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On the moment of inertia of a proto neutron star*

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Abstract The influences of σ^* and Φ mesons, temperature and coupling constants of nucleons on the moment of inertia of the proto neutron star (PNS) are examined in the framework of relativistic mean field theory for the baryon octet {n, p, Λ, Σ^-, Σ^0, Σ^+,Ξ^-, Ξ^0} system. It is found that, compared with that without considering σ^* and Φ mesons, the moment of inertia decreases. It is also found that the higher the temperature, the larger the incompressibility and symmetry energy coefficient, and the larger the moment of inertia of a PNS. The influence of temperature and coupling constants of the nucleons on the moment of inertia of a PNS is larger than that of the σ^* and Φ mesons.

Key words neutron star, moment of inertia, relativistic mean field theory

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1 Introduction

After a supernova implodes with subsequent bouncing at the center, the proto-neutron star (PNS) comes into being. A newly formed PNS, where the temperature may be as high as 10–20 MeV [1], may consist of neutrons, protons, electrons, muons and trapped neutrinos. However, in a few seconds, the neutrinos will escape and then the temperature will drop down to the order of several MeV [2]. Therefore, the PNS is a compact stellar object with high temperature and highly rotating angular velocity. So the moment of inertia of a PNS will play an important role during its evolution.

In 1967, J. B. Hartle et al. derived the structure equation of slowly rotating neutron stars from the general relativistic theory [3], and they calculated the equilibrium structure of rotating white dwarfs and neutron stars in 1968 [4]. In the last few years, much work has been done on the moment of inertia of neutron stars [5–7], but very little on that of PNSs.

The relativistic mean field theory (RMF) provides a good description of the bulk properties of nuclear matter as well as a large number of single-particle properties of finite nuclei [8–10]. It can only be adapted to static neutron stars with spherical symmetry. For very slowly rotating neutron stars, the spherical symmetry could be approximately looked upon as not being broken and the RMF can be adapted to them.

In this paper, we calculate the moment of inertia of a PNS within the RMF approach considering the baryon octet {n, p, Λ, Σ^-, Σ^0, Σ^+, Ξ^-, Ξ^0}. We mainly study how the scalar meson $f_0(975)$ (denoted as σ^*) and the vector meson $\phi(1020)$ (denoted as ϕ) [11], which only interact between hyperons, affect the moment of inertia of a PNS. On the other hand, the effects of temperature and the coupling constants of the nucleons on the moment of inertia are calculated too.

2 Relativistic mean field theory (RMF) and the moment of inertia of neutron stars

The Lagrangian density of hadron matter reads as follows [12],

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$$\mathcal{L} = \sum_{\mathrm{B}} \overline{\Psi}_{\mathrm{B}} \Big(\mathrm{i}\gamma_\mu \partial^\mu - m_{\mathrm{B}} + g_{\sigma\mathrm{B}}\sigma - g_{\omega\mathrm{B}}\gamma_\mu\omega^\mu - \frac{1}{2} g_{\rho\mathrm{B}}\gamma_\mu \tau\cdot\rho^\mu \Big) \Psi_{\mathrm{B}} + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - U_\sigma + \sum_{\lambda=\mathrm{e},\mu} \overline{\Psi}_\lambda (\mathrm{i}\gamma_\mu\partial^\mu - m_\lambda)\Psi_\lambda - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\cdot\rho^\mu + \mathcal{L}^{YY}. \quad (1)$$

The last term, which represents the contribution of the σ^* and ϕ mesons and only couples to hyperons, is given by

$$\mathcal{L}^{YY} = \sum_{\mathrm{B}} g_{\sigma^*\mathrm{B}} \overline{\Psi}_{\mathrm{B}}\Psi_{\mathrm{B}}\sigma^* - \sum_{\mathrm{B}} g_{\phi\mathrm{B}}\overline{\Psi}_{\mathrm{B}}\gamma_\mu\Psi_{\mathrm{B}}\phi^\mu + \frac{1}{2}(\partial_\mu\sigma^*\partial^\mu\sigma^* - m_{\sigma^*}^2\sigma^{*2}) - \frac{1}{4}S_{\mu\nu}S^{\mu\nu} + \frac{1}{2}m_\phi^2\phi_\mu\phi^\mu. \quad (2)$$

Here, $S_{\mu\nu} = \partial_\mu\phi_\nu - \partial_\nu\phi_\mu$.

The energy density and pressure of a neutron star are given by

$$\varepsilon = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_{\sigma^*}^2\sigma^{*2} + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 + \sum_{\mathrm{B}}\frac{2J_{\mathrm{B}}+1}{2\pi^2}\int_0^\infty \sqrt{k^2+(m_{\mathrm{B}} - g_{\sigma\mathrm{B}}\sigma - g_{\sigma^*\mathrm{B}}\sigma^*)^2} \times(\exp[(\varepsilon_{\mathrm{B}}(k) - \mu_{\mathrm{B}})/T]+1)^{-1}k^2\mathrm{d}k + \sum_\lambda \frac{2J_\lambda+1}{2\pi^2}\int_0^\infty \sqrt{k^2+m_\lambda^2}(\exp[\varepsilon_\lambda(k) - \mu_\lambda)/T]+1)^{-1}k^2\mathrm{d}k, \quad (3)$$

$$p = -\frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - \frac{1}{2}m_{\sigma^*}^2\sigma^{*2} + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 + \frac{1}{3}\sum_{\mathrm{B}}\frac{2J_{\mathrm{B}}+1}{2\pi^2} \times\int_0^\infty \frac{k^2}{\sqrt{k^2+(m_{\mathrm{B}}^2 - g_{\sigma\mathrm{B}}^\sigma - g_{\sigma^*\mathrm{B}}\sigma^*)^2}}(\exp[(\varepsilon_{\mathrm{B}}(k) - \mu_{\mathrm{B}})/T]+1)^{-1}k^2\mathrm{d}k + \frac{1}{3}\sum_\lambda\frac{2J_\lambda+1}{2\pi^2} \times\int_0^\infty \frac{k^2}{\sqrt{k^2+m_\lambda^2}}(\exp[(\varepsilon_\lambda(k) - \mu_\lambda)/T]+1)^{-1}k^2\mathrm{d}k. \quad (4)$$

The above equations of state will be used to solve the matter distribution of a neutron star.

We use the Oppenheimer-Volkoff (O-V) equation [13] to obtain the mass and the radius of neutron stars,

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{(p+\varepsilon)(M+4\pi r^3\mathrm{p})}{r(r-2M)}, \quad (5)$$

$$M = 4\pi\int_0^r \varepsilon r^2\mathrm{d}r. \quad (6)$$

In the following, we will derive the moment of inertia of a slowly rotating star [14]. Because of the rotation of the local inertial frames, the structure of a rotating star depends in a complicated way on its frequency. The centrifugal force acting on a fluid element of the star does not depend on the global frequency, Ω, but rather on the difference between Ω and the local frequency, $\omega(r)$, in the local inertial frame at the location of the fluid element,

$$\overline{\omega}(r) = \Omega - \omega(r,\theta). \quad (7)$$

From the Einstein equation,

$$G_\phi^t = 8\pi T_\phi^t, \quad (8)$$

Hartle [3] obtained the relevant equation

$$\frac{1}{r^4}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^4 j\frac{\mathrm{d}\overline{\omega}}{\mathrm{d}r}\right) + \frac{4}{r}\frac{\mathrm{d}j}{\mathrm{d}r}\overline{\omega} = 0. \quad (9)$$

We will use his result in the following.

The quantity $j(r)$ used above is defined in terms of the metric for a Schwarzschild star,

$$j(r) = \mathrm{e}^{-(\nu+\lambda)} = \mathrm{e}^{-\nu}\sqrt{1-2M(r)/r}, \quad r<R, \quad (10)$$

$$j(r) = 1, \quad r\leqslant R. \quad (11)$$

After some algebras and by use of

$$\frac{\mathrm{d}\nu}{\mathrm{d}r} = \frac{M(r)+4\pi r^3 p(r)}{r(r-2M(r))}, \quad (12)$$

we obtain

$$\frac{\mathrm{d}j}{\mathrm{d}r} = -4\pi r(p+\varepsilon)\mathrm{e}^{-\nu}/\sqrt{1-2M(r)/r}. \quad (13)$$

Integrating (9) within the interval $(0,R)$, we have

$$\left(r^4\frac{\mathrm{d}\overline{\omega}}{\mathrm{d}r}\right)_{\mathrm{R}} = \int_0^R 4r^3\frac{\mathrm{d}j}{\mathrm{d}r}\overline{\omega}\mathrm{d}r. \quad (14)$$

According to

$$\omega(r) \sim \frac{J}{r^3} = \frac{I}{r^3}\Omega, \quad r>R, \quad (15)$$

one obtains the angular momentum,

$$J = -\frac{2}{3}\int_0^R \mathrm{d}r r^3\frac{\mathrm{d}j}{\mathrm{d}r}\overline{\omega}, \quad (16)$$

where the proportionality constant has been set to 2/3.

From the above results, we find the moment of inertia of a slowly rotating star as follows,

$$I=\frac{8\pi}{3}\int_0^R \mathrm{d}rr^4\frac{\varepsilon+p}{\sqrt{1-2M(r)/r}}\frac{[\Omega-\omega(r)]}{\Omega}\mathrm{e}^{-\nu}, \quad (17)$$

where ν is defined as

$$-\frac{\mathrm{d}\nu(r)}{\mathrm{d}r}=\frac{1}{\varepsilon+p}\frac{\mathrm{d}p}{\mathrm{d}r}. \quad (18)$$

The mass distribution $M(r)$ is obtained by solving first the O-V Eqs. (5) and (6). Then, combining with Eqs. (3) and (4), we can obtain the moment of inertia of the neutron star by solving Eqs. (9), (10), (17) and (18). The solutions have to satisfy the following boundary conditions,

$$\frac{\mathrm{d}\overline{\omega}}{\mathrm{d}r}|_{r=0} = 0, \quad (19)$$

$$\nu(\infty) = 0, \quad (20)$$

$$\overline{\omega}(R) = \Omega-\frac{R}{3}\frac{\mathrm{d}\overline{\omega}}{\mathrm{d}r}\bigg|_{r=R}. \quad (21)$$

3 Parameters

The parameters in our calculation are chosen according to three cases. For the study of the influence of the σ^*, ϕ mesons on the moment of inertia of the PNS, we assume a temperature of T=15 MeV. The corresponding coupling constants of the nucleons are the GL85 [14] constants listed in Table 1.

We define the following ratios: $x_{\sigma\mathrm{h}}=g_{\sigma\mathrm{h}}/g_{\sigma}$, $x_{\omega\mathrm{h}}=g_{\omega\mathrm{h}}/g_{\omega}$, $x_{\rho\mathrm{h}}=g_{\rho\mathrm{h}}/g_{\rho}$. For the ω coupling constants g, we use ratios

$$g_{\omega\mathrm{N}}/3=g_{\omega\Sigma}/2=g_{\omega\Lambda}/2=g_{\omega\Xi}, \quad (22)$$

which are given by the constituent quark model [$SU(6)$ symmetry]. The σ couplings are then determined by fitting the Λ and Ξ well depth in nuclear matter, $U_{\Lambda}^{(\mathrm{N})}=U_{\Sigma}^{(\mathrm{N})}\approx 30$ MeV and $U_{\Xi}^{(\mathrm{N})}\approx 28$ MeV.

For the ϕ couplings, we use the quark model relationships,

$$g_{\phi\Xi}=2g_{\phi\Lambda}=-2\sqrt{2}g_{\omega\mathrm{N}}/3. \quad (23)$$

For the σ^* mesons, we use the mass of the obtained $f_0(975)$ meson, but treat its couplings purely phenomenologically such as to satisfy the equation for the potential depths,

$$U_{\Lambda}^{(\Xi)}\approx U_{\Xi}^{(\Xi)}\approx 2U_{\Lambda}^{(\Lambda)}\approx 40\ \mathrm{MeV}.$$

This leads to

$$g_{\sigma^*\Lambda}/g_{\sigma\mathrm{N}}=g_{\sigma^*\Sigma}/g_{\sigma\mathrm{N}}=0.69,\ g_{\sigma^*\Xi}/g_{\sigma\mathrm{N}}=1.25\ [15].$$

Next, the influence of the temperature on the moment of inertia of a PNS is calculated. In this case, the contribution of the σ^*, ϕ mesons has to be considered. The nucleon coupling constants are listed in Table 1 as GL85. The temperature is chosen as T=10 MeV, 15 MeV, 20 MeV and 25 MeV.

Table 1. Parameters for the nucleon interactions.

	m	m_σ	m_ω	m_ρ	g_σ	g_ω	g_ρ	g_2
GL85 [14]	939	500	782	770	7.9955	9.1698	9.7163	10.07
GL97 [16]	939	500	782	770	7.9835	8.7	8.5411	20.966
	g_3	C_3	ρ_0	B/A	K	a_{sym}	m^*/m	
GL85 [14]	29.262	0	0.145	15.95	285	36.8	0.77	
GL97 [16]	−9.835	0	0.153	16.3	240	32.5	0.78	

Finally we studied the influence of the coupling constants of the nucleons on the moment of inertia of a PNS. In this case, the coupling constants of nucleons GL85 and GL97 were used. The temperature is T=15 MeV and the contribution of the σ^*, ϕ mesons is considered.

4 Theoretical results and analysis

The effect of the σ^*, ϕ mesons on the field strength of mesons and the chemical potentials of neutrons and electrons are shown in Fig. 1. The solid curves show the results without considering the contribution of the σ^*, ϕ mesons, and the dotted ones show the results including their contribution. The temperature and the coupling constants of the nucleons are chosen as T=15 MeV and GL85, respectively.

From Fig. 1 it can be seen that the influence of the σ^*, ϕ mesons becomes visible around 0.43 fm^{-3} and the fields of the σ_0, ω_0 and ρ_0 mesons are decreasing if the contribution of the σ^*, ϕ mesons is taken into acount. From Fig. 1 we can also see that the chemical potential of the neutrons grows rapidly as the baryon density increases. Compared with those without considering the σ^*, ϕ mesons, the chemical potentials of neutrons and electrons obviously decrease. The reason is that the degrees of freedom of the σ^* and ϕ mesons lowers the Fermi momentum of the baryons and leptons.

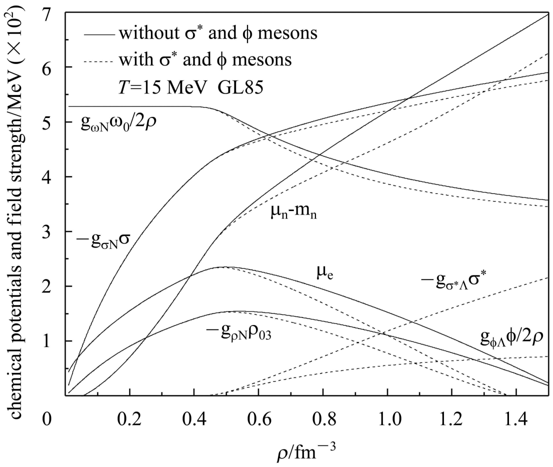

Fig. 1. The field strengths of σ, ω_0, ρ_{03}, σ^* and ϕ, and the chemical potential of neutrons and electrons as functions of the baryon density. The solid curves show the results without the contribution of the σ^*, ϕ mesons. The dotted ones show the results with their contribution included. The temperature is 15 MeV and the coupling constants of the nucleons are GL85.

The effect of the σ^*, ϕ mesons on the moment of inertia of a PNS is shown in Fig. 2, from which we can see that a maximum value $I_{\max}$ occurs. Including the σ^*, ϕ mesons, $I_{\max}$ decreases from 2.1565×10^{45} g·cm^2 to 2.1551×10^{45} g·cm^2, i.e. a reduction of $\Delta I_{\max} = 0.0014\times10^{45}$ g·cm^2 or roughly 0.06%. The reason is that including the contribution of the σ^*, ϕ mesons, the equation of state becomes softer and the moment

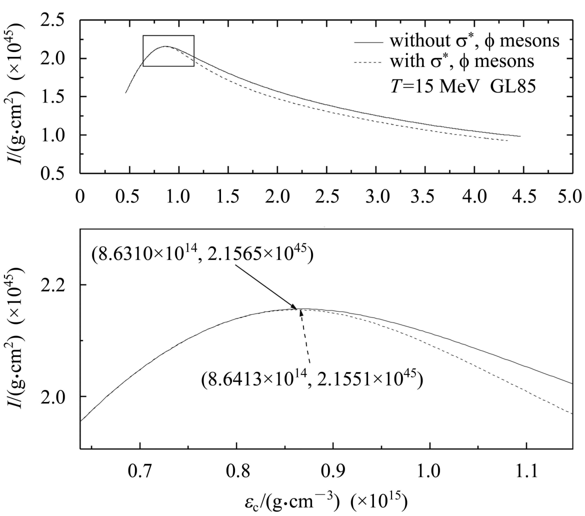

Fig. 2. The moment of inertia of a PNS as a function of the central energy density. The solid curves show the results without the contribution of the σ^*, ϕ mesons. The dotted ones show the results with their contribution included. The temperature and the coupling constants of the nucleons are chosen as T=15 MeV and GL85.

of inertia of the PNS decreases. But the influence is not very pronounced.

Secondly, the influence of the temperature on the moment of inertia of a PNS is examined. The results are shown in Fig. 3 and Fig. 4. Here we choose GL85 and consider the contribution of the σ^*, ϕ mesons. The temperatures are chosen as T=10 MeV, 15 MeV, 20 MeV and 25 MeV.

From Fig. 3 we see that the higher the temperature, the lower the field strengths of σ_0, ω_0 and ρ_0 mesons and the chemical potential of the neutrons and electrons.

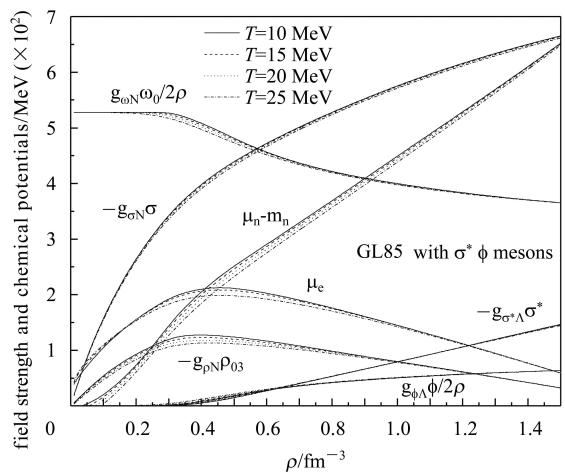

Fig. 3. The field strengths of σ, ω_0, ρ_{03}, σ^*, ϕ and the chemical potentials of neutrons and electrons as functions of the baryon density. The coupling constants of the nucleons are GL85 and the contribution of the σ^* and ϕ mesons is included.

The effect of the temperature on the moment of inertia of a PNS is shown in Fig. 4, from which we see that there is a maximum value $I_{\max}$ for each temperature. With growing temperature, the maximum value $I_{\max}$ increases. For the temperatures T=10 MeV, 15 MeV, 20 MeV and 25 MeV, we obtain the corresponding maximum values $I_{\max}$: 2.0961×10^{45} g·cm^2, 2.1535×10^{45} g·cm^2, 2.2648×10^{45} g·cm^2 and 2.4539×10^{45} g·cm^2 and the increments 0.0574×10^{45} g·cm^2, 0.1113×10^{45} g·cm^2 and 0.1891×10^{45} g·cm^2, or roughly 2.7%, 5.2% and 8.3%. Obviously, a higher temperature corresponds to a larger maximum value $I_{\max}$. From Ref. [13] we know that the higher the temperature, the larger the radius and the mass. So it is easy to understand why a higher temperature corresponds to a larger moment of inertia.

Finally, the influence of the coupling constants of the nucleons on the moment of inertia is shown in

Fig. 5 and Fig. 6. In this case, the contribution of the σ*, ϕ mesons is included and the temperature is chosen as T=15 MeV.

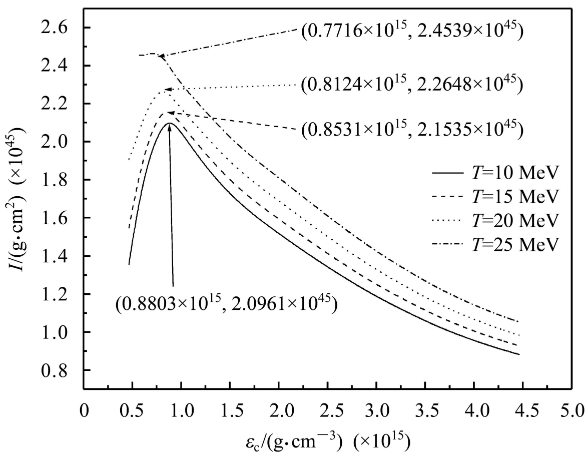

Fig. 4. The moment of inertia of a PNS as a function of the central energy density. The coupling constants of the nucleons are GL85 and the contribution of the σ* and ϕ mesons is included.

In Fig. 5 we see that the coupling constants GL85 and GL97 of the nucleons lead to different field strengths and chemical potentials. For GL97, the influence of the σ*, ϕ mesons appears at a higher baryon density.

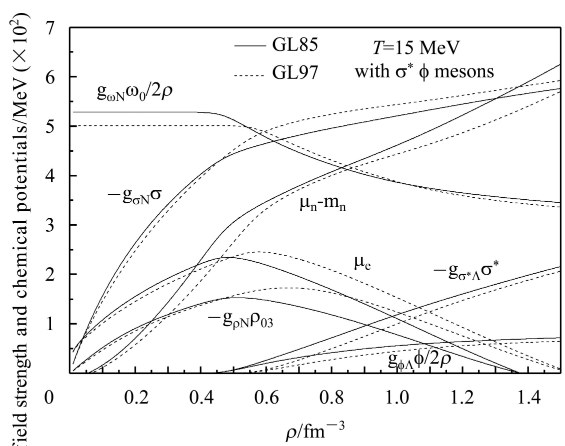

Fig. 5. The field strengths of σ, ω_0, ρ_{03}, σ* and ϕ and the chemical potential of the neutrons and electrons as functions of the baryon density. The temperature is T=15 MeV and the contribution of the σ* and ϕ mesons is included.

From Fig. 6 we can see that for GL97 the maximum value of the moment of inertia $I_{\max}$ is lower than that for GL85. The individual maximum values $I_{\max}$ for GL85 and GL97 are 2.1530×10^{45} g·cm^2 and 1.8025×10^{45} g·cm^2, and their difference is 0.3505×10^{45} g·cm^2, i.e. about 19%. We know that the compression modulus and the symmetry energy coefficient of GL85 and GL97, respectively, are

$$K=285\ \text{MeV},\quad a_{\text{sym}}=36.8\ \text{MeV}$$

and

$$K=240\ \text{MeV},\quad a_{\text{sym}}=32.5\ \text{MeV}.$$

It is evident that the former is larger than the latter. This shows that the larger the compression modulus and symmetry energy coefficient, the larger the maximum of the moment of inertia.

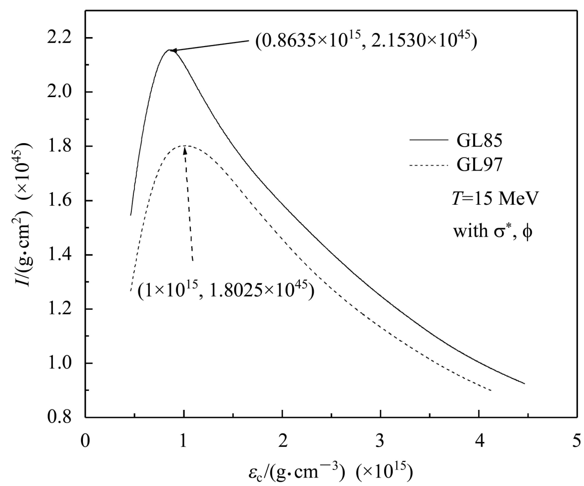

Fig. 6. The moment of inertia of a PNS as a function of the central energy density. The temperature is T=15 MeV and the contribution of the σ* and ϕ mesons is included.

In conclusion, the effects of the σ* and ϕ mesons, the temperature and the coupling constants of the nucleons on the moment of inertia of a proto neutron star are quite different. The influence of the temperature and coupling constants of the nucleons on the moment of inertia of a proto neutron star is lager than that of the σ* and ϕ mesons.

5 Summary

In conclusion, in this paper the influence of the σ* and ϕ mesons, the temperature and the coupling constants of the nucleons on the moment of inertia of a PNS have been investigated within the framework of relativistic mean field theory for the baryon octet {n,p, Λ, Σ$^-$, Σ^0, Σ$^+$, Ξ$^-$, Ξ^0} system. It is found that, compared with the case without considering the σ*, ϕ mesons, the moment of inertia decreases. It is also found that the higher the temperature, the larger the incompressibility and symmetry energy coefficient, and the larger the moment of inertia of a PNS. The influence of the temperature and the coupling constants of the nucleons on the moment of inertia of a PNS is larger than that of the σ^* and ϕ mesons.

In forthcoming work, we shall study other very important physical properties of PNSs, such as the angular velocity and the kinetic energy.

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