

Probing new physics in $B \rightarrow J/\psi\pi^0$ decay

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Abstract We calculate the branching ratio of $B \rightarrow J/\psi\pi^0$ with a mixed formalism that combines the QCD-improved factorization and the perturbative QCD approaches. The result is consistent with experimental data. The quite small penguin contribution in $B \rightarrow J/\psi\pi^0$ decay can be calculated with this method. We suggest two methods to extract the weak phase β . One is through the dependence of the mixing induced CP asymmetry $S_{J/\psi\pi^0}$ on the weak phase β , the other is from the relation of the total asymmetry A_{CP} with the weak phase β . Our results show that the deviation $\Delta S_{J/\psi\pi^0}$ of the mixing induced CP asymmetry from $\sin(-2\beta)$ is of $\mathcal{O}(10^{-3})$ and has much less uncertainty. The above $\mathcal{O}(10^{-3})$ deviation can provide a good reference for identifying new physics.

Key words new physics, weak phase, B meson decay, CP asymmetry

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1 Introduction

B physics is entering the era of precision measurement and is not far from revealing new physics beyond the Standard Model(SM). Many authors have studied the topic and suggested some windows for looking for new physics (NP) [1–9]. Because flavour-changing neutral current (FCNC) processes only occur at the loop-level in the SM, they are particularly sensitive to NP interactions. It has been pointed out that $B_q^0-\bar{B}_q^0$ mixing and decays are good places for new physics to enter through the exchange of new particles in the box diagrams, or through new contributions at the tree level [10–12], so the $B_q^0-\bar{B}_q^0$ system has been studied in many papers for probing new physics [13–16]. $B \rightarrow J/\psi\pi^0$ decay is a good mode for looking for new physics and extracting the weak phase β . The direct CP asymmetry $C_{J/\psi\pi^0}$ and the deviation $\Delta S_{J/\psi\pi^0} \equiv S_{J/\psi\pi^0} - \sin(-2\beta)$ of the mixing-induced CP asymmetry from $\sin(-2\beta)$ in this decay arise from quite small penguin contribution in the SM, so these quantities are sensitive to new physics effects. Comparing the prediction of CP asymmetry in the SM with the experimental data, one can find a new physics signal. Thus it is essential to calculate

the $\Delta S_{J/\psi\pi^0}$ and $C_{J/\psi\pi^0}$ in $B \rightarrow J/\psi\pi^0$ in the SM accurately.

The deviation $\Delta S_{J/\psi\pi^0} = S_{J/\psi\pi^0} - \sin(-2\beta)$ or direct CP asymmetry $C_{J/\psi\pi^0}$ in $B \rightarrow J/\psi\pi^0$ decay have been studied in Ref. [17] by fitting to the current experimental data. The result is $C_{J/\psi\pi^0} = 0.09 \pm 0.19$, which has very large uncertainty. In that case we cannot say anything about new physics effects.

In order to reveal new physics effects, we need both better theoretical prediction and experimental measurement with fewer uncertainties. That is the aim of our present paper.

In what follows, we first evaluate the penguin pollution effect by a method which has been used to explain many B decays into charmonia successfully [18, 19]. We find the penguin pollution in the $B \rightarrow J/\psi\pi^0$ decay is quite small, the deviation $\Delta S_{J/\psi\pi^0} = S_{J/\psi\pi^0} - \sin(-2\beta)$ in $B \rightarrow J/\psi\pi^0$ decay is $\mathcal{O}(10^{-3})$, which means that the measured deviation $\Delta S_{J/\psi\pi^0}$ at 1% will indicate the presence of new physics.

The latest experimental data of $\Delta S_{J/\psi\pi^0}$ are $S_{J/\psi\pi^0} = -0.4 \pm 0.4$ [20], which has large error, so we are expecting to have more precise measurements in the near future.

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2 Factorization formulas

The decay rate of $B \rightarrow J/\psi\pi^0$ can be written as

$$\Gamma = \frac{1}{32\pi m_B} G_F^2 \left(1 - r_2^2 + \frac{1}{2}r_2^4 - r_3^2\right) |\mathcal{A}|^2, \quad (1)$$

with $r_2 = m_{J/\psi}/m_B$, $r_3 = m_\pi/m_B$.

The amplitude \mathcal{A} consists of a factorizable part and a nonfactorizable part. It can be written as

$$\mathcal{A} = \mathcal{A}_{\text{NF}} + \mathcal{A}_{\text{VERT}} + \mathcal{A}_{\text{HS}}, \quad (2)$$

where \mathcal{A}_{NF} denotes the factorizable contribution in Naive Factorization Assumption (NF), $\mathcal{A}_{\text{VERT}}$ is the vertex corrections from Fig. 1(a)–(d), and \mathcal{A}_{HS} is the spectator correction from Fig. 1(e)–(f).

The factorizable part \mathcal{A}_{NF} in Eq. (2) for $B \rightarrow J/\psi\pi^0$ decay cannot be calculated reliably in the pQCD approach because its characteristic scale is around 1 GeV. We parameterize the sum of the factorizable part \mathcal{A}_{NF} and the vertex corrections $\mathcal{A}_{\text{VERT}}$ as

$$\mathcal{A}_{\text{NF}} + \mathcal{A}_{\text{VERT}} = a_{\text{eff}} m_B^2 f_{J/\psi} F_1^{B \rightarrow \pi}(m_{J/\psi}^2)(1 - r_2^2), \quad (3)$$

where $f_{J/\psi}$ is the decay constant of J/ψ meson,

For the $B \rightarrow \pi$ transition form factors, we employ the models derived from the light-cone sum rules [21], which have been parameterized as

$$F_1^{B \rightarrow \pi}(q^2) = \frac{r_1}{1 - q^2/m_1^2} + \frac{r_2}{1 - q^2/m_{\text{fit}}^2} \quad (4)$$

with $r_1 = 0.744$, $r_2 = -0.486$, $m_1 = 5.32$ GeV, $m_{\text{fit}}^2 = 40.73$ GeV for $B \rightarrow \pi$ transition.

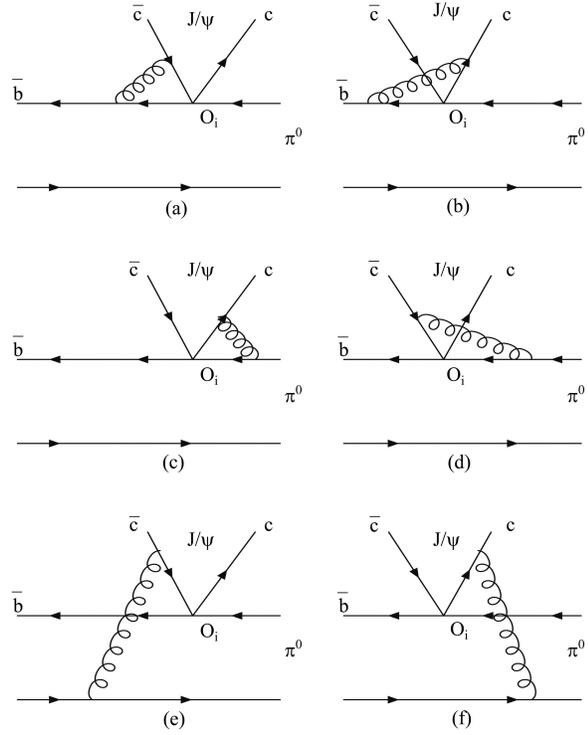


Fig. 1. Nonfactorizable contribution to the $B^0 \rightarrow J/\psi\pi^0$ decay.

The factorization and vertex correction from Fig. 1(a)–(d) can be calculated in the QCDF [22]. Summing up the factorizable part and vertex correction, we can get the Wilson coefficient a_{eff} ,

$$\begin{aligned} a_{\text{eff}} = & V_c^* \left[C_1 + V_c^* \frac{C_2}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_2 \left(-18 + 12 \ln \frac{m_b}{\mu} + f_1 \right) \right] - V_t^* \left[C_3 + \frac{C_4}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_4 \left(-18 + 12 \ln \frac{m_b}{\mu} + f_1 \right) \right. \\ & \left. + C_5 + \frac{C_6}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_6 \left(6 - 12 \ln \frac{m_b}{\mu} - f_1 \right) + C_7 + \frac{C_8}{N_c} + C_9 + \frac{C_{10}}{N_c} \right] \end{aligned} \quad (5)$$

with the function

$$f_1 = \frac{2\sqrt{2N_c}}{f_{J/\psi}} \int dx_3 \Psi^L(x_2) \left[\frac{3(1-2x_2)}{1-x_2} \ln x_2 - 3\pi i + 3 \ln(1-r_2^2) + \frac{2r_2^2(1-x_2)}{1-r_2^2 x_2} \right]. \quad (6)$$

The spectator corrections \mathcal{A}_{HS} from Fig. 1(e)–(f) can be calculated reliably in the pQCD as in Ref. [18, 19],

$$\mathcal{A}_{\text{HS}} = V_c^* \mathcal{M}_1^{(J/\psi\pi)} - V_t^* \mathcal{M}_4^{(J/\psi\pi)} - V_t^* \mathcal{M}_6^{(J/\psi\pi)}, \quad (7)$$

where the amplitudes $\mathcal{M}_{1,4}^{(J/\psi\pi)}$ and $\mathcal{M}_6^{(J/\psi\pi)}$ result from the $(V-A)(V-A)$ and $(V-A)(V+A)$ operators in the effective Hamiltonian, respectively. Their factorization formulas are given by the pQCD approach. In the calculation of $\mathcal{M}_{1,4}^{(J/\psi\pi)}$ and $\mathcal{M}_6^{(J/\psi\pi)}$, because J/ψ is heavy, we reserve the power terms of r_2 up to $\mathcal{O}(r_2^4)$, and the power terms of r_3 up to $\mathcal{O}(r_3^2)$.

$$\begin{aligned}
\mathcal{M}_{1,4}^{(J/\psi\pi)} &= 16\pi m_B^2 C_F \sqrt{2N_c} \int_0^1 [dx] \int_0^\infty b_1 db_1 \Phi_B(x_1, b_1) \times \left\{ \left[(1 - 2r_2^2 + r_2^4)(1 - x_2) \Phi_\pi(x_3) \Psi^L(x_2) \right. \right. \\
&+ \frac{1}{2}(r_2^2 - r_2^4) \Phi_\pi(x_3) \Psi^t(x_2) - r_\pi(1 - r_2^2)x_3 \Phi_\pi^p(x_3) \Psi_L(x_2) + r_\pi(2r_2^2(1 - x_2) \\
&+ (1 - r_2^2)x_3) \Phi_\pi^t(x_3) \Psi^L(x_2) \left. \right] \times E_{1,4}(t_d^{(1)}) h_d^{(1)}(x_1, x_2, x_3, b_1) - \left[(x_2 - x_2 r_2^4 + x_3 - 2r_2^2 x_3 + r_2^4 x_3) x_3 \right. \\
&\times \Phi_\pi(x_3) \Psi^L(x_2) + r_2^2(2r_\pi \Phi_\pi^t(x_3) - \frac{1}{2}(1 - r_2^2) \Phi_\pi(x_3)) \Psi^t(x_2) - r_\pi(1 - r_2^2)x_3 \Phi_\pi^p(x_3) \Psi_L(x_2) \\
&\left. \left. - r_\pi(2r_2^2 x_2 + (1 - r_2^2)x_3) \Phi_\pi^t(x_3) \Psi^L(x_2) \right] \times E_{1,4}(t_d^{(2)}) h_d^{(2)}(x_1, x_2, x_3, b_1) \right\}, \tag{8}
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_6^{(J/\psi\pi)} &= 16\pi m_B^2 C_F \sqrt{2N_c} \int_0^1 [dx] \int_0^\infty b_1 db_1 \times \Phi_B(x_1, b_1) \times \left\{ \left[(1 - x_2 + r_2^4 x_2 + x_3 - 2r_2^2 x_3 \right. \right. \\
&+ r_2^4 x_3 - r_2^4) \Phi_\pi(x_3) \Psi^L(x_2) + r_2^2(2r_\pi \Phi_\pi^t(x_3) - \frac{1}{2}(1 - r_2^2) \Phi_\pi(x_3)) \Psi^t(x_2) - r_\pi(1 - r_2^2) \\
&\times x_3 \Phi_\pi^p(x_3) \Psi^L(x_2) - r_\pi(2r_2^2(1 - x_2) + (1 - r_2^2)x_3) \Phi_\pi^t(x_3) \Psi^L(x_2) \left. \right] \times E_6(t_d^{(1)}) \\
&\times h_d^{(1)}(x_1, x_2, x_3, b_1) - \left[(1 - 2r_2^2 + r_2^4)x_2 \times \Phi_\pi(x_3) \Psi^L(x_2) + \frac{1}{2}(r_2^2 - r_2^4)r_2^2 \Phi_\pi(x_3) \right. \\
&\times \Psi^t(x_2) - r_\pi(1 - r_2^2)x_3 \Phi_\pi^p(x_3) \Psi^L(x_2) + r_\pi(2r_2^2 x_2 + (1 - r_2^2)x_3) \Phi_\pi^t(x_3) \Psi^L(x_2) \left. \right] \\
&\left. \times E_6(t_d^{(2)}) h_d^{(2)}(x_1, x_2, x_3, b_1) \right\}, \tag{9}
\end{aligned}$$

with the color factor $C_F = 4/3$, the number of colors $N_c = 3$, the symbol $[dx] \equiv dx_1 dx_2 dx_3$ and the mass ratio $r_\pi = m_\pi^2/m_B^2$, m_π^2 being the chiral scale associated with the π meson.

The evolution factor E_i and hard function h_d in Eq. (9) can be found in Ref. [19]. In the derivation of spectator correction in the pQCD, we need to take the wave function of relevant mesons. We list the wave functions in Appendix A.

3 Numerical results and conclusion

For the B^0 decay, the CP asymmetry is time dependent,

$$\begin{aligned}
A_{CP}(t) &= \frac{\Gamma(\bar{B}^0(t) \rightarrow J/\psi\pi^0) - \Gamma(B^0(t) \rightarrow J/\psi\pi^0)}{\Gamma(\bar{B}^0(t) \rightarrow J/\psi\pi^0) + \Gamma(B^0(t) \rightarrow J/\psi\pi^0)} \\
&= S_{J/\psi\pi^0} \sin(\Delta Mt) - C_{J/\psi\pi^0} \cos(\Delta Mt), \tag{10}
\end{aligned}$$

where the mixing-induced asymmetry $S_{J/\psi\pi^0}$ and direct CP asymmetry are defined as

$$\begin{aligned}
S_{J/\psi\pi^0} &= \frac{2\text{Im} \lambda_{J/\psi\pi^0}}{1 + |\lambda_{J/\psi\pi^0}|^2}, \\
C_{J/\psi\pi^0} &= \frac{1 - |\lambda_{J/\psi\pi^0}|^2}{1 + |\lambda_{J/\psi\pi^0}|^2}, \tag{11}
\end{aligned}$$

where

$$\lambda_{CP} = \frac{V_{tb}^* V_{td} \langle J/\psi\pi^0 | H_{\text{eff}} | \bar{B}^0 \rangle}{V_{tb} V_{td}^* \langle J/\psi\pi^0 | H_{\text{eff}} | B^0 \rangle}. \tag{12}$$

According to Eq. (10), we can get the dependence of the mixing-induced asymmetry $S_{J/\psi\pi^0}$ and the deviation $\Delta S_{J/\psi\pi^0}$ on the weak phase β shown in Fig. 2. We can find that the penguin pollution in the decays is quite small from Fig. 2.

There are two ways to extract the weak phase β through $B^0 \rightarrow J/\psi \pi^0$ decay. The first way is through the dependence of the mixing-induced CP asymmetry on weak phase β . The $S_{J/\psi\pi^0}$ is not sensitive to input parameters, as shown in Fig. 4. That means that the theoretical uncertainties of $S_{J/\psi\pi^0}$ are quite small. If we measure the mixing-induced asymmetry $S_{J/\psi\pi^0}$, we can determine the weak phase β through the dependence of $S_{J/\psi\pi^0}$ on β , as shown in Fig. 3 and Table 1.

Another way is to use the relation of the total asymmetry A_{CP} with the weak phase β . By integrating $A_{CP}(t)$ with respect to the time variable t , we can get the total asymmetry A_{CP} ,

$$A_{CP} = \frac{x}{1+x^2} S_{J/\psi\pi^0} - \frac{1}{1+x^2} C_{J/\psi\pi^0}, \tag{13}$$

with $x = \Delta m/\Gamma \approx 0.723$ for the B^0 - \bar{B}^0 mixing in the SM [20].

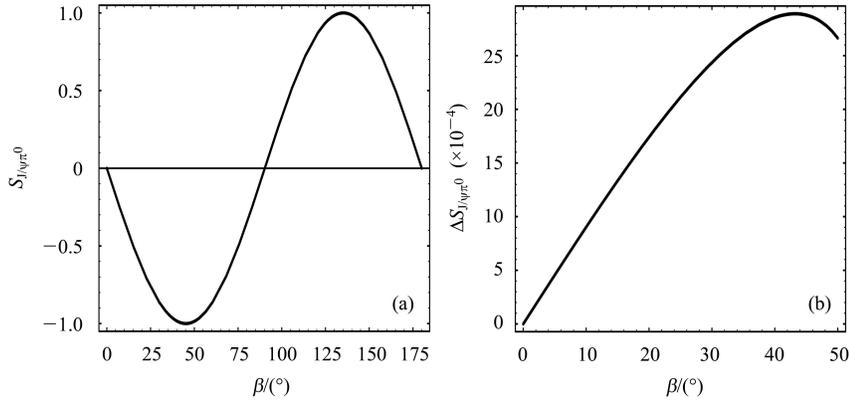


Fig. 2. The dependence of the mixing-induced asymmetry $S_{J/\psi\pi^0}$ for $B^0 \rightarrow J/\psi\pi^0$ on the weak phase β in Diagram (a). The dependence of the deviation $\Delta S_{J/\psi\pi^0}$ of the mixing-induced asymmetry from $\sin(-2\beta)$ on the weak phase β in Diagram (b).

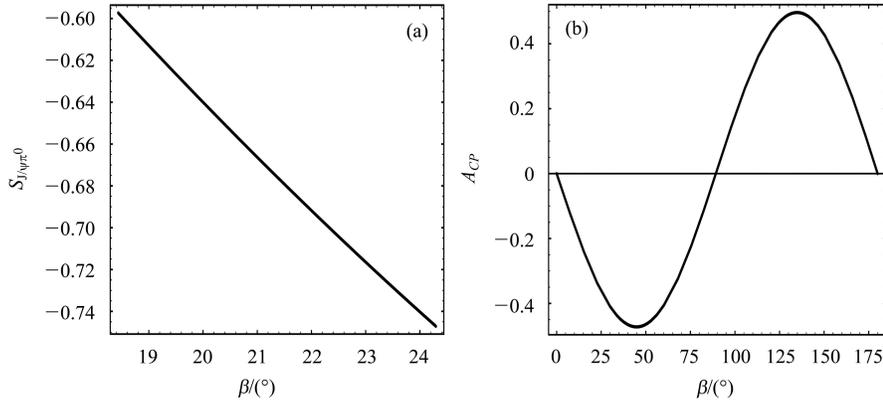


Fig. 3. The dependence of the the mixing-induced asymmetry $S_{J/\psi\pi^0}$ for $B^0 \rightarrow J/\psi\pi^0$ on the weak phase β in Diagram (a) can be used to extract the weak phase β . The dependence of total CP asymmetry A_{CP} on the weak phase β in Diagram (b) can be used to extract the weak phase β also.

Table 1. Determination of weak phase β through mixing-induced CP asymmetry $S_{J/\psi\pi^0}$.

$\beta/(\circ)$	18.0	18.3	18.6	18.9	19.2	19.5	19.8	20.1
$S_{J/\psi\pi^0}$	-0.58515	-0.59357	-0.60192	-0.61021	-0.61843	-0.62658	-0.63467	-0.64269
$\beta/(\circ)$	20.4	20.7	21	21.3	21.6	21.9	22.2	22.5
$S_{J/\psi\pi^0}$	-0.65063	-0.65851	-0.66631	-0.67404	-0.68170	-0.68929	-0.69680	-0.70424
$\beta/(\circ)$	22.8	23.1	23.4	23.7	24.0	24.3	24.6	24.9
$S_{J/\psi\pi^0}$	-0.71160	-0.71888	-0.72608	-0.73321	-0.74025	-0.74722	-0.75410	-0.76090

$$\begin{aligned}
 Br(B^0 \rightarrow J/\psi\pi^0) &= [1.89_{-0.21}^{+0.182}(\omega b)_{-0.02}^{+0.0496}(\mu)_{-0.171}^{+0.193}(F_1)_{-0.014}^{+0.015}(f_{J/\psi})_{-0.059}^{+0.04}(\lambda)_{-0.068}^{+0.04}(A)] \times 10^{-5}, \\
 C_{J/\psi\pi^0} &= [-9.936_{-3.093}^{+0.866}(\omega b)_{-2.368}^{+1.173}(\gamma)_{-0.289}^{+6.914}(\mu)_{-1.18}^{+1.34}(F_1)_{-0.56}^{+0.54}(\beta)] \times 10^{-3}, \\
 \Delta S_{J/\psi\pi^0} &= [2.84_{-1.00}^{+4.07}(\omega b)_{-0.35}^{+0.72}(\gamma)_{-0.17}^{+2.1}(\mu)_{-0.20}^{+0.29}(F_1)_{-0.05}^{+0.03}(\beta)] \times 10^{-3}.
 \end{aligned} \tag{14}$$

Like the mixing-induced asymmetry, the total asymmetry is also not sensitive to the input parameters, so we can determine the weak phase through the

relation of the total CP asymmetry with the weak phase β shown in Fig. 3.

The numerical calculation needs some parameters

and meson distribution amplitudes as input. We list them in the appendix.

With the parameters and meson distribution amplitude in the appendix, we get the branching ratios of $B \rightarrow J/\psi\pi^0$ decays, $\Delta S_{J/\psi\pi^0}$ and $C_{J/\psi\pi^0}$.

The main theoretical errors of the branching ratio are induced by the uncertainties below. The first error is from $\omega b = 0.4 \pm 0.04$ GeV, the second one is due to the renormalization scale μ taken from $mb/2$ to mb , the third one is induced by 15% uncertainty of $B \rightarrow \pi$

form factor $F_1^{B \rightarrow \pi}$, the fourth one arises from the decay constant $f_{J/\psi} = 0.405 \pm 0.05$ GeV, the fifth error is from the CKM matrix parameter $\lambda = 0.2272 \pm 0.001$ and the sixth one is from the CKM matrix parameter $A = 0.818_{-0.017}^{+0.007}$.

Compared with the experimental data [20],

$$Br(B^0 \rightarrow J/\psi\pi^0) = (2.2 \pm 0.4) \times 10^{-5}, \quad (15)$$

our prediction of the branching ratio for $B \rightarrow J/\psi\pi^0$ is consistent with it.

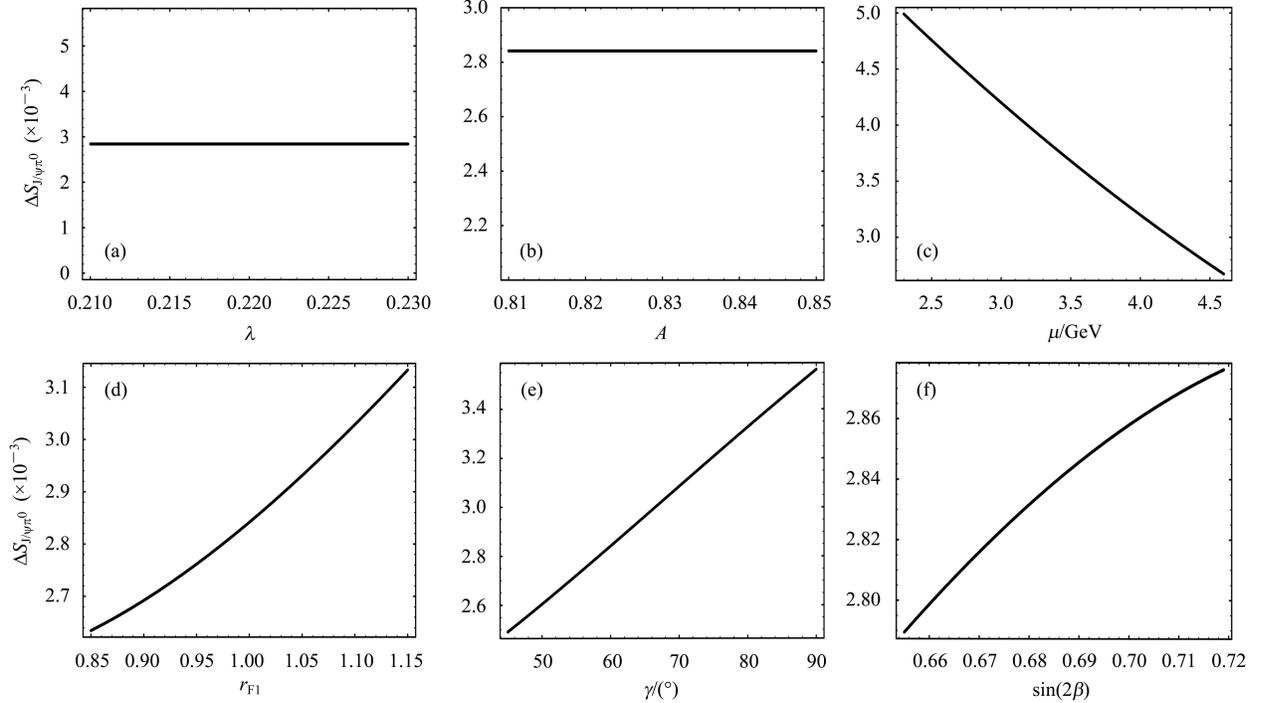


Fig. 4. The uncertainties of $\Delta S_{J/\psi\pi^0}$ of the mixing-induced asymmetry from $\sin(-2\beta)$ are induced by those of renormalization scale μ in (c), those of $B \rightarrow \pi$ form factor in (d), those of the weak phase γ in (e) and those of $\sin(2\beta)$ in (f).

Unlike the branching ratio, $\Delta S_{J/\psi\pi^0}$ and $C_{J/\psi\pi^0}$ are not sensitive to the CKM matrix parameter λ or A , because these parameter dependences cancel out. The independence of $\Delta S_{J/\psi\pi^0}$ and $C_{J/\psi\pi^0}$ on some CKM parameters is shown in Fig. 4(a), (b), and Fig. 5(a), (b).

To find new physics and extract the weak phase β , we need reliable evaluation for the direct CP asymmetry $C_{J/\psi\pi^0}$ and $\Delta S_{J/\psi\pi^0}$, so we now consider the dependence of the direct CP asymmetry $C_{J/\psi\pi^0}$ and $\Delta S_{J/\psi\pi^0}$ with all parameters of input.

The main uncertainties of $C_{J/\psi\pi^0}$ and $\Delta S_{J/\psi\pi^0}$ are induced by the uncertainties of shape parameter ωb , CKM matrix phase γ , renormalization scale μ , $B \rightarrow \pi$ form factor $F_1^{B \rightarrow \pi}$ and the weak phase β . The uncer-

tainties of $\Delta S_{J/\psi\pi^0}$ and $C_{J/\psi\pi^0}$ are shown in Fig. 4(c)–(f) and Fig. 5(c)–(f).

Compared with the result in Ref. [17],

$$C_{J/\psi\pi^0} = 0.09 \pm 0.19 \quad (16)$$

$$S_{J/\psi\pi^0} = -0.47 \pm 0.30, \quad (17)$$

our results of $\Delta S_{J/\psi\pi^0}$ and $C_{J/\psi\pi^0}$ have much less theoretical uncertainty. So we conclude that if the measured deviation $\Delta S_{J/\psi\pi^0}$ of the mixing-induced asymmetry is at 1% or the direct asymmetry $C_{J/\psi\pi^0}$ is at the level of percentage, then we can say that there should be new physics. We are expecting precise measurement of the CP asymmetry of $B^0 \rightarrow J/\psi\pi^0$ in the near future.

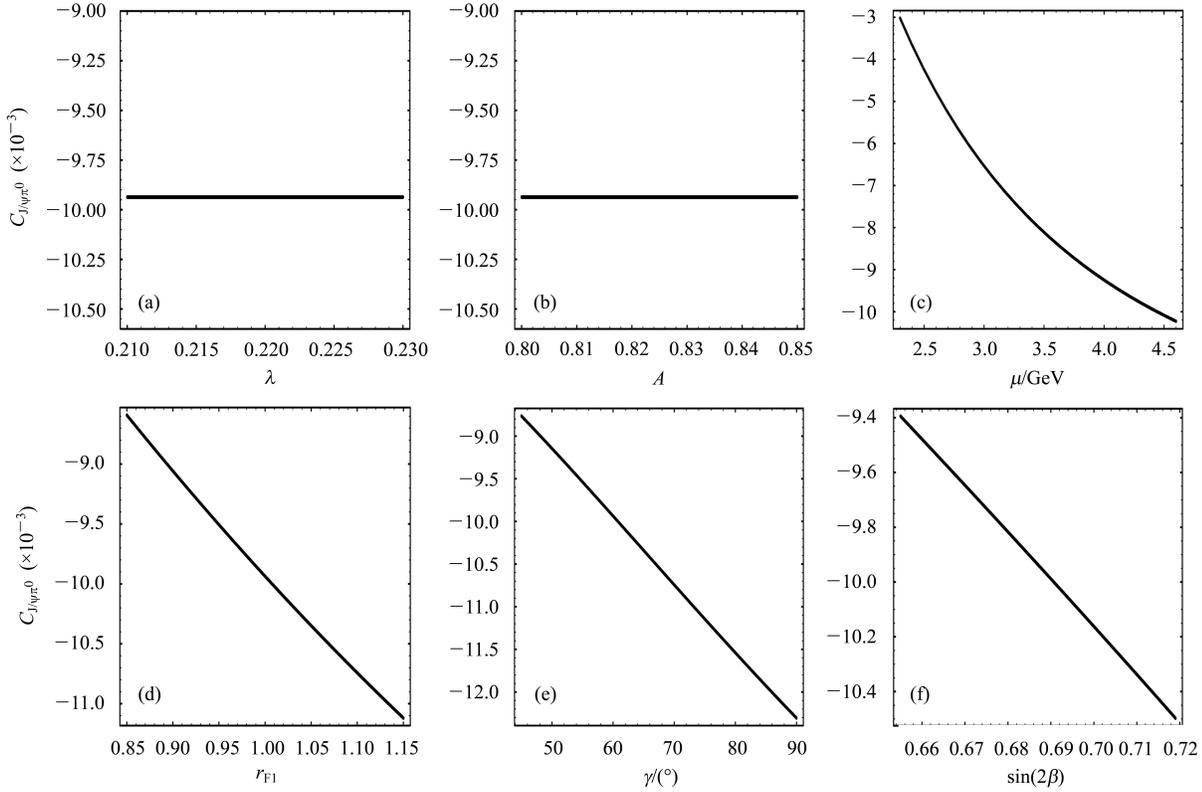


Fig. 5. The uncertainties of the direct CP asymmetry $C_{J/\psi\pi^0}$ are induced by those of renormalization scale μ in (c), those of $B \rightarrow \pi$ form factor in (d), those of the weak phase γ in (e) and those of $\sin(2\beta)$ in (f).

Appendix A

Input parameters and wave functions

We use the following input parameters in the numerical calculations,

$$\begin{aligned} A_{\overline{\text{MS}}}^{(f=4)} &= 250 \text{ MeV}, & f_\pi &= 130 \text{ MeV}, \\ f_B &= 190 \text{ MeV}, & m_0^\pi &= 1.4 \text{ GeV}, \\ M_B &= 5.2792 \text{ GeV}, & \tau_{B^0} &= 1.53 \times 10^{-12} \text{ s}. \end{aligned} \quad (\text{A1})$$

For the CKM matrix elements, we adopt the wolfenstein parametrization for the CKM matrix up to $\mathcal{O}(\lambda^3)$ [20],

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \quad (\text{A2})$$

with the parameters $\lambda = 0.2272$, $A = 0.818$, $\rho = 0.221$ and $\eta = 0.340$.

For the B meson distribution amplitude, we adopt the

model [23]

$$\phi_B(x, b) = N_B x^2 (1-x)^2 \exp\left[-\frac{M_B^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right], \quad (\text{A3})$$

where ω_b is a free parameter, and we take $\omega_b = 0.4 \pm 0.05$ GeV in numerical calculations, and $N_B = 91.745$ is the normalization factor for $\omega_b = 0.4$.

The J/ψ meson asymptotic distribution amplitudes are given by [24]

$$\begin{aligned} \Psi^L(x) &= \Psi^T(x) \\ &= 9.58 \frac{f_{J/\psi}}{2\sqrt{2}N_c} x(1-x) \left[\frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7}, \\ \Psi^t(x) &= 10.94 \frac{f_{J/\psi}}{2\sqrt{2}N_c} (1-2x)^2 \left[\frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7}, \\ \Psi^V(x) &= 1.67 \frac{f_{J/\psi}}{2\sqrt{2}N_c} [1 + (2x-1)^2] \\ &\quad \times \left[\frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7}. \end{aligned} \quad (\text{A4})$$

For the light meson wave function, we neglect the b dependent part, which is not important in numerical analysis.

We choose the wave function of π meson [25]:

$$\begin{aligned}\Phi_{\pi}(x) &= \frac{3}{\sqrt{6}}f_{\pi}x(1-x)[1+0.44C_2^{3/2}(2x-1) \\ &\quad +0.25C_4^{3/2}(2x-1)] \\ \Phi_{\pi}^P(x) &= \frac{f_{\pi}}{2\sqrt{6}}[1+0.43C_2^{1/2}(2x-1) \\ &\quad +0.09C_4^{1/2}(2x-1)], \\ \Phi_{\pi}^t(x) &= \frac{f_{\pi}}{2\sqrt{6}}(1-2x)[1+0.55(10x^2-10x+1)].\end{aligned}\quad (\text{A5})$$

The Gegenbauer polynomials are defined by

$$\begin{aligned}C_2^{1/2}(t) &= \frac{1}{2}(3t^2-1), \\ C_4^{1/2}(t) &= \frac{1}{8}(35t^4-30t^2+3), \\ C_2^{3/2}(t) &= \frac{3}{2}(5t^2-1), \\ C_4^{3/2}(t) &= \frac{15}{8}(21t^4-14t^2+1).\end{aligned}\quad (\text{A6})$$

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