A new mechanism for the electromagnetic transition $\gamma^* N \rightarrow N^* (1535)^*$

AN Chun-Sheng(安春生)^{1,2;1)} ZOU Bing-Song(邹冰松)^{1,3;2)}

¹ Institute of High Energy Physics, CAS, P.O.Box 918, Beijing 100049, China

 $^{\ 2}$ Graduate University of Chinese Academy of Sciences, Beijing 100049, China

³ Theoretical Physics Center for Science Facilities, CAS, P.O.Box 918, Beijing 100049, China

Abstract The helicity amplitude $A_{1/2}^{p}$ for the electromagnetic transition $\gamma^* N \to N^*(1535)$ is investigated. It is found that a new mechanism $\gamma^* \to q\bar{q}$ plays an important role in order to improve the description of this transition. On one hand, the $A_{1/2}^{p}$ is decreased to fall in the data range at the photon point $Q^2 = 0$, while on the other hand, the new mechanism makes the function $A_{1/2}^{p}(Q^2)$ to decrease more slowly vs increasing Q^2 , as required by the data.

Key words N*(1535), electromagnetic transition, helicity amplitudes

PACS 14.20.Gk, 13.40.Gp, 12.39.Jh

1 Introduction

Among the low-lying nucleon excitations, the S_{11} state N^{*}(1535) plays a special role due to the fact that it has a sizeable N η decay branch [1], even though its energy is very close to the N η threshold. To understand its nature, the electromagnetic transition $\gamma^* N \rightarrow N^*(1535)$ is of great interests. While data is getting more and more precise [2–4], more and more phenomenological models are employed to describe this transition. For instance, the naive nonrelativistic constituent quark model (NQM) [5, 6], the relativistic constituent quark model (RQM) [6, 7], light front model [8] and so on.

It is well known that there are two limitations once the NQM is employed. First, at the photon point $Q^2 = 0$, the calculated value for the helicity amplitude $A_{1/2}^{\rm p}$ is much larger than the experimental data [5]. Secondly, the function $A_{1/2}^{\rm p}(Q^2)$ decreases too fast when the transferred momentum Q^2 is increasing [5]. Consequently, the theoretical results obtained by using the NQM cannot fit the data at both the lower and higher Q^2 sides. If one takes the relativistic corrections into account, the result may be a bit better, but it is still not very good. This indicates that there may be some new mechanisms in addition to these quark models to describe this electromagnetic transition.

In this work we want to show that a new mechanism $\gamma^* \to q\bar{q}$ may play a very important role in the electromagnetic transition $\gamma^* N \to N^*(1535)$. This has been ignored by both NQM and RQM, which limit the effective degrees of freedom to be three constituent quarks. To show what role this new mechanism plays we first give the result obtained by using the NQM.

2 Formalism

For pointlike quarks the electromagnetic transition operator for elastic transitions between states with $n_{\rm q}$ quarks is given by

$$\hat{T} = -\sum_{i=1}^{n_{\rm q}} \frac{e_i}{2m_i} \left[\sqrt{2} \hat{\sigma}_{i+} k_{\gamma} + (p'_{i+} + p_{i+}) \right].$$
(1)

Here e_i and m_i denote the electric charge and constituent mass, respectively, of the quark absorbing the photon. p_{i+} and p'_{i+} are defined as the initial and

Received 27 August 2009

^{*} Supported by National Natural Science Foundation of China (10435080, 10521003) and Chinese Academy of Sciences (KJCX3-SYW-N2)

¹⁾ E-mail: ancs@ihep.ac.cn

²⁾ E-mail: zoubs@ihep.ac.cn

 $[\]odot$ 2009 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

final momenta of the quark absorbing the photon:

$$p_{i+}^{(\prime)} \equiv \frac{1}{\sqrt{2}} (p_{ix}^{(\prime)} + i p_{iy}^{(\prime)}), \qquad (2)$$

and $\hat{\sigma}_{i+}$ is defined as

$$\hat{\sigma}_{i+} \equiv \frac{1}{2} (\hat{\sigma}_{ix} + \mathrm{i}\hat{\sigma}_{iy}), \qquad (3)$$

which raises the spin of the quark. Here we consider a right-handed virtual-photon in the operator \hat{T} , with the momentum of the photon set to : $\vec{k} = (0, 0, k_{\gamma})$ in the center of mass frame of the final resonance N*(1535). It is related to the initial and final momentum of the hit quark as $k_{\gamma} = \vec{p}_i - \vec{p}_i$, and the magnitude of the four-momentum transfer $Q = \sqrt{-k^2}$ by:

$$k_{\gamma}^2 = Q^2 + \frac{(M^{*2} - M_{\rm N}^2 - Q^2)^2}{4M^{*2}}, \qquad (4)$$

where M^* and M_N denote the mass of the N^{*}(1535) resonance and proton, respectively.

With the transition operator given by Eq. (1), the helicity amplitude of the electromagnetic transition $\gamma^* p \rightarrow N^*(1535)$, may be expressed as

$$A_{1/2}^{\rm p} = \frac{1}{\sqrt{2K_{\gamma}}} \left\langle N^*(1535), \frac{1}{2}, \frac{1}{2} |\hat{T}| p, \frac{1}{2}, -\frac{1}{2} \right\rangle . (5)$$

Here K_{γ} is the real-photon three-momentum in the center of mass frame of N^{*}(1535). Once the wave functions of the proton and N^{*}(1535) are given, we can get the helicity amplitude $A_{1/2}^{\rm p}$ directly. Here we employ the wave functions given by Glozman and Riska [9]. After some simple calculations we get

$$A_{1/2}^{\rm p} = \frac{1}{\sqrt{2K_{\gamma}}} \frac{e}{2m} \left(\frac{k_{\gamma}^2}{3\omega_3} + \frac{2\omega_3}{3} \right) \exp\left\{ -\frac{k_{\gamma}^2}{6\omega_3^2} \right\}.(6)$$

The model parameters are the constituent masses of the light quarks m, the oscillator parameter ω_3 . Here we set the constituent quark mass to be m =340 MeV. The oscillator parameter ω_3 may be determined by the nucleon radius as $\omega_3 = \frac{1}{\sqrt{\langle r^2 \rangle}}$, which yields the value 246 MeV, while the empirical value for this parameter falls into the range 110–410 MeV [10, 11]. We give the numerical results as functions of Q^2 by setting $\omega_3 = 340$ MeV, $\omega_3 = 246$ MeV and $\omega_3 = 200$ MeV, respectively, which are shown in Fig. 1 compared to the experimental data extracted from Refs. [1–4]. As shown, all of the three curves cannot fit the data very well.



Fig. 1. The helicity amplitude $A_{1/2}^{\rm p}$ for $\gamma^* {\rm p} \rightarrow N^*(1535)$ in the qqq model. Here the solid line is obtained by taking $\omega_3 = 340$ MeV, and the dashed and dotted lines are obtained by taking $\omega_3 = 246$ MeV and $\omega_3 = 200$ MeV, respectively. The data point at $Q^2 = 0$ (square) is from Ref. [1], the other points are taken from Refs. [2] (triangles), [3] (open circles) and [4] (filled circles).

Then we should consider the effect of the new mechanism $\gamma^* \rightarrow q\bar{q}$. This process is shown in Fig. 2. Once this mechanism is considered, the key issue is how to deal with the final $q\bar{q}$ state. Here we assume the state of the N^{*}(1535) resonance to be a composition of the conventional qqq state and a higher Fock state. The wave function of the higher Fock state may be expressed as

$$|\mathbf{N}^{*}(1535), s_{z}\rangle_{5\mathbf{q}} = \sum_{abc} C^{[1^{4}]}_{[31]_{a}[211]_{a}} C^{[31]_{a}}_{[211]_{b}[22]_{c}} [4]_{X} \times [211]_{F}(b) [22]_{S}(c) [211]_{C}(a) \bar{\chi}_{s_{z}} \varphi(\{\vec{\kappa}_{i}\}).$$
(7)

Here $C_{[31]_a[211]_a}^{[11]_a}$ and $C_{[211]_b[22]_c}^{[31]_a}$ are the SU(3) C-G coefficients. $[4]_X$, $[211]_F(b)$, $[22]_S(c)$ and $[211]_C(a)$ denote the orbital, flavor, spin and color states, respectively. The explicit forms of the flavor-spin wave functions are given in appendix. And $\bar{\chi}_{s_z}$ is the spin state of the final anti-quark, $\varphi(\{\vec{\kappa}_i\})$ the orbital wave functions of the quarks and anti-quark. The flavor configuration $[211]_F$ implies that the final q \bar{q} can only be s \bar{s} . This indicates that there are appreciable s \bar{s} components in the higher Fock states of N*(1535), which is consistent with the strong couplings of N*(1535)KA [12] and N*(1535)N ϕ [13].





Considering the new mechanism, the operator for the $\gamma^* \to q\bar{q}$ process may be expressed as

$$\hat{T}_{\text{new}} = -\sum_{i=1}^{4} \sqrt{2} e_i \hat{\sigma}_{i+} \,.$$
 (8)

An explicit calculation leads to the result:

$$A_{1/2}^{\rm p(new)} = -\delta \frac{1}{\sqrt{2K_{\gamma}}} \frac{\sqrt{2}}{6} eC_{35} \exp\left\{-\frac{3k_{\gamma}^2}{20\omega_5^2}\right\}.$$
 (9)

Here the factor C_{35} denotes the orbital overlap factor:

$$\langle \varphi_{00}(\vec{\kappa}_1)\varphi_{00}(\vec{\kappa}_2)|\varphi_{00}(\vec{\kappa}_1)\varphi_{00}(\vec{\kappa}_2)\rangle = \left(\frac{2\omega_3\omega_5}{\omega_3^2 + \omega_5^2}\right)^3.$$
(10)

The factor δ is the phase factor between the conventional qqq state and the higher Fock state in the N^{*}(1535) resonance. Besides the constituent quark masses of u,d,s quarks, there is another new parameter ω_5 , which is the oscillator parameter for the higher Fock state. As it is shown in Refs. [14, 15], ω_5 favors a larger value than ω_3 .

3 Results of the new mechanism

The numerical result is shown in Fig. 3 with several different values for the oscillator parameters. Note, if the higher Fock state is considered, the constituent quark masses should change, here we take the value $m_{\rm u} = m_{\rm d} = m = 290$ MeV for the light quarks, and $m_{\rm s} = 430$ MeV for the strange quark. As we can see, the result is much better than that obtained from the conventional NQM. The main reason may be that the new mechanism $\gamma^* \to q\bar{q}$ gives a small but very significant contributions to the electromagnetic transition $\gamma^* p \rightarrow N^*(1535)$. The absolute value of this contribution is shown by the dash-dotted line in Fig. 3. Here we have set the phase factor to be $\delta = +1$. In principle, this factor can also be -1, but if we take this value, the result will be less good. With the probability of the higher Fock state being about 80%, the obtained curve is also very good, but this probability may be too large [16].

As we can see in Fig. 3, the result obtained from the process $\gamma^* \to q\bar{q}$ is small and negative, so the calculated value for $A_{1/2}^p$ at the photon point should be decreased naturally when this process is considered. On the other hand, the curve obtained from this process decreases more slowly than that from the NQM. Consequently, with a certain probability of this contributions, the total result may also be improved. The main reason for this may be that the oscillator parameter ω_5 is larger ω_3 and principally ω_3 cannot be adjusted to a larger value becaouse it is restricted to a certain range. On the other hand, the result is much more sensitive to ω_3 than it is to ω_5 .





As a test of this model we also calculate the helicity amplitude $A_{1/2}^n$. As we know, the ratio of the helicity amplitudes at the photon point for proton and neutron is not a trivial issue. Because of isospin symmetry, the wave functions for the neutron and n^{*}(1535) can be obtained from the wave functions we have given for the proton and p^{*}(1535). Consequently, we can calculate the matrix elements directly. After some calculations we get the following results:

$$A_{1/2}^{n(3q)} = -\frac{1}{\sqrt{2K_{\gamma}}} \frac{e}{2m} \left(\frac{k_{2\gamma}^2}{3\omega_3} + \frac{2\omega_3}{9} \right) \exp\left\{ -\frac{k_{\gamma}^2}{6\omega_3^2} \right\},$$

$$A_{1/2}^{n(new)} = -\delta \frac{1}{\sqrt{2K_{\gamma}}} \frac{\sqrt{2}}{6} eC_{35} \exp\left\{ -\frac{3k_{\gamma}^2}{20\omega_5^2} \right\}.$$
(11)

As we can see from Eq. (11) the contribution of the new mechanism to the helicity amplitude $A_{1/2}^n$ is same as that to $A_{1/2}^p$. If we only consider the contributions obtained by the conventional NQM with the oscillator parameter $\omega_3 = 340$ MeV, we get $A_{1/2}^n = -0.123/\sqrt{\text{GeV}}$, which is much larger than the data, $A_{1/2}^n = -0.046 \pm 0.027/\sqrt{\text{GeV}}$ [1]. But if the contributions of the new mechanism are taken into account, with the same parameter employed for the best fit of $A_{1/2}^p$, we get a helicity amplitude of $A_{1/2}^{\rm n} = -0.074/\sqrt{\text{GeV}}$, which falls well into the range of the data. The same is true for the ratio $A_{1/2}^{\rm n}/A_{1/2}^{\rm p}$, which is then -0.82, and also falls well into the range of the data 0.84 ± 0.15 .

In summary, the new mechanism $\gamma^* \to q\bar{q}$ plays a very important role in the electromagnetic transition $\gamma^* N \to N^*(1535)$. It can decreases the calculated value for $A_{1/2}^p$ to fall well in the data range at the photon point. Considering this effect, we can give a good description for $A_{1/2}^p(Q^2)$ and also of the ratio $A_{1/2}^n/A_{1/2}^p$ at the photon point.

Appendix A

The explicit form of the flavor-spin configuration $[31]_{FS}[211]_F[22]_S$

The explicit decomposition of the flavor-spin configuration $[31]_{FS}[211]_F[22]_S$ may be expressed as [17]

$$|[31]_{FS}\rangle_1 = \frac{1}{2} \{ \sqrt{2} | [211] \rangle_{F1} | [22] \rangle_{S1} - | [211] \rangle_{F2} | [22] \rangle_{S1} + | [211] \rangle_{F3} | [22] \rangle_{S2} \},$$
(A1)

$$|[31]_{FS}\rangle_2 = \frac{1}{2} \{\sqrt{2} | [211] \rangle_{F1} | [22] \rangle_{S2} + | [211] \rangle_{F2} | [22] \rangle_{S2} + | [211] \rangle_{F3} | [22] \rangle_{S1} \},$$
(A2)

$$|[31]_{FS}\rangle_3 = \frac{1}{\sqrt{2}} \{ -|[211]\rangle_{F2} |[22]\rangle_{S2} + |[211]\rangle_{F3} |[22]\rangle_{S1} \},$$
(A3)

and the explicit forms of the flavor symmetry $[211]_F$ and spin symmetry $[22]_S$ are

$$[211]\rangle_{F1} = \frac{1}{4} \{2|\mathrm{uuds}\rangle - 2|\mathrm{uusd}\rangle - |\mathrm{duus}\rangle - |\mathrm{udus}\rangle - |\mathrm{sudu}\rangle - |\mathrm{usdu}\rangle + |\mathrm{suud}\rangle + |\mathrm{udsu}\rangle + |\mathrm{udsu}\rangle + |\mathrm{udsu}\rangle \},$$
(A4)

$$|[211]\rangle_{F2} = \frac{1}{\sqrt{48}} \{3|udus\rangle - 3|duus\rangle + 3|suud\rangle - 3|usud\rangle + 2|dsuu\rangle - 2|sduu\rangle - |sudu\rangle + |usdu\rangle + |dusu\rangle - |udsu\rangle\},$$
(A5)

$$|[211]\rangle_{F3} = \frac{1}{\sqrt{6}} \{|\operatorname{sudu}\rangle + |\operatorname{udsu}\rangle + |\operatorname{dsuu}\rangle - |\operatorname{usdu}\rangle - |\operatorname{dusu}\rangle - |\operatorname{sduu}\rangle\},$$
(A6)

$$|[22]\rangle_{S1} = \frac{1}{\sqrt{12}} \{2|\uparrow\uparrow\downarrow\downarrow\rangle + 2|\downarrow\downarrow\uparrow\uparrow\rangle - |\downarrow\uparrow\downarrow\downarrow\rangle - |\uparrow\downarrow\uparrow\downarrow\rangle - |\downarrow\uparrow\downarrow\downarrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle\},$$
(A7)

$$|[22]\rangle_{S2} = \frac{1}{2} \{|\uparrow\downarrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\downarrow\downarrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle\}.$$
(A8)

References

- 1 Particle Group Data. J. Phys. G, 2006, 33: 1
- 2 Aznauryan I G et al. Phys. Rev. C, 2005, 71: 015201
- 3 The CLAS Collaboration. Phys. Rev. Lett., 2001, 86: 1702
- 4 The CLAS Collaboration. Phys. Rev. C, 2007, **76**: 015204
- Koniuk R, Isgur N. Phys. Rev. D, 1982, 21: 1868; Erratumibid. D, 1981, 23: 818; Close F E, LI Z. Phys. Rev. D, 1990, 42: 2194
- 6 Capstick S, Roberts W. Pro. Par. Nucl. Phys., 2000, **45**: S241
- 7 Capstick S. Phys. Rev. D, 1992, 46: 2864
- 8 Konen W, Weber H J. Phys. Rev. D, 1992, 41: 2201
- 9 Glozman L Ya, Riska D O. Phys. Rept., 1996, 268: 263

- 10 LI Q B, Riska D O. Phys. Rev. C, 2006, 74: 015202
- 11 Koniuk R, Isgur N. Phys. Rev. D, 1980, 21: 1868; Erratumibid. D, 1981, 23: 818
- 12 LIU B C, ZOU B S. Phys. Rev. Lett., 2006, 96: 042002
- 13 XIE J J, ZOU B S, CHIANG H C. Phys. Rev. C, 2008, 77: 015206
- 14 AN C S, Riska D O, ZOU B S. Phys. Rev. C, 2006, 73: 035207
- AN C S, LI Q B, Riska D O, ZOU B S. Phys. Rev. C, 2006, 74: 055205; Erratum-ibid. C, 2007, 79: 069901
- 16 AN C S, Riska D O. Eur. Phys. J. A, 2008, 37: 263
- 17 CHEN J Q. Group Representation Theory for Physicists. Singapore: World Scientific, 1989