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Rigorous spectral representation of relativistic random phase approximation for finite nuclei*

LI Ning(李宁) YAO Hai-Bo(姚海波) CHEN Xi(陈曦) WU Shi-Shu(吴式枢)

Center for Theoretical Physics and School of Physics, Jilin University, Changchun 130023, China

Abstract By using the rigorous spectral representation of relativistic random phase approximation, the low-lying excitation of finite nuclei and its longitudinal response function for quasielastic electron scattering are calculated in the σ-ω model of quantum hadrodynamics. It is shown that the reproduction of the correct order of the 1^- and 3^- excitation states of ^{16}O is due to the contribution of the exchange vertex. There is no significant influence of the retardation effect on the low-lying excitation states. In contrast, the retardation effect plays an important role in the electron scattering process of nuclei. The theoretical longitudinal responses of ^{12}C and ^{40}Ca, including the contributions of the exchange vertex and the retardation effect, are suppressed and reproduce the experimental data better than the results excluding them.

Key words exchange vertex, retardation effect, relativistic random phase approximation, quasielasic electron scattering

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1 Introduction

The relativistic random phase approximation (RRPA) is one of the most successful tools for treating nuclear many-body effects in quantum hadrodynamics [1, 2]. It is a relativistic extension of the nonrelativistic random phase approximation and represents the small amplitude limit of the time-dependent relativistic mean-field theory. In the framework of RRPA, one can solve the relativistic integral equation, for example the Bethe-Salpeter equation, and obtain the properties of the nuclear many-body system. In the last decade, the RRPA not only yielded good results of the excited state of spherical nuclei around the stable region, but was also widely applied to the description of the isotopes far from the β-stability line.

The study of the relativistic random phase approximation was pioneered by Chin [3] in the late of 1970s. Summing a set of ring diagrams, collective modes could be generated in nuclear matter within the Walecka model. Horowitz and Serot [4] have pointed out that the retardation effects may cause a significant change in the results in relativistic nuclear matter calculations. According to the Baym-Kadanoff [5] formalism or perturbation theory, the integral equation depends at least on two energy-variables, so that it is difficult to solve unless approximations are made. Blunden and McCorquodale [6] considered the contribution of both ring and ladder diagrams. To simplify the calculation, they neglected the retardation effects of the exchange vertex. Crecca and Walker [7] established a simple model with ladder diagrams in which the motion of the baryon was treated nonrelativistically. Bauer E et al. [8, 9] theoretically studied the electron scattering response, including the contribution of the exchange vertex, using the standard Laudan-Migdal parameters plus a $(\pi+\rho)$-meson exchange as a model interaction and evaluated the ladder diagram up to the second level.

One of the authors established a three-dimensional relativistic two-body wave equation to replace the four dimensional Bethe-Salpeter equation [10]. The three-dimensional equation is relativistic self-closed and it takes the full retardation effects into account. With the method of Ref. [10], a rigorous spectral representation of the RRPA was given [11, 12]. In this RRPA, the explicit expressions for the annihilation term $K^{(\mathrm{an})}$, the exchange term $K^{(\mathrm{ex})}$ and the

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kernel for the fermion line renormalization $K^{(\mathrm{fr})}$ were obtained rigorously in a form that depends only on one energy variable and the retardation effect was taken into account properly. This scheme provides a simpler and more systematic treatment of the RRPA. In order to compare the relative contributions of the above subkernels and the retardation effects, it is desirable to investigate the properties of the finite nuclei using this scheme. In this paper, the rigorous spectral representation of the RRPA is applied to calculate the low-lying excitation states of finite nuclei and its longitudinal response function for quasielastic electron scattering in the σ-ω model.

The paper is organized as follows. In Section 2, we review the derivation of the full expression of the RRPA and its relation with the response function. In Section 3, the numerical results of the low-lying negative parity excitations of ^{16}O, the longitudinal quasielastic functions of ^{12}C and ^{40}Ca are discussed and compared with experiments. Finally, a summary is given in Section 4.

2 Formalism

In the σ-ω model of quantum hadrodynamics [13–17], the Lagrangian density for the system of the nucleons and mesons has the following form,

$$\mathcal{L}=\bar{\psi}[\mathrm{i}\gamma_\mu\partial^\mu-M]\psi+\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma-m_\sigma^2\sigma^2)-\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu}+\frac{1}{2}m_\omega^2\omega_\mu\omega^\mu+\bar{\psi}[g_\sigma\sigma-g_\omega\gamma^\mu\omega_\mu]\psi, \tag{1}$$

with $\Omega_{\mu\nu}=\partial_\mu\omega_\nu-\partial_\nu\omega_\mu$, where m_σ and m_ω are the rest masses of the mesons and M is the mass of nucleon. The field operators are denoted by σ and ω for the meson fields and ψ is for the nucleon field. All fields are functions of $x=(x,\ t)$.

From Eq. (1), one may write the Hamiltonian of the system as $H=H_0+H_{\mathrm{int}}$. If a single particle potential u is introduced into the Hamiltonian to facilitate a many-body calculation, it takes the form $H=H_0+u+H_{\mathrm{int}}-u$. In the basis of the single nucleon spectrum, the nucleon-related Hamiltonian H_{nucl} can be exactly expanded as

$$H_{\mathrm{nucl}}=\sum_\alpha E_\alpha c_\alpha^\dagger c_\alpha+\sum_{\eta\lambda kr}[b_r(\eta\lambda k)-u_r(\eta\lambda)]\,c_\lambda^\dagger c_\eta, \tag{2}$$

with

$$\begin{aligned}b_r(\eta\lambda k)&=f_r(\eta\lambda k)a_{rk}+f_r^\dagger(\lambda\eta k)a_{rk}^\dagger,\\ f_r(\eta\lambda k)&=\int\mathrm{d}^3x\bar{\psi}_\lambda(x)\Gamma_r\psi_\eta(x)\phi_k(x),\\ f_r^\dagger(\eta\lambda k)&=\int\mathrm{d}^3x\bar{\psi}_\eta(x)\Gamma_r\psi_r(x)\phi_k^*(x),\end{aligned}$$

where ψ^α is the nucleon Dirac spinor and all of the quantum numbers are denoted by α. The basis functions $\phi_k(x)$ satisfy the Klein-Gordon equation for mesons and all quantum numbers of the mesons meson are abbreviated with k. $\{E_\alpha\}$ is the eigenvalue spectrum, $c_\alpha(c_\alpha^\dagger)$ are the nucleon annihilation (creation) operators, $a_{rk}(a_{rk}^\dagger)$ are the meson operators and the subscript r denotes the type of meson. The Dirac field operator has the form $\psi(x,t)=\sum_\alpha\psi_\alpha(x)c_\alpha(t)$. For finite nuclei, $\psi_\alpha(x)$ and $\phi_k(x)$ are expressed in polar coordinates as in Ref. [13].

To get the rigorous spectral representation of the relativistic random phase approximation, we start from the two time Green function, which is defined as

$$\begin{aligned}G_{\alpha\beta\gamma\delta}(t_1-t_2)&=\langle 0|T[c_\beta^\dagger(t_1)c_\alpha(t_1)c_\gamma^\dagger(t_2)c_\delta(t_2)]|0\rangle\\&\quad-\langle c_\beta^\dagger c_\alpha\rangle\langle c_\gamma^\dagger c_\delta\rangle.\end{aligned} \tag{3}$$

The equation satisfied by $G_{\alpha\beta\gamma\delta}(\omega)$ is

$$\begin{aligned}&(\omega\delta_{\alpha\beta\xi\zeta}-\alpha_{\alpha\beta\xi\zeta}-u_{\alpha\beta\xi\zeta})G_{\xi\zeta\gamma\delta}(\omega)\\&\quad=-g_{\alpha\beta\gamma\delta}+G(M_{\alpha\beta},\gamma\delta;\omega),\end{aligned} \tag{4}$$

where $\alpha_{\alpha\beta\xi\zeta}=(E_\alpha-E_\beta)\delta_{\alpha\xi}\delta_{\beta\zeta}$, $u_{\alpha\beta\xi\zeta}=u_{\beta\zeta}\delta_{\alpha\xi}-u_{\alpha\xi}\delta_{\beta\zeta}$, and

$$G(M_{\alpha\beta},\gamma\delta;\omega)=\sum_{\xi\zeta}[b_r(\xi\alpha k)\delta_{\beta\zeta}-b_r(\beta\zeta k)\delta_{\alpha\xi}]G_{\xi\zeta,\gamma\delta}(\omega). \tag{5}$$

To make Eq. (4) closed, we introduce the kernel K through the following equation,

$$G(M_{\alpha\beta},\gamma\delta;\omega)=g_{\alpha\beta,\xi'\zeta'}K_{\xi'\zeta',\xi\zeta}(\omega)G_{\xi\zeta,\gamma\delta}(\omega). \tag{6}$$

If $G^{-1}_{\xi\zeta,\gamma\delta}(\omega)$ exists, then the kernel $K_{\xi'\zeta',\xi\zeta}$ is well defined. Following the method described in Ref. [10], $K_{\alpha\beta,\gamma\delta}$ is given by

$$\begin{aligned}&K_{\alpha\beta,\gamma\delta}(\omega)=g^{-1}_{\alpha\beta\xi\zeta}[-h_{\xi\zeta\eta\lambda}+G(M_{\xi\zeta},M^\dagger_{\eta\lambda};\omega)\\&-G(M_{\xi\zeta},\xi'\zeta';\omega)G^{-1}_{\xi'\zeta'\eta'\lambda'}(\omega)G(\eta'\lambda',M^\dagger_{\eta\lambda};\omega)]g^{-1}_{\eta\lambda\gamma\delta},\end{aligned} \tag{7}$$

where

$$h_{\xi\zeta\eta\lambda}=\sum_{\xi'\zeta'rk}[b_r(\xi'\xi k)\delta_{\zeta\zeta'}-b_r(\zeta\zeta'k)\delta_{\xi\xi'}]\langle c^\dagger_{\zeta'}c_{\xi'},c^\dagger_\eta c_\lambda\rangle. \tag{8}$$

Eq. (7) is rigorous if $g^{-1}_{\alpha\beta\xi\zeta}$ exists. It was proved in Ref. [10] that the complicated term $G(M_{\xi\zeta},\xi'\zeta';\omega)G^{-1}_{\xi'\zeta'\eta'\lambda'}(\omega)G(\eta'\lambda',M^\dagger_{\eta\lambda};\omega)$ is a useful term rather than a bothering one, because it exactly cancels the particle-hole reducible part of $G(M_{\xi\zeta},M^\dagger_{\eta\lambda};\omega)$, which can be cut into two separate parts by breaking a particle and a hole line at the same horizontal level in the Feynman diagram. So $K_{\alpha\beta,\gamma\delta}(\omega)$ is obtained by calculating only the irreducible diagrams of $G(M_{\xi\zeta},M^{\dagger}_{\eta\lambda};\omega)$, which can be given straightforwardly by perturbation theory. Defining $G_{ir}(M_{\xi\zeta},M^{\dagger}_{\eta\lambda};\omega)$ as the irreducible term of $G(M_{\xi\zeta},M^{\dagger}_{\eta\lambda};\omega)$, the kernel $K_{\alpha\beta,\gamma\delta}(\omega)$ has a simpler form,

$$K_{\alpha\beta,\gamma\delta}(\omega)=g^{-1}_{\alpha\beta\xi\zeta}[-h_{\xi\zeta\rho\varrho}+G_{ir}(M_{\xi\zeta},M^{\dagger}_{\rho\varrho};\omega)]g^{-1}_{\rho\varrho\gamma\delta}. \tag{9}$$

Considering the lowest nonzero approximation, Eq. (9) provides a simple and systematic way to calculate the RRPA and its higher order corrections. Finding the zero order of G and the first order of h, $K_{\alpha\beta,\gamma\delta}(\omega)$ is written by $K_{\alpha\beta,\gamma\delta}(\omega)=K^{(\text{an})}_{\alpha\beta,\gamma\delta}(\omega)+K^{(\text{ex})}_{\alpha\beta,\gamma\delta}(\omega)+K^{(\text{sf})}_{\alpha\beta,\gamma\delta}(\omega)$, where $K^{(\text{an})}_{\alpha\beta,\gamma\delta}(\omega)$ is the annihilation term and $K^{(\text{ex})}_{\alpha\beta,\gamma\delta}(\omega)$ is the exchange term. Their corresponding Feynman diagrams are shown in Fig. 1. By iterating, $K^{(\text{an})}_{\alpha\beta,\gamma\delta}(\omega)$ and $K^{(\text{ex})}_{\alpha\beta,\gamma\delta}(\omega)$ produce the ring diagrams and ladder diagrams. $K^{(\text{sf})}_{\alpha\beta,\gamma\delta}(\omega)$ is a singular fermion line renormalization term.

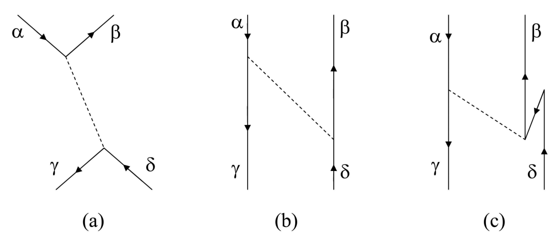

Fig. 1. Feynman diagrams of irreducible kernels. Fig. 1(a) is the Feynman diagram of $\tilde{K}^{(\text{an})}_{\alpha\beta,\gamma\delta}(\omega)$ and Fig. 1(b) and Fig. 1(c) are the Feynman diagrams of $\tilde{K}^{(\text{ex})}_{\alpha\beta,\gamma\delta}(\omega)$. The solid and dotted lines denote the nuclear and meson propagators, respectively.

The kernel $K_{\alpha\beta,\gamma\delta}(\omega)$ depends only on the one variable ω. The quantity ω in the kernel corresponds to the retardation of the interaction between the two nucleons in real time. This retardation effect is properly taken into account in our RRPA kernels and can be calculated self-consistently in our scheme.

Defining $\tilde{K}_{\alpha\beta,\gamma\delta}=K_{\alpha\beta,\gamma\delta}+g^{-1}_{\alpha\beta,\alpha'\beta'}u_{\alpha'\beta',\gamma\delta}$, Eq. (4) turns to a more clear form,

$$\begin{aligned}&(\omega\delta_{\alpha\beta,\xi\zeta}-\alpha_{\alpha\beta,\xi\zeta})G_{\xi\zeta,\gamma\delta}(\omega)\\ &=-g_{\alpha\beta\gamma\delta}+g_{\alpha\beta,\eta\lambda}\tilde{K}_{\eta\lambda,\xi\zeta}(\omega)G_{\xi\zeta,\gamma\delta}(\omega).\end{aligned} \tag{10}$$

If a suitable u is adopted, $\tilde{K}$ can be simplified. In this paper, the effective potential u is determined by a nucleon mean field. The various kernels $\tilde{K}$ are listed below,

$$\begin{aligned}\tilde{K}^{(\text{an})}_{ir}(\alpha\beta,\gamma\delta;\omega)=&-\sum_{rk}f_r(\beta\alpha k)f^{\dagger}_r(\delta\gamma k)\\ &\times D^{(\text{an})}_r(\alpha\beta,\gamma\delta,k;\omega),\end{aligned} \tag{11}$$

$$\begin{aligned}&\tilde{K}^{(\text{ex})}_{ir}(\alpha\beta,\gamma\delta;\omega)\\ &=\sum_{rk}f_r(\gamma\alpha k)f^{\dagger}_r(\delta\beta k)\times[D^{(\text{ex})}_r(\alpha\beta,\gamma\delta,k;\omega)\\ &\quad-D^{(\text{ex})}_r(\alpha\beta,\delta\gamma,k;0)],\end{aligned} \tag{12}$$

$$\begin{aligned}&\tilde{K}^{(\text{sf})}_{ir}(\alpha\beta,\gamma\delta;\omega)\\ &=-\sum_{rk}\Big\{\sum_{\lambda}f_r(\lambda\alpha k)f^{\dagger}_r(\lambda\xi k)\\ &\quad\times[D^{\text{sf}}_r(\beta\lambda k;\omega)-D^{\text{sf}}_r(\xi\lambda k;0)]\delta_{\beta\zeta}\\ &\quad+\sum_{\lambda}f_r(\beta\lambda k)f^{\dagger}_r(\zeta\lambda k)\\ &\quad\times[D^{\text{sf}}_r(\lambda\alpha k;\omega)-D^{\text{sf}}_r(\lambda\zeta k;0)]\delta_{\xi\alpha}\Big\},\end{aligned} \tag{13}$$

where

$$\begin{aligned}D^{(\text{an})}_r(\alpha\beta,\gamma\delta,k;\omega)=&\,\Delta^{+}_r(\alpha\beta,\gamma\delta,k;\omega)\\ &-\Delta^{-}_r(\alpha\beta,\gamma\delta,k;\omega),\end{aligned} \tag{14}$$

$$\begin{aligned}D^{(\text{ex})}_r(\alpha\beta,\gamma\delta,k;\omega)=&\,D^{+}_r(\beta\gamma k,\omega)-D^{-}_r(\delta\alpha k,\omega)\\ &+D^{+}_r(\delta\alpha k,\omega)-D^{-}_r(\beta\gamma k,\omega),\end{aligned} \tag{15}$$

$$D^{(\text{sf})}_r(\beta\lambda k;\omega)=D^{+}_r(\beta\lambda k;\omega)-D^{-}_r(\beta\lambda k;\omega), \tag{16}$$

$$\Delta^{\pm}_r(\alpha\beta,\gamma\delta,k;\omega)=\frac{(N_\beta-N_\alpha)(N_\delta-N_\gamma)}{\omega\mp E_{rk}\pm\mathrm{i}\epsilon}, \tag{17}$$

$$D^{+}_r(\alpha\beta k;\omega)=\frac{N_\alpha(1-N_\beta)}{\omega-E_{rk}+E_\alpha-E_\beta+\mathrm{i}\epsilon}, \tag{18}$$

$$D^{-}_r(\alpha\beta k;\omega)=\frac{(1-N_\alpha)N_\beta}{\omega+E_{rk}+E_\alpha-E_\beta-\mathrm{i}\epsilon}. \tag{19}$$

There is no contribution of $h_{\alpha\beta,\gamma\delta}$ in $\tilde{K}^{(\text{an})}_{\alpha\beta,\gamma\delta}(\omega)$ because the single particle potential u is equal to $h^{(\text{an})}_{\alpha\beta,\gamma\delta}$ and they eliminate each other.

Our above results have taken all the terms and retardation effects into account and give a full, rigorous description of the RRPA. When substituting $\tilde{K}$ into Eq. (10), the series obtained by iteration contains all the forward and backward propagating diagrams produced by $\tilde{K}^{(\text{an})}_{\alpha\beta,\gamma\delta}(\omega)$, $\tilde{K}^{(\text{ex})}_{\alpha\beta,\gamma\delta}(\omega)$ and $\tilde{K}^{(\text{sf})}_{\alpha\beta,\gamma\delta}(\omega)$ and their cross combinations.

According to the Lehmann representation of the particle-hole Green function, Eq. (4) can be rewritten as the RRPA eigenequation,

$$(\omega\delta_{\alpha\beta,\xi\zeta}-\alpha_{\alpha\beta,\xi\zeta})\chi_{\xi\zeta}=-g_{\alpha\beta,\eta\lambda}\tilde{K}_{\eta\lambda,\xi\zeta}(\omega)\chi_{\xi\zeta}, \tag{20}$$

where $\chi_{\xi\zeta}=\langle 0|c^{\dagger}_{\zeta}c_{\xi}|f\rangle$. In the numerical calculation, Eq. (20) is transformed into the angular and isospin coupling representation and the RRPA equation has the following matrix form,

$$\begin{bmatrix} A^{JT}_{php'h'}(\omega) & B^{JT}_{phh'p'}(\omega) \\ -B^{JT}_{hpp'h'}(\omega) & -A^{JT}_{hph'p'}(\omega) \end{bmatrix} \begin{pmatrix} \chi^{JT}_{p'h'} \\ \chi^{JT}_{h'p'} \end{pmatrix} = \omega \begin{pmatrix} \chi^{JT}_{p'h'} \\ \chi^{JT}_{h'p'} \end{pmatrix}, \tag{21}$$

with $A^{JT}_{\alpha\beta,\xi\zeta}(\omega) = \alpha_{\alpha\beta,\xi\zeta} - \tilde{K}^{JT}_{\alpha\beta,\xi\zeta}(\omega)$ and $B^{JT}_{\alpha\beta,\xi\zeta}(\omega) = -\tilde{K}^{JT}_{\alpha\beta,\xi\zeta}(\omega)$, where the subscripts p and h denote the particle and hole states, respectively.

The response function for quasi-elastic electron scattering is the imaginary part of the polarization with three momentum transfer q and energy loss ω,

$$S_\eta(q,\omega) = 2^{|\theta|}\frac{1}{\pi}\mathrm{Im}\Pi(J_\theta, J_\theta, q, \omega), \tag{22}$$

where $\Pi(J_\theta, J_\theta, q, \omega)$ is the polarization propagator, which depends on the baryon currents J_θ. The baryon current J_η is longitudinal for $\theta = 0$, while J_θ is transverse for $\theta = \pm 1$. In general, the baryon current [2] is given by $J_\theta = e^\mu(q,\theta)J_\mu$ with

$$\begin{aligned} J_\mu(q,\omega) \ = \ & F_1(q^2)\int \mathrm{d}^3x \mathrm{e}^{\mathrm{i}qx}\bar{\psi}(x)\gamma_\mu\psi(x) \\ & + F_2(q^2)\int \mathrm{d}^3x \mathrm{e}^{\mathrm{i}qx}\bar{\psi}(x)\frac{\lambda_M}{2M}\mathrm{i}\sigma_{\mu\nu}q^\nu\psi(x), \end{aligned} \tag{23}$$

where F_1 and F_2 are the Dirac and Pauli form factors of the nucleons, respectively. λ_M is the anomalous moment. The response function is represented through J_θ as

$$S_\theta(q,\omega) = 2^{|\theta|}\frac{1}{\pi}\mathrm{Im}[J_\theta(q,\omega)G(\omega)J^\dagger_\theta(q,\omega)], \tag{24}$$

where $G(\omega)$ is the particle-hole Green function. Substituting the solution of Eq. (21) into Eq. (24), one gets the response function $S_\theta(q,\omega)$.

3 Results and discussions

The numerical results of the low lying collective states in ^{16}O and the longitudinal response of ^{12}C and ^{40}Ca are presented in this section. In order to concentrate on the exchange term and the retardation effects, the simple σ-ω model and the mean-field parametrization are adopted [18]: $g^2_\sigma = 109.6$, $m_\sigma = 520$ MeV, $g^2_\omega = 190.4$ and $m_\omega = 783$ MeV. The ground-state properties are evaluated by the relativistic mean field theory without Dirac sea. The continua of the single particle excited states are discretized by imposing boundary conditions at a radius several times the nuclear radius. Theoretically, the contribution of the retardation effects can be taken into account by iteration. But the calculation of response functions is too formidable to accomplish. So we develop an algorithm in which the range of eigenvalues of Eq. (21) is divided into N parts, and the median of each part is regarded as the ω in the kernels. If the part is small enough to ignore the difference between the median and the eigenvalues in it, the median of the part will be regarded as the energy-variable in the kernel.

The low-lying negative-parity states of ^{16}O are shown in Fig. 2. The first column shows the unperturbed levels from the ground-state calculation. The second and third columns are the spectra obtained from the RRPA, including $K^{(\mathrm{an})}$ and $K^{(\mathrm{an})} + K^{(\mathrm{ex})}$, respectively. The forth column is the experimental data of Ref. [19].

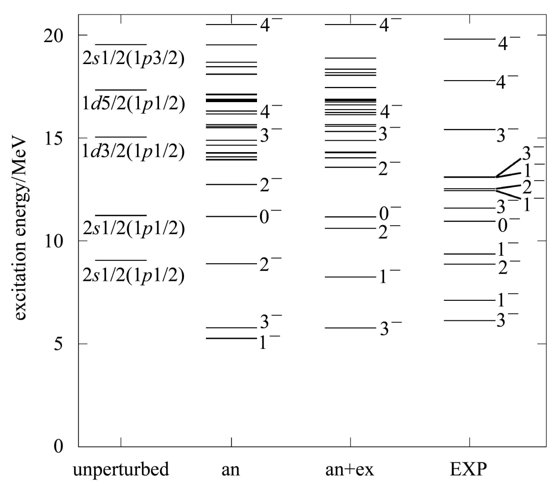

Fig. 2. Energy levels of the low-lying states in ^{16}O.

If only the contribution of $K^{(\mathrm{an})}$ is considered, the energy of the 3^- state is higher that of the 1^- state, which is not in agreement with experiment results. The exchange vertex $K^{(\mathrm{ex})}$ may lead to an increase in the energy of the 1^- state above that of the 3^- state, while the 3^- state is not changed. The latter order of the the 1^- and 3^- states is comparable to experiment. Our RRPA spectrum with $K^{(\mathrm{an})}$ and $K^{(\mathrm{ex})}$ is very similar to the results in Ref. [6]. However, they did not show clearly that the exchange vertex plays a crucial role in the order of the collective 1^- and 3^- states.

Dawson and Furnstahl [20] obtained the correct order of the 1^- and 3^- states due to the negative-energy states included in the configuration spaces. However, at the same time, a spurious 1^- state appears. According to our results, the exchange vertex $K^{(\mathrm{ex})}$ can reproduce the correct order of these two states and no spurious state exists, which is another candidate for the explanation of the state inversion.

The retardation effects (RE) for the low-lying states are shown in Table 1. The first and second columns show the excitation energies with (+RE) and without retardation effects (no RE), respectively. One finds that there is no significant influence of the retardation effects on the low-lying excited states. However, for the longitudinal response function, the contribution of the retardation effects is important, as we show below.

Table 1. Energies of the low-lying excited states.

J	+RE/MeV	no RE/MeV	EXP/MeV
3	5.7732	5.7746	6.13043
1	8.2423	8.2408	7.11685
2	10.6125	10.6192	8.8719
0	11.1672	11.1672	9.632
2	13.5806	13.5834	12.531
4	16.1304	16.1307	17.788

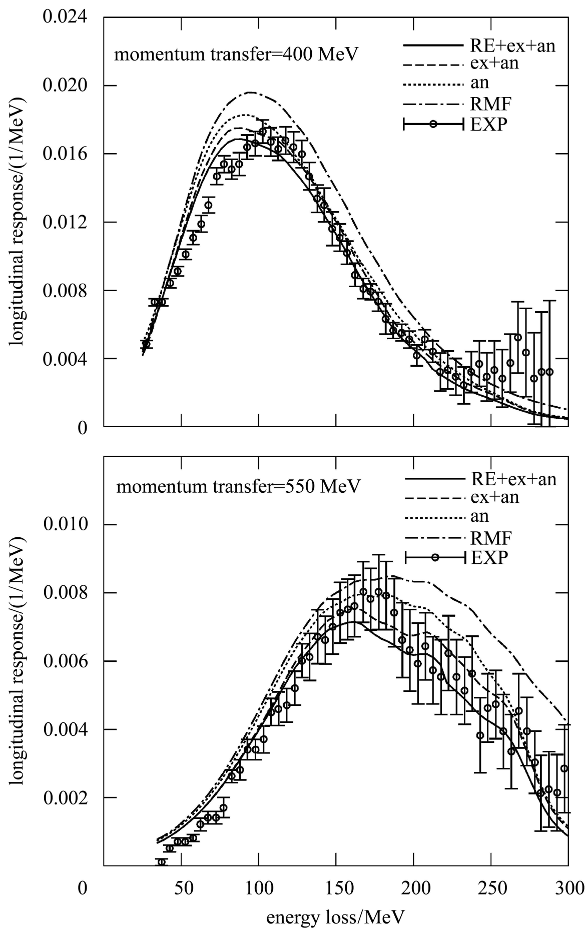

Fig. 3. Longitudinal response function for ^{12}C at q = 400 MeV and 550 MeV. The solid (dashed) line shows the results of the calculation, including $K^{(\mathrm{an})}$ and $K^{(\mathrm{ex})}$ with (without) retardation effects. The dotted line shows the corresponding results using the RRPA, which only includes the annihilation interaction, and the dashed-dotted line shows the response in the relativistic mean field (RMF). The experimental data are from Ref. [21].

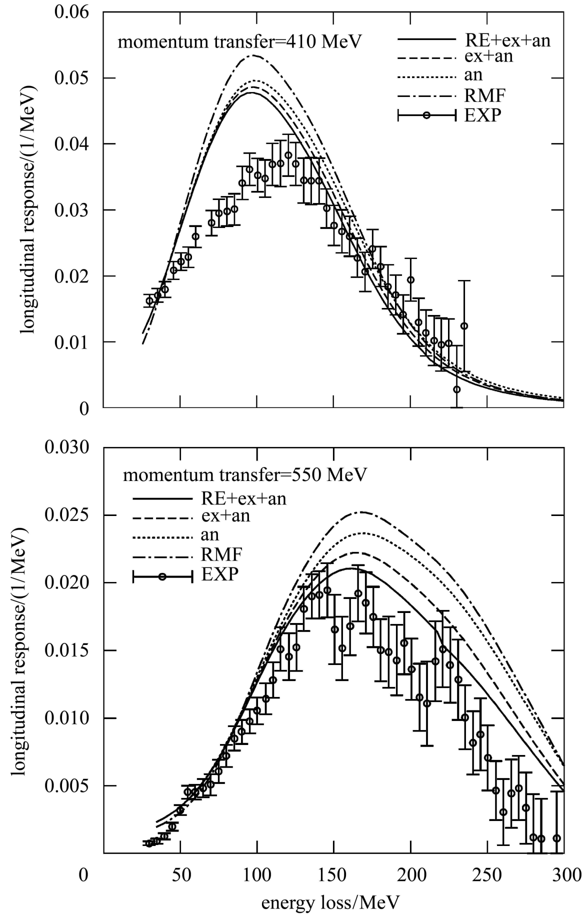

Fig. 4. Longitudinal response function for ^{40}Ca at q = 410 MeV and 550 MeV. The solid (dashed) line shows the results of the calculation, including $K^{(\mathrm{an})}$ and $K^{(\mathrm{ex})}$ with (without) retardation effects. The dotted line shows the corresponding results using the RRPA, which only includes the annihilation interaction, and the dashed-dotted line shows the response in the relativistic mean field (RMF). The experimental data are from Ref. [21].

The longitudinal response function is obtained from Eq. (24) with the $G(\omega)$ derived from Eq. (21) including Dirac and Pauli currents. In Fig. 3 and Fig. 4, we plot our results for ^{12}C at q = 400 MeV and q = 550 MeV, and ^{40}Ca at q = 410 MeV and $q = 550$ MeV. Comparing with the calculations in the relativistic mean field theory and the RRPA with only $K^{(\mathrm{an})}$, one notices that the exchange vertex $K^{(\mathrm{ex})}$ is not only responsible for a decline in the amplitude of the response function but also shifts the maximum towards a lower energy transfer. For ^{12}C at q = 550 MeV, a good agreement is obtained when taking $K^{(\mathrm{ex})}$ into account. At $q = 400$ MeV, the position of the maximum is slightly lower and the amplitude is in better agreement with the experimental data than that obtained by the method with only $K^{(\mathrm{an})}$. Though all of the responses of ^{40}Ca are overestimated, as in Ref. [22] and Ref. [23], the compressibility owing to $K^{(\mathrm{ex})}$ and the retardation effects have significant contributions and the response fits better to the experimental data. This feature becomes more obvious at $q = 550$ MeV.

Until now, experimental data of response functions for ^{12}C, ^{40}Ca and ^{56}Fe are available. The number of protons in ^{56}Fe have been different from the number of neutrons, and it needs the nonlinear model with the isovector meson coupling for a descripation. Our RRPA scheme is independent of the model and it can be extended to the nonlinear model, which is beyond the scope of this paper.

The treatment of the subkernel $K^{(\mathrm{sf})}$ in our RRPA scheme involves the renormalization and the selection of a single particle potential. In this paper, we focus on the exchange vertex and retardation effects for finite nuclei, and the effects of the subkernel $K^{(\mathrm{sf})}$ are not discussed here.

4 Summary

The low-lying excited states of ^{16}O and the longitudinal response functions for quasielastic electron scattering off ^{12}C and ^{40}Ca are calculated in the framework of the σ-ω model of quantum hadrodynamics. By using the rigorous spectral representation of the relativistic random phase approximation, the exchange vertex and retardation effects are taken into account simultaneously and properly. We have pointed out that the reproduction of the correct order of the 1^- and 3^- excited states of ^{16}O is due to the contribution of the exchange vertex, and there is no significant influence of the retardation effect on the low-lying excited states. In contrast, the retardation effect plays an important role in the electron scattering process of nuclei. The theoretical longitudinal responses of ^{12}C and ^{40}Ca, including the contributions of the exchange vertex and retardation effects, are suppressed and reproduce the experimental data better than the results excluding them.

References

1 Lipparini E, Stingari S. Phys. Rep., 1989, **175**(3,4): 103–261

2 Wehrberger K. Phys. Rep., 1993, **225**(5,6): 273–362

3 Chin S A. Ann. Phys. (N. Y.), 1977, **108**: 301–367

4 Horowitz C J, Serot B D. Nucl. Phys. A, 1987, **464**: 613–699

5 Baym G, Kadanoff L P. Phys. Rev., 1961, **124**: 287–299

6 Blunden P G, McCorquodale P. Phys. Rev. C, 1988, **38**: 1861–1869

7 Crecca M A, Walker G E. Phys. Rev. C, 1991, **43**: 1709–1724

8 Bauer E. Phys. Rev. C, 1996, **54**, 2959–2966

9 Bauer E, Polls A, Ramos A. Phys. Rev. C, 1998, **58**: 1052–1065

10 WU Shi-Shu. J. Phys. G, 1990, **16**: 1447–1460

11 WU Shi-Shu, YAO Yu-Jie, SU Jun-Chen et al. Chinese Phys. Lett., 1992, **9**: 345–348

12 WU Shi-Shu, ZHANG Hai-Xia, YAO Yu-Jie. Study of Consistency in the Relativistic Random Phase Approximation. In: Kuo T T S, Lee T -S H, WU Shi-Shu, ed. Nuclear Many-Body Problem and Sub-Nucleonic Degrees of Freedom in Nuclei. Changchun: Jilin Pepole's Press. 2008. 141–155

13 Serot B D, Walecka J D. Adv. Nucl. Phys., 1985, **16**: 1–327

14 Reinhard P G. Rep. Prog. Phys. 1989, **52**: 439–514

15 Ring P. Prog. Part. Nucl. Phys., 1996, **37**: 193–263

16 Vretenar D, Afanasjev A, Lalazissis G, Ring P. Phys. Rep., 2005, **409**: 101–259

17 MENG J, Toki H, ZHOU S G, ZHANG S Q, LONG W H, GENG L S. Prog. Part. Nucl. Phys., 2006, **57**: 470–563

18 Horowitz C J, Serot B D. Nucl. Phys. A, 1981, **368**: 503–528

19 Ajzenberg-Selove F et al. Nucl. Phys. A, 1982, **375**: 1–168

20 Dawson J F, Furnstahl R J. Phys. Rev. C, 1990, **42**: 2009–2022

21 Meziani Z E et al. Phys. Rev. Lett., 1984, **52**: 2130–2133

22 Wehrverger K, Beck F. Phys. Rev. C, 1988, **37**: 1148–1154

23 Shepard J R, Rost E, McNeil J A. Phys. Rev. C, 1989, **40**: 2320–2336