# Pseudoscalar mixing in $J/\psi$ and $\psi(2S)$ decay<sup>\*</sup>

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Abstract Based on the branching fractions of  $J/\psi(\psi(2S)) \rightarrow VP$  from different collaborations, pseudoscalar mixing is extensively discussed with a well established phenomenological model. The mixing angle is determined to be  $-14^{\circ}$  by fitting to the new world average if only quark content is considered. After taking into account the gluonic content in  $\eta$  and  $\eta'$  simultaneously, the investigation shows that  $\eta$  favors only consisting of light quarks, while the gluonic content of  $\eta'$  is  $Z^{2}_{\eta'} = 0.30 \pm 0.24$ .

Key words pseudoscalar mixing,  $J/\psi$  decay, branching fraction

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# 1 Introduction

As the ground pseudoscalar nonet,  $\pi$ , K,  $\eta$  and  $\eta'$ , in the constituent quark model, their masses and widths are determined with high precision and the main decay modes are also observed [1] in addition to the forbidden and rare decays. However, there is one issue, pseudoscalar mixing, that remains to be completely settled, which has been discussed on many occasions with different transitions. The linear Gell-Mann-Okubo (GMO) mass relation [2] gives a mixing angle,  $\theta_{\rm P} = -11^{\circ}$ , which is hardly consistent with the value,  $\theta_{\rm P} = -24.6^{\circ}$ , obtained from the quadratic GMO mass formula by replacing the meson masses by their squares. The full set of  $J/\psi$  decays into a vector and a pseudoscalar was measured by MarkIII, and the phenomenological analysis of mixing angle is determined to be  $\theta_{\rm P} = (-19.2 \pm 1.4)^{\circ}$  [3], which was confirmed by DM2 [4]. Both of them reached the conclusion that  $\eta$  and  $\eta'$  consist of light quarks, with no contribution from gluonium or radial excitation states. After that, important work was performed by Bramon and Scadron [5, 6]. Taking into account  $\omega$ - $\phi$  mixing in the analysis for  $J/\psi \rightarrow VP$ , a weighted  $\theta_{\rm P}$  is calculated to be  $(-15.5 \pm 1.3)^{\circ}$  based on many different transitions. For a nice review based on the discussions before 2000, see Ref. [7], in which the reasonable range of  $\eta$ - $\eta'$  mixing angle is believed to be  $-20^{\circ}-10^{\circ}$ .

Recently, the new experimental data on  $J/\psi \rightarrow$ 

VP and  $\psi(2S) \rightarrow VP$  were reported by BES [8–14], BABAR [15–18] and CLEO [19]. It is worth pointing out that some of the new measurements are not very consistent with the previous works. Take  $J/\psi \rightarrow \rho\pi$ , for example. The branching fractions measured by BES is  $(2.10\pm0.12)\%$  [8], subsequently confirmed by BABAR [15], which is larger than the world average  $(1.28 \pm 0.10)\%$  [20] of about 64%. This significant change stimulates new interest in this issue [21– 24]. The analysis in Ref. [23] indicates that it is difficult to get reasonable results with the updated branching fractions of  $J/\psi \to \rho \pi$ . However, the results in Ref. [24] performed with the same data and phenomenological model seem reasonable. This discrepancy motivated us to reanalyze the full set of  $J/\psi \rightarrow VP$  data. Actually, it is difficult for us to compare the results obtained with different sets of parameters at one time. In this paper we would like to discuss this issue for different cases (e.g. fix SU(3)) breaking term x to 0.64, 0.82 or 1).

# 2 Notation

The physical eigenstates  $\eta,\,\eta'$  are the mixture of octet, singlet and gluonium. They are defined as,

$$|\eta\rangle = X_{\eta}|N\rangle + Y_{\eta}|S\rangle + Z_{\eta}|G\rangle,$$
  
$$|\eta'\rangle = X'_{\eta}|N\rangle + Y'_{\eta}|S\rangle + Z'_{\eta}|G\rangle, \qquad (1)$$

where 
$$N = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$
,  $S = s\bar{s}$  and G for gluonium;

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and  $X_i$ ,  $Y_i$  and  $Z_i$  denote the magnitude of nonstrange, strange contents and gluonium in  $\eta$  and  $\eta'$ .

The above form can be written in terms of the three Euler angles, with

$$\begin{aligned} X_{\eta} &= \cos \phi_{\rm P} \cos \phi_{\rm G1}, \\ Y_{\eta} &= -\sin \phi_{\rm P} \cos \phi_{\rm G1}, \\ Z_{\eta} &= -\sin \phi_{\rm G1}, \\ X_{\eta'} &= \cos \phi_{\rm P} \cos \phi_{\rm G2} - \sin \phi_{\rm P} \sin \phi_{\rm G2} \sin \phi_{\rm G1}, \\ Y_{\eta'} &= \sin \phi_{\rm P} \cos \phi_{\rm G2} + \cos \phi_{\rm P} \sin \phi_{\rm G2} \sin \phi_{\rm G1}, \\ Z_{\eta'} &= -\sin \phi_{\rm G2} \cos \phi_{\rm G1}. \end{aligned}$$

$$(2)$$

If we only consider the simplest case and neglect possible mixing of the  $\eta$  and  $\eta'$  with other pseudoscalar states,  $\eta$ - $\eta'$  mixing is characterized by a single mixing angle  $\theta_{\rm P}$ ,

$$\begin{aligned} |\eta\rangle &= \cos\theta_{\rm P} |\eta_8\rangle - \sin\theta_{\rm P} |\eta_0\rangle, \\ |\eta'\rangle &= \sin\theta_{\rm P} |\eta_8\rangle + \cos\theta_{\rm P} |\eta_0\rangle, \end{aligned} \tag{3}$$

where  $\eta$  and  $\eta'$  are the orthogonal mixture of the respective singlet and octet iso-spin zero states.  $\eta_0$  and  $\eta_8$  are SU(3) quark basis states which are denoted as  $\eta_0 = \frac{1}{\sqrt{3}} |u\bar{u} + d\bar{d} + s\bar{s}\rangle$  and  $\eta_8 = \frac{1}{\sqrt{6}} |u\bar{u} + d\bar{d} - 2s\bar{s}\rangle$  respectively.

In terms of quark basis, the  $\eta$  and  $\eta'$  include non-strange and strange contents. In the flavor SU(3) quark model, they are defined through quarkantiquark (q $\bar{q}$ ) basis states as

$$X_{\eta} = Y_{\eta'} = \sqrt{\frac{1}{3}}\cos\theta_{\rm P} - \sqrt{\frac{2}{3}}\sin\theta_{\rm P} = \cos\phi_{\rm P},$$
$$X_{\eta'} = -Y_{\eta} = \sqrt{\frac{1}{3}}\sin\theta_{\rm P} + \sqrt{\frac{2}{3}}\cos\theta_{\rm P} = \sin\phi_{\rm P}, \quad (4)$$

where  $\theta_{\rm P} = \phi_{\rm P} - 54.7^{\circ}$ .

## 3 Phenomenological model

 $J/\psi$  and  $\psi(2S)$  have a similar decay mechanism and are suppressed by the Okubo-Zweig-Iizuka (OZI) rule. Both of them decay into a vector and pseudoscalar meson via three gluon annihilation and electromagnetic decays. Therefore, in this paper, the phenomenological model for  $J/\psi \rightarrow VP$  in Ref. [25] is simply applied in  $\psi(2S)$  decays to discuss the  $\eta$ - $\eta'$  mixing and other physics.

A first-order parameterization of the amplitudes appears in Ref. [25] and is described there in detail. The amplitude, which has contributions from both the three gluon annihilation and electromagnetic processes, can be expressed in terms of an SU(3) symmetric single-OZI(SOZI) amplitude g, an electromagnetic amplitude e (the coupling strength e has a relative phase  $\theta_e$  to the strength g because these are produced from different origins) and the nonetsymmetry-breaking double-OZI(DOZI) amplitude r, relative to g. SU(3) violation has been accounted for by a pure octet SU(3) breaking term. The SU(3)breaking term in strong interaction and electromagnetic process is expressed by (1-s) and x, respectively. A factor (1-s) represents every strange quark contributing to g and a factor for x for a strange quark contributing to e. The factor  $s_v(s_P)$  is for the strange vector(pseudoscalar) contributing to r.

In spite of these simplified assumptions, this phenological model contains a rather large number of parameters  $(g, e, r, s, s_{\rm P}, s_{\rm v}, x, \theta_{\rm P} \text{ and } \theta_{\rm e})$ . This x can be determined via  $V \to P\gamma$  and  $P \to V\gamma$  data. We reanalyzed it using the phenomenological model in Ref. [26] and the branching fractions of  $V \to P\gamma$  and  $P \to V\gamma$  in Ref. [1]. x is determined to be  $0.82\pm0.05$ ,  $\theta_{\rm V} = (3.2\pm0.9)^{\circ}$  and  $\theta_{\rm P} = (-12.9\pm0.5)^{\circ}$ , which are in good agreement with those in Ref. [21]. To further simplify it again,  $s_{\rm P}$  is ignored in this paper and  $s_{\rm v}$ is discussed below with two assumptions ( $s_{\rm v} = 0$  and  $s_{\rm v} = s$ ).

# 4 Results

The experimental data sets shown in Table 1 are analyzed with the least squares method to determine the coupling strengths and mixing angle. To clarify the results obtained from different data sets, we divided it into several subsections to investigate the pseudoscalar mixing.

# 4.1 Analysis of $J/\psi \rightarrow VP$ from MarkIII and DM2

We performed the fit to experimental data by starting with the simplest case. The  $\omega$ - $\phi$  mixing and gluon content are ignored. Actually, the treatment on the SU(3)-breaking parameter x and the second order corrections  $s_v$  in Ref. [3] and Ref. [4] are different. The x is set to 1 and the correction term  $s_v = 0$ , is ignored in MarkIII analysis, while x is fixed to 0.64 and the correction term  $s_v = s$  is included in DM2 analysis. To clearly compare the difference between them, all the possible combinations are considered to perform the fit.

A fit to the data without considering SU(3) breaking as well as in MarkIII analysis yields  $\theta_{\rm P} = (-13.95\pm 2.39)^{\circ}$  with  $\chi^2/\text{d.o.f} = 9.0/4$ , which is obviously inconsistent with the value  $(-19.2\pm 1.4)^{\circ}$  [3]. After tuning the parameter, we also get reasonable results, which are the same as those in Ref. [3],

 $g = 1.10 \pm 0.03, \ s = 0.12 \pm 0.03, \ e = 0.122 \pm 0.005,$ 

 $\theta_{\rm e} = 1.25 \pm 0.12, \ \theta_{\rm P} = (-19.34 \pm 1.40)^{\circ}, \ r = -0.15 \pm 0.09.$ But the goodness of fit,  $\chi^2/\text{d.o.f} = 10.1/4$ , seems slightly worse. Compared with the results listed in the first column of Table 2, s and r also change significantly. The results of the fits performed with x = 0.64 and x = 0.82 are also given in Table 2. Apart from the mixing angle, the values of other parameters are also consistent with the previous fit.

Table 1.	Branching	fractions	of $J/\psi \rightarrow$	$VP(\times 10^{-4}).$	
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decay modes	MarkIII	DM2	BES	BABAR	PDG2010
ρπ	$142\pm1\pm9$	$132\pm20$	$210 \pm 12 \pm 20.1$	$218\pm19$	$169\pm15$
ρη	$1.93 \pm 0.13 \pm 0.29$	$1.94 \pm 0.17 \pm 0.29$			$1.93 \pm 0.23$
ρη′	$1.14 \pm 0.14 \pm 0.16$	$0.83 \pm 0.30 \pm 0.12$			$1.05\pm0.18$
$\phi\pi^0$	< 0.068		< 0.064		< 0.064
φη	$6.61 \pm 0.45 \pm 0.78$	$6.4 \!\pm\! 0.4 \!\pm\! 1.1$	$8.98 \pm 0.24 \pm 0.89$	$14\pm 6\pm 1$	$7.5 \pm 0.8$
φη′	$3.08 \pm 0.34 \pm 0.36$	$4.1\!\pm\!0.3\!\pm\!0.8$	$5.46 \pm 0.31 \pm 0.56$		$4.0\pm0.7$
$\omega\pi^0$	$4.82 \pm 0.19 \pm 0.64$	$3.6\!\pm\!0.28\!\pm\!0.54$	$5.38 \pm 0.12 \pm 0.65$		$4.5 \pm 0.5$
ωη	$17.1 \pm 0.8 \pm 2.0$	$14.3 \pm 1.0 \pm 2.1$	$23.52 \pm 2.73$	$14.4 \pm 4.0 \pm 1.4$	$17.4 \pm 2.0$
ωη′	$1.66 \pm 0.17 \pm 0.19$	$1.8^{+1.0}_{-0.8}\pm0.3$	$2.26 \pm 0.43$		$1.82 \pm 0.21$
$K^{*-}K^{+} + c.c.$	$52.6 \pm 1.3 \pm 5.3$	$45.7 \pm 1.7 \pm 7.0$		$52 \pm 4 \pm 1$	$51.2 \pm 3.0$
$\mathrm{K}^{*0}\bar{\mathrm{K^{o}}} + \mathrm{c.c.}$	$43.3 \pm 1.2 \pm 4.5$	$39.6 \pm 1.5 \pm 6.0$		$48 \pm 5 \pm 1$	$43.9 \pm 3.1$

Table 2. Results of fit to MarkIII data.

parameter	$s_{\rm v} = 0, x = 1$	$s_{\rm v} = 0, x = 0.64$	$s_{\rm v} = 0, \ x = 0.82$	$s_v = s, x = 1$	$s_v = s, x = 0.64$	$s_v = s, x = 0.82$
g	$1.30 \pm 0.04$	$1.31 \pm 0.04$	$1.30\pm0.04$	$1.12 \pm 0.04$	$1.11 \pm 0.04$	$1.11 \pm 0.04$
s	$0.27 \pm 0.02$	$0.28 \pm 0.02$	$0.27\pm0.02$	$0.13 \pm 0.03$	$0.13 \pm 0.02$	$0.13 \pm 0.03$
e	$0.124 \pm 0.005$	$0.123 \pm 0.05$	$0.124 \pm 0.05$	$0.123 \pm 0.005$	$0.123 \pm 0.005$	$0.123 \pm 0.005$
$ heta_{ m e}$	$1.21 \pm 0.12$	$1.29 \pm 0.12$	$1.27\pm0.12$	$1.27\pm0.12$	$1.30 \pm 0.12$	$1.29 \pm 0.12$
r	$-0.37 \pm 0.01$	$-0.37 \pm 0.01$	$-0.37 \pm 0.01$	$-0.16 \pm 0.01$	$-0.15\pm0.01$	$-0.15 \pm 0.01$
$ heta_{ m P}$	$-13.95 \pm 2.39$	$-13.17 \pm 2.40$	$-13.49 \pm 2.38$	$-18.29 \pm 1.43$	$-18.59 \pm 1.40$	$-18.47 \pm 1.41$
$\chi^2/d.o.f$	9.0/4	7.9/4	8.3/4	8.1/4	9.0/4	8.6/4

If  $s_{\rm v}$  is replaced with s and x is fixed to 0.64, the fit gives  $\theta_{\rm P} = (-18.59 \pm 1.40)^{\circ}$  with  $\chi^2/\text{d.o.f} = 9.0/4$ , which is in good agreement with DM2's result  $\theta_{\rm P} =$  $(-19.1 \pm 1.4)^{\circ}$ . Meanwhile, we also checked the fits with x = 1 and x = 0.82, and the results are listed in Table 2. Compared with the results without considering the contribution of  $s_{\rm v}$ , the results change significantly, in particular for s, r and  $\theta_{\rm P}$ . This is reasonable because the two phenomenological models are slightly different. The fit to DM2 data is also performed to check the discrepancy discussed above. In the DM2's analysis, the common error of the branching fractions is removed, so the fitting error here is larger than those in Ref. [4]. Here it is clear that the reasonable results can also be obtained,  $\theta_{\rm P} = (-14.84 \pm 4.35)^{\circ}$ , with  $\chi^2/d.o.f = 1.9/4$  in the case of  $s_v = 0$  and x = 1.

Based on the above results, we can reach the conclusion that  $s_v$  plays an important role in the fit to

extract the mixing angle. The mixing angle in DM2 analysis is consistent with that in MarkIII because the latter is not from the best fit.

# 4.2 Analysis of $J/\psi \rightarrow VP$ from BES, BABAR and PDG2010

Until now, the pseudoscalar mixing is investigated with well established models and the data measured about 20 years ago. The new measurements reported by BES, BABAR and the new world average of 2010 are listed in Table. 1. Each branching fraction is regarded as one constraint in the fit to BES and BABAR data. The amplitude of  $J/\psi \rightarrow \rho\eta$  and  $J/\psi \rightarrow \rho\eta'$  is removed from the fit because no new measurements are available. The results of the fits with  $s_v = 0$  yields the mixing angle  $\theta_P \sim -17^\circ$ , which is still consistent with the above results within one standard deviation. This value is also in agreement with the previous work in Refs. [21, 22, 24]. The fit with  $s_v = s$  is performed, but the quality of fit is very poor.

A further check is performed using the world average of 2010 [1], and the results are shown in Table 3.

As we expected, the results are fine for the fit with  $s_v = 0$  and the mixing angle  $\theta_p$  favors ~ 14°. The goodness of the fit with  $s_v = s$  is still worse because of the weight of new measurements in the world average.

	Table 3.	Results of fit to	PDG2010 data.
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parameter	$s_{\rm v} = 0, x = 1$	$s_{\rm v} = 0, x = 0.64$	$s_{\rm v} = 0, \ x = 0.82$	$s_{\mathrm{v}}=s, x=1$	$s_{\rm v} = s, x = 0.64$	$s_{\rm v}{=}s,x{=}0.82$
g	$1.35\pm0.04$	$1.36\pm0.04$	$1.36 \pm 0.04$	$1.15 \pm 0.04$	$1.14 \pm 0.04$	$1.14 \pm 0.04$
s	$0.30\pm0.02$	$0.30\pm0.03$	$0.30 \pm 0.02$	$0.15 \pm 0.03$	$0.14 \pm 0.03$	$0.14 \pm 0.03$
e	$0.120 \pm 0.005$	$0.121\pm0.04$	$0.121 \pm 0.04$	$0.119 \pm 0.005$	$0.119 \pm 0.005$	$0.119 \pm 0.005$
$ heta_{ m e}$	$1.31\pm0.12$	$1.36\pm0.12$	$1.34 \pm 0.12$	$1.35 \pm 0.12$	$1.38 \pm 0.12$	$1.36\pm0.12$
r	$-0.37 \pm 0.01$	$-0.37 \pm 0.01$	$-0.37 \pm 0.01$	$-0.16 \pm 0.01$	$-0.15 \pm 0.01$	$-0.15 \pm 0.01$
$ heta_{ m P}$	$-14.27 \pm 2.44$	$-13.90 \pm 2.35$	$-14.04 \pm 2.37$	$-17.66 \pm 1.81$	$-17.96 \pm 1.77$	$-17.84 \pm 1.78$
$\chi^2/{ m d.o.f}$	3.1/4	3.5/4	3.3/4	16.5/4	18.1/4	17.4/4

### 4.3 Analysis of $\psi(2S) \rightarrow VP$

We now turn to examine the full set of  $\psi(2S) \rightarrow \psi(2S)$ VP to get the pseudoscalar mixing using the same phenomenological model. At present, the measurements of  $\psi(2S) \rightarrow VP$  mainly come from BES and CLEO's reports, which are shown in Table 4. We have omitted the known upper limit for the  $\psi(2S) \rightarrow \phi \pi$ and  $\psi(2S) \rightarrow \omega \eta$  branching fractions in our analysis because they are the upper limits at the 90% confidence level rather than branching fractions. As previously stated, we just consider the mixing angle between  $\eta$  and  $\eta'$ , and assume that the mixing of  $\omega$  and  $\phi$  is ideal. The results listed in Table 5 indicate that both of the above two slightly different models are reasonable and the  $\theta_{\rm P}$  favors  $\sim -12^{\circ}$  with large uncertainty. Without considering the branching fraction of  $\psi(2S) \to \rho \pi$ , the fit was also performed in Ref. [24]. The mixing angle is calculated to be  $-10^{+7}_{-8}$ , which is in agreement with our result. But the branching fraction of  $\psi(2S) \rightarrow K^{*+}K^{-}$ ,  $(8.5 \pm 4.0) \times 10^{-5}$ , applied in the analysis is not correct. Therefore the values of the parameters listed in Table 5 are inconsistent with those in Ref. [24].

Table 4. Branching fractions of  $\psi(2S) \rightarrow VP(\times 10^{-5})$ .

decay modes	BES	CLEO	PDG2010
ρπ	$5.1 \pm 0.7 \pm 1.1$	$2.4 \pm 0.8 \pm 0.2$	$3.2\pm1.2$
ρη	$1.78^{+0.67}_{-0.62}\pm0.17$	$3.0^{+1.1}_{-0.9}\pm0.2$	$2.2\pm0.6$
ρη′	$1.87^{+1.64}_{-1.11}\pm0.33$		$1.9^{+1.7}_{-1.2}$
$\phi\pi^0$	< 0.4	< 0.7	< 0.4
φη	$3.3 \!\pm\! 1.1 \!\pm\! 0.5$	$2.0^{+1.5}_{-1.1}\pm0.4$	$2.8^{+1.0}_{-0.8}$
φη′	$3.1 \pm 1.4 \pm 0.7$		$3.1\pm1.6$
$\omega\pi^0$	$1.87^{+0.68}_{-0.62}\pm0.28$	$2.5^{+1.2}_{-1.0}\pm0.2$	$2.1\pm0.6$
ωη	< 3.1	< 1.1	< 1.1
ωη′	$3.2^{+2.4}_{-2.0}\pm0.7$		$3.2^{+2.5}_{-2.1}$
$\mathrm{K}^{*-}\mathrm{K}^{+}+\mathrm{c.c}$	$2.9^{+1.3}_{-1.7} \pm 0.4$	$1.3^{+1.0}_{-0.7}\pm0.3$	$1.7^{+0.8}_{-0.7}$
$\mathrm{K}^{*0}\bar{\mathrm{K}^{\mathrm{o}}}\!+\!\mathrm{c.c.}$	$13.3^{+2.4}_{-2.8}\pm\!1.7$	$9.2^{+2.7}_{-2.2}\pm0.9$	$10.9\pm2.0$

Table 5. Results of fit to PDG2010 data of  $\psi(2S) \rightarrow VP$ .

parameter	$s_{\rm v} = 0, x = 1$	$s_{\rm v} = 0, \ x = 0.64$	$s_{\rm v} = 0, \ x = 0.82$	$s_v = s, x = 1$	$s_v = s, x = 0.64$	$s_v = s, x = 0.82$
g	$0.64 \pm 0.11$	$0.65 \pm 0.10$	$0.65 \pm 0.10$	$0.64 \pm 0.11$	$0.65 \pm 0.04$	$0.65\pm0.04$
8	$0.003 \pm 0.18$	$-0.01 \pm 0.19$	$-0.05 \pm 0.18$	$0.02 \pm 0.18$	$-0.10 \pm 0.19$	$-0.10 \pm 0.20$
e	$0.23\pm0.02$	$0.23 \pm 0.02$	$0.23 \pm 0.02$	$0.23 \pm 0.02$	$0.23 \pm 0.02$	$0.23\pm0.02$
$ heta_{ m e}$	$2.73 \pm 0.62$	$2.81\pm0.60$	$2.79 \pm 0.63$	$2.75 \pm 0.64$	$2.83 \pm 0.64$	$2.83 \pm 0.62$
r	$0.18 \pm 0.28$	$0.17 \pm 0.27$	$0.16 \pm 0.28$	$0.14 \pm 0.28$	$0.14 \pm 0.29$	$0.14 \pm 0.31$
$ heta_{ m P}$	$-12.07 \pm 10.42$	$-11.94 \pm 10.48$	$-11.99 \pm 10.46$	$-11.80 \pm 10.63$	$-12.19 \pm 11.59$	$-12.19 \pm 12.18$
$\chi^2/{\rm d.o.f}$	4.4/3	4.5/3	4.4/3	4.4/3	4.5/3	4.5/3

## 4.4 $\omega$ - $\phi$ mixing

In the above analysis, the  $\omega$ - $\phi$  mixing is ignored to simplify the model. This fit in the case of  $s_v = 0$ and x = 0.82 is an attempt to account for the  $\omega$ - $\phi$ mixing. If the  $\omega$ - $\phi$  mixing angle is left as a free parameter, the fit to the world average of 2010 leads to a minimum  $\chi^2 = 3.3$  for three degrees of freedom,

$$\begin{split} g &= 1.36 \pm 0.04, \; s = 0.30 \pm 0.03, \; e = 0.121 \pm 0.005, \\ \theta_{\rm e} &= 1.33 \pm 0.12, \; \theta_{\rm P} = (-14.06 \pm 2.37)^{\circ}, \\ r &= -0.37 \pm 0.02, \; \theta_{\rm V} = (0.09 \pm 4.13)^{\circ}. \end{split}$$

If we assumed  $s_v = s$ , then the fit with  $\chi^2/d.o.f$  of 17.4/3 gives

$$\begin{split} g = 1.14 \pm 0.05, \ s = 0.14 \pm 0.04, \ e = 0.119 \pm 0.005, \\ \theta_{\rm e} = 1.36 \pm 0.12, \ \theta_{\rm P} = (-17.78 \pm 2.70)^{\circ}, \\ r = -0.15 \pm 0.01, \ \theta_{\rm V} = (0.11 \pm 3.78)^{\circ}. \end{split}$$

 $\theta_{\rm V}$  is very close to zero and the uncertainty is very large compared with other parameters. This means that there is not a significant constraint on it. Among  $J/\psi \rightarrow VP$  decays, the amplitude of  $J/\psi \rightarrow \phi \pi^0$  is directly related to the  $\omega$ - $\phi$  mixing, but it is still not observed yet. No observation of  $J/\psi \rightarrow \phi \pi^0$  shows that the contribution of  $\omega$ - $\phi$  is small. On the other hand, the values of other parameters are almost the same as those listed in Table 3 without considering  $\omega$ - $\phi$  mixing. Therefore, it is reasonable that  $\omega$ - $\phi$  mixing is assumed to be ideal and could be ignored in the above analysis. A further check is done by fixing the  $\omega$ - $\phi$ mixing angle to 3.2° obtained from the V  $\rightarrow \gamma P$  and  $P \rightarrow \gamma V$  process. The fit with  $\chi^2/d.o.f=3.3/4$  gives

 $g = 1.36 \pm 0.04, \ s = 0.30 \pm 0.02, \ e = 0.121 \pm 0.004,$ 

 $\theta_{\rm e} = 1.33 \pm 0.13, \ \theta_{\rm P} = (-14.05 \pm 2.36)^{\circ}, \ r = -0.37 \pm 0.01.$ 

These values are also in good agreement with those in the hypothesis of the ideal  $\omega$ - $\phi$  mixing.

#### 4.5 Gluon content in $\eta$ and $\eta'$

At present,  $\eta$  is believed to be well-understood as an SU(3) flavor octet with a small quarkonium singlet admixture, and not much room for a significant gluonium admixture [21, 24]. Therefore the analyses [24] are usually performed to determine the gluonic content in  $\eta'$  with the assumption of no gluonic content in  $\eta$ . After taking into account the gluonic content in  $\eta$  and  $\eta'$  simultaneously, we present the fit with the above two slightly different models. In the first case,  $s_v$  is assumed to be zero and the fit to the world average in 2010 yields

$$g = 1.32 \pm 0.06, \ s = 0.27 \pm 0.04, \ e = 0.126 \pm 0.007,$$

$$\theta_{\rm e} = 1.34 \pm 0.12, \ \theta_{\rm P} = (-10.21 \pm 4.48)^{\circ}, \ r = -0.45 \pm 0.08,$$

$$\phi_{\rm g1} = 0.04 \pm 0.05, \ \phi_{\rm g2} = 0.53 \pm 0.24, \ r' = -0.77 \pm 0.46,$$

with  $\chi^2$ /d.o.f = 1.56/1.

The second fit is performed under the hypothesis of  $s_v = s$ , and the results with  $\chi^2/d.o.f = 3.5/1$  are

$$g = 1.28 \pm 0.06, s = 0.24 \pm 0.03, e = 0.128 \pm 0.007,$$

$$\theta_{\rm e} = 1.35 \pm 0.11, \ \theta_{\rm P} = (-9.17 \pm 4.67)^{\circ}, \ r = -0.67 \pm 0.08,$$

$$\phi_{\rm g1} = 0.11 \pm 0.04, \ \phi_{\rm g2} = 0.50 \pm 0.22, \ r' = -0.85 \pm 0.56.$$

The goodness of the second fit is still worse than the first fit. Based on the results of the first fit, the magnitudes of gluon components in  $\eta$  and  $\eta'$  are calculated to be  $Z_{\eta}^2 = 0.002 \pm 0.002$  and  $Z_{\eta'}^2 = 0.30 \pm$ 0.24, respectively. The small gluonic contribution in  $\eta$  shows that there is not much room for gluonium admixture, which is consistent with the results presented in Ref. [22]. It seems that 30% of the  $\eta'$  component could be attributed to gluonium, but further investigation with more precise data needs to be carried out due to the large uncertainty.

#### 5 Summary and outlook

A wide set of data on  $J/\psi \rightarrow VP$  and  $\psi(2S) \rightarrow VP$ decays are analyzed in terms of a rather general phenomenological model in an attempt to determine the magnitudes of components in  $\eta$  and  $\eta'$ . The data include the branching fractions of  $J/\psi \rightarrow VP$ , which were measured nearly 20 years ago, and the recent measurements by BES and BABAR. The measurements of MarkIII and DM2 are reanalyzed. We found that the results obtained from the two different phenomenological models are inconsistent. The fit to the new measurements by BES and BABAR indicates that the assumption of  $s = s_v$  is not a good approximation in accordance with the goodness of fit. And the mixing angle is determined to be  $-14^\circ$ , which is in good agreement with previous work.

The content of  $\eta$  and  $\eta'$  is also examined in this paper. After considering the gluonium content in the model, the fit to data of the world average in 2010 yields  $Z_{\eta}^2 = 0.002 \pm 0.002$  and  $Z_{\eta'}^2 = 0.30 \pm 0.24$ , which are the contribution of gluonium content in  $\eta$  and  $\eta'$ , respectively. Although the possibility of gluonic content cannot be excluded, it is a reasonable description for  $\eta$  in terms of pure  $q\bar{q}$  meson, and not much room for a significant gluonium admixture. The magnitude of gluonium contamination in  $\eta'$  shows that  $\eta'$  has room for the gluonium admixture, but the large uncertainty prevents us from definitely saying whether gluonium content is present or not.

As previously stated, the latest results from BES and BABAR are not consistent with those previous works. The branching fractions shown in Table 1 still have a large error, including statistical and systematic errors. The main reason is that  $J/\psi$  and  $\psi(2S)$  sam-

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ples are not enough and the performance of the detector needs to be improved. A modern detector, BESIII [27], has been built to meet the above requirements. Up to now, about  $2.3 \times 10^8 \text{ J/\psi}$  and  $1.2 \times 10^8 \text{ }\psi(2S)$  events have been accumulated at BESIII, which provide a unique chance to study the  $\eta$ - $\eta'$  mixing and further improve these measurements with much higher sensitivities.

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