}_\alpha + \mathrm{H}$ decays (for $i = 1, 2, 3$ and $\alpha = \mathrm{e}, \mu, \tau$) depend on the phases of $M_{\rm D}$ (or Y_ν) in the following way [19]:

$$\varepsilon_{i\alpha} = \frac{1}{8\pi v^2}\sum_{j\neq i}\left\{\mathcal{F}(x_{ij})\frac{\mathrm{Im}\left[(M_{\rm D}^\dagger M_{\rm D})_{ij}(M_{\rm D}^*)_{\alpha i}(M_{\rm D})_{\alpha j}\right]}{|(M_{\rm D})_{\alpha i}|^2} + \frac{1}{1-x_{ij}}\cdot\frac{\mathrm{Im}\left[(M_{\rm D}^\dagger M_{\rm D})_{ji}(M_{\rm D}^*)_{\alpha i}(M_{\rm D})_{\alpha j}\right]}{|(M_{\rm D})_{\alpha i}|^2}\right\}, \qquad (25)$$

where the loop function $\mathcal{F}(x_{ij})$ with $x_{ij} \equiv M_j^2/M_i^2$ has been given below Eq. (4). Taking account of $M_{\rm D} \approx R\widehat{M}_{\rm N}$, we find

$$\mathrm{Im}\left[(M_{\rm D}^\dagger M_{\rm D})_{ij}(M_{\rm D}^*)_{\alpha i}(M_{\rm D})_{\alpha j}\right] \approx M_i^2 M_j^2\ \mathrm{Im}\left[(R^\dagger R)_{ij}R^*_{\alpha i}R_{\alpha j}\right]\ ,$$
$$\mathrm{Im}\left[(M_{\rm D}^\dagger M_{\rm D})_{ji}(M_{\rm D}^*)_{\alpha i}(M_{\rm D})_{\alpha j}\right] \approx M_i^2 M_j^2\ \mathrm{Im}\left[(R^\dagger R)^*_{ij}R^*_{\alpha i}R_{\alpha j}\right]\ . \qquad (26)$$

It has been shown in Eq. (24) that the quantities $(R^\dagger R)_{ij}$ (for $i \neq j$) rely on six independent phase combinations of R. On the other hand, it is easy to check that the quantities $R^*_{\alpha i}R_{\alpha j}$ (for $\alpha = \mathrm{e}, \mu, \tau$ and $i \neq j$) depend on the same phase combinations. Hence non-vanishing $\varepsilon_{i\alpha}$ in Eq. (25) and non-zero ε_i in Eq. (4) originate from the same source of CP violation, no matter whether there are flavor effects or not. This point keeps unchanged even if thermal resonant leptogenesis [18] is taken into account.

If the CI parametrization in Eq. (3) is applied to the description of flavored leptogenesis, then V will show up in the expression of $\varepsilon_{i\alpha}$. The reason is simply that the elements of V cannot cancel out in $(M_{\rm D}^*)_{\alpha i}(M_{\rm D})_{\alpha j}$, although they can cancel out in $(M_{\rm D}^\dagger M_{\rm D})_{ij}$. This observation has been used by a number of authors to support the argument that viable flavored leptogenesis may result from V even in the case of O being a real orthogonal matrix (see, e.g., Refs. [8–11]). Such an argument or observation is certainly not wrong, but it is not profound either [20]. In view of Eq. (17), we find that O can be real only when nontrivial CP-violating phases in V and R delicately combine to make $V^\dagger R$ purely imaginary. This extremely special case means nothing but a very special correlation between V and R. While one may argue that flavored leptogenesis is linked to the neutrino mixing matrix V in this contrived case, one should keep in mind that both $\varepsilon_{i\alpha}$ and the CP-violating phases of V actually originate from R and their direct connection can only be established when some (or most) of the phase parameters of R are switched off. In general, however, "there is no correlation between successful leptogenesis and the low-energy CP phase" [20][1)].

5 Summary

The CI parametrization, in which the neutrino mixing matrix V and an orthogonal matrix O are unjustifiedly assumed to be independent of each other, has often been applied to the phenomenology of neutrino mixing and leptogenesis in the type-I seesaw mechanism. In the present work, we have clarified the physical meaning of O by establishing a relationship between O and the observable quantities in a generic type-I seesaw model without any special assumptions. We find that O depends not only on V but also on

1) This conclusion was drawn in Ref. [20] from a very detailed analysis of the sensitivity of leptogenesis to the neutrino mixing matrix V by using the CI parametrization and allowing the elements of O to take arbitrary values in the parameter space. Here we arrive at the same conclusion by clarifying the physical meaning of O in an analytic way.

R, the matrix responsible for the charged-current interactions of heavy Majorana neutrinos. The CP-violating phases of R govern the strength of CP violation at low energies and that in leptogenesis. We have examined the dependence of unflavored or flavored leptogenesis on R and analyzed the correlation between R and V. Our general conclusion is that both unflavored leptogenesis and flavored leptogenesis have no direct connection with low-energy CP violation.

Let us finally give some remarks on R, which makes more sense than O in the analysis of leptogenesis. If the type-I seesaw mechanism could be realized at the TeV scale, it might be possible to measure or constrain the mixing angles of R at the Large Hadron Collider and probe the CP-violating phases of R at a neutrino factory [21]. Because non-vanishing R is a clean signature of the unitarity violation of V, it can actually lead to rich phenomenology of lepton-flavor-violating and lepton-number-violating processes. In particular, R bridges a gap between high-energy neutrino physics (e.g., heavy neutrino decays and leptogenesis) and low-energy neutrino physics (e.g., neutrino mixing and neutrino oscillations).

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