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Supernova neutrino detection on earth*

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Abstract In this paper, we first discuss the detection of supernova neutrinos on earth. Then we propose a possible method to acquire information about θ_{13} smaller than 1.5° by detecting the ratio of the event numbers of different flavor supernova neutrinos. Such an sensitivity cannot yet be achieved by the Daya Bay reactor neutrino experiment.

Key words supernova, neutrino, collective effects, MSW effects, earth matter effects, Daya Bay

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1 Introduction

A supernova (SN) explosion is one of the most spectacular cosmic events and a source of new physics ideas [1, 2]. Observable effects of SN neutrinos in underground detectors have been a subject of intense investigation in astroparticle physics, both on general grounds and in relation to the SN event like 1987A. In particular, flavor oscillations in the SN may shed light on the problem of neutrino masses and mixing by means of the associated matter effects. Several neutrino laboratories, including the Daya Bay reactor neutrino underground laboratory [3] which is under construction, can be used to detect possible neutrino events from SN explosion and serve as a SN Earth Warning System. Hence theoretical predictions for the detection of SN neutrinos in the Daya Bay and other neutrino experiments are very desirable.

We discuss here the realistic scenario that neutrinos from a type II SN explosion would be detected at Daya Bay. There are three effects that need to be considered, namely the collective effects arising from neutrino-neutrino interactions [4–12], the well-known Mikheyev-Smirnov-Wolfenstein (MSW) effects [13–16], and earth matter effects [17–19]. Using the Landau-Zener formula [20, 21], one can calculate the crossing probability P_{H}, which is the neutrino jump probability from mass eigenstate ν_1 to ν_3 at the high resonance region inside the SN [22–24]. Using a relation between P_{H} and the mixing angle θ_{13}, we can predict the SN neutrino event numbers N as a function of θ_{13}. Therefore, we can propose a possible method to acquire information about small θ_{13} through the detection of SN neutrinos [19].

2 Detection of SN neutrinos on earth

SN are extremely powerful explosions in the universe which terminate the life of some stars [1, 25–27]. They mark the catastrophic end of stars more massive than 8 solar masses, leaving behind compact remnants such as neutron stars or black holes which may be observed. For historical reasons, SN are divided into two wide categories (type I and type II) characterized by the absence or presence of hydrogen lines. However, the most important physical characteristics is the mechanism that produces the SN, which distinguishes SN of type Ia from SN of type Ib, Ic, and II. This difference becomes noticeable in the light spectrum some months after maximum luminosity, when the ejecta become optically thin and the innermost regions become visible: the spectrum

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of SN Ia is dominated by Fe emission lines, while SN Ib, Ic, and II show O and C emission lines. From the point of view of neutrino physics, type Ib, Ic, and II SN are much more important than type Ia SN, simply because they produce a huge flux of neutrinos and antineutrinos of all flavors.

The type II SN is thought to be generated by the core collapse of red (or blue as SN1987A) giant stars with a mass between about 8–9 and 40–60 solar masses. The total energy release (about 3×10^{53} erg) is approximately the gravitational binding energy of the core. It generates intensive neutrinos which take away about 99% of this total energy. The explosion itself consumes about 1% of this total energy. The vast amount of neutrinos are produced in two bursts. In the first burst which lasts for only a few milliseconds, electron neutrinos are generated via the electron capture by nuclei $\mathrm{e}^-+\mathrm{N(Z,A)}\rightarrow\mathrm{N(Z-1,A)}+\nu_{\mathrm{e}}$ and the inverse beta-decay $\mathrm{e}^-+\mathrm{p}\rightarrow\mathrm{n}+\nu_{\mathrm{e}}$. In the second burst which lasts longer, neutrinos of all flavors (ν_α and $\bar{\nu}_\alpha$ with α being e, μ, τ) are produced through the electron-positron pair annihilation $\mathrm{e}^-+\mathrm{e}^+\rightarrow\nu_\alpha+\bar{\nu}_\alpha$, electron-nucleon bremsstrahlung $\mathrm{e}^\pm+\mathrm{N}\rightarrow\mathrm{e}^\pm+\mathrm{N}+\nu_\alpha+\bar{\nu}_\alpha$, nucleon-nucleon bremsstrahlung $\mathrm{N}+\mathrm{N}\rightarrow\mathrm{N}+\mathrm{N}+\nu_\alpha+\bar{\nu}_\alpha$, plasmon decay $\gamma\rightarrow\nu_\alpha+\bar{\nu}_\alpha$, and photoannihilation $\gamma+\mathrm{e}^\pm\rightarrow\mathrm{e}^\pm+\nu_\alpha+\bar{\nu}_\alpha$ [25–27]. When SN neutrinos of a definite flavor are produced they are approximately in an effective mass eigenstate due to the extremely high matter density environment. While they propagate outward to the surface of the SN they could experience collective effects [4–12] and MSW effects [13–16]. After travelling the cosmic distance to reach earth, the arriving neutrinos are mass eigenstates, which then oscillate in flavors while going through earth matter. Therefore, we also have to consider earth matter effects [17–19] when we compute the event numbers of the various flavors of neutrinos.

Let $P_{\nu\nu}$ represent the collective effects of neutrino-neutrino interactions which is a stepwise flavor conversion probability of neutrinos at a critical energy E_{C}. In order to obtain a simple expression for $P_{\nu\nu}$ for the neutrino and $\bar{P}_{\nu\nu}$ for the antineutrino, we take a constant matter density and box-spectra for both the neutrino and antineutrino [8–10]. An analysis of the collective effects in the case of three flavors has been made in Ref. [11], and it allows us to characterize the collective oscillation effects and to write down the flavor spectra of the neutrino and antineutrino arriving at earth. Following Ref. [11], we have $P_{\nu\nu}=\bar{P}_{\nu\nu}=1$ for the normal hierarchy; and $\bar{P}_{\nu\nu}=1$, while

$$P_{\nu\nu}=\begin{cases}1 & (E<E_{\mathrm{C}}),\\ 0 & (E>E_{\mathrm{C}}),\end{cases}\tag{1}$$

for the inverted hierarchy, where $E_{\mathrm{C}}=7$ MeV [11, 12].

There are two MSW resonance regions: the high resonance region and the low resonance region. Let us denote the probability that the neutrinos jump from one mass eigenstate to another at the high (low) resonance layer by P_{H} (P_{L}). Using the Landau-Zener formula [20, 21], one obtains

$$P_{\mathrm{H}}=\frac{\exp\left(-\dfrac{\pi}{2}\gamma F\right)-\exp\left[-\dfrac{\pi}{2}\gamma\left(\dfrac{F}{\sin^2\theta_{13}}\right)\right]}{1-\exp\left[-\dfrac{\pi}{2}\gamma\left(\dfrac{F}{\sin^2\theta_{13}}\right)\right]},\tag{2}$$

with

$$\gamma=\frac{|\Delta m_{13}^2|}{2E}\frac{\sin^2 2\theta_{13}}{\cos 2\theta_{13}}\frac{1}{|\mathrm{d}\ln N_{\mathrm{e}}/\mathrm{d}r|_{\mathrm{res}}},$$

where N_{e} is the electron density and F can be calculated by Landau's method [22]. Using the SN matter density profile $\rho\approx C\cdot(10^7\ \mathrm{cm}/r)^3\cdot10^{10}\ \mathrm{g/cm}^3$ where C is a structure constant between 1 and 15 [28–30] and $F\approx1$ (small θ_{13}), we can obtain an simple expression for P_{H} [19, 22–24]

$$P_{\mathrm{H}}=\exp\left\{-\frac{\pi}{12}\left[\frac{10^{10}\ \mathrm{MeV}}{E}\left(\frac{\sin^3 2\theta_{13}}{\cos^2 2\theta_{13}}\right)\times\left(\frac{|\Delta m_{32}^2|}{1\ \mathrm{eV}^2}\right)C^{1/2}\right]^{2/3}\right\},\tag{3}$$

where $|\Delta m_{32}^2|=2.6\times10^{-3}\ \mathrm{eV}^2$. Similarly, we can calculate the expression of the crossing probability at the low resonance region inside the SN, P_{L}. However, due to the large angle solution of the neutrino mixing, $P_{\mathrm{L}}\approx0$.

Suppose a neutrino reaches the detector at the incident angle θ (see Fig. 1). Then the distance that the neutrino travels through earth is

$$L=(-R_{\mathrm{E}}+h)\cos\theta+\sqrt{R_{\mathrm{E}}^2-(R_{\mathrm{E}}-h)^2\sin^2\theta},\tag{4}$$

where h ($\approx$ 0.4 km for the Daya Bay experiment) is the depth of the detector below the surface and $R_{\mathrm{E}}=6400$ km is the radius of the earth. Let x be the distance that the neutrino travels into earth, then the distance of the neutrino to the center of the earth, $\tilde{x}$, is given by

$$\tilde{x}=\sqrt{(-R_{\mathrm{E}}+h)^2+(L-x)^2+2(R_{\mathrm{E}}-h)(L-x)\cos\theta}.$$

Let $P_{i\mathrm{e}}$ be the probability that a neutrino mass eigenstate ν_i enters the surface of earth and arrives at the detector as an electron neutrino ν_{e}, then one obtains

[18, 19]

$$P_{2\mathrm{e}}=\sin^2\theta_{12}+\frac{1}{2}\sin^2 2\theta_{12}\int_{x_0}^{x_\mathrm{f}}\mathrm{d}x V(x)\sin\phi^m_{x\to x_\mathrm{f}}, \quad (5)$$

where $\theta_{12}=32.5°$, the potential $V(x)$ that the electron neutrino experiences in earth is $\sqrt{2}G_\mathrm{F}\rho(x)/(m_\mathrm{p}+m_\mathrm{n})$ and $\phi^m_{a\to b}$ is defined as

$$\begin{aligned}\phi^m_{a\to b} &= \int_a^b \mathrm{d}x\varDelta_m(x),\\ \varDelta_m(x) &= \frac{\Delta m^2_{21}}{2E}\sqrt{(\cos 2\theta_{12}-\varepsilon(x))^2+\sin^2 2\theta_{12}}\,,\end{aligned}$$

where $\varepsilon(x)=2EV(x)/\Delta m^2_{21}$ and $\rho(x)$ is the matter mass density inside the earth [31].

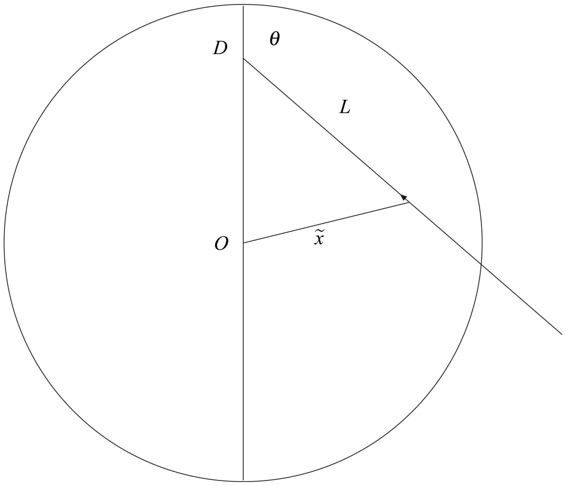

Fig. 1. Illustration of the path of the SN neutrino reaching the detector underground in earth. D is the location of the detector, θ is the incident angle of the neutrino, O is the center of earth, L is the distance the neutrino travels through earth, and $\tilde{x}$ is the distance of the neutrino to the center of earth.

In the following we calculate the event numbers $N(i)$ of SN neutrinos that can be observed through various reaction channels "i". This is done by integrating over the neutrino energy E of the product of the target number N_T, the cross section of each channel $\sigma(i)$ and the neutrino flux function at the detector $F^\mathrm{D}_\alpha(E)/4\pi D^2$,

$$N(i)=N_\mathrm{T}\int\mathrm{d}E\cdot\sigma(i)\cdot\frac{1}{4\pi D^2}\cdot F^\mathrm{D}_\alpha, \quad (6)$$

where α stands for the neutrino or antineutrino of a given flavor, and D (10 kpc in the present discussion) is the distance between the SN and the earth. After a straightforward calculation, the fluxes at the detector can be obtained [11, 12] ($x=\mu,\tau$):

$$\begin{aligned}F^{\mathrm{D(N)}}_{\nu_\mathrm{e}} &= P_{2\mathrm{e}}P_\mathrm{H}F^{(0)}_{\nu_\mathrm{e}}+(1-P_{2\mathrm{e}}P_\mathrm{H})F^{(0)}_{\nu_\mathrm{x}},\\ F^{\mathrm{D(N)}}_{\bar{\nu}_\mathrm{e}} &= (1-P_{2\mathrm{e}})F^{(0)}_{\bar{\nu}_\mathrm{e}}+P_{2\mathrm{e}}F^{(0)}_{\bar{\nu}_\mathrm{x}},\\ 2F^{\mathrm{D(N)}}_{\nu_\mathrm{x}} &= (1-P_{2\mathrm{e}}P_\mathrm{H})F^{(0)}_{\nu_\mathrm{e}}+(1+P_{2\mathrm{e}}P_\mathrm{H})F^{(0)}_{\nu_\mathrm{x}},\\ 2F^{\mathrm{D(N)}}_{\bar{\nu}_\mathrm{x}} &= P_{2\mathrm{e}}F^{(0)}_{\bar{\nu}_\mathrm{e}}+(2-P_{2\mathrm{e}})F^{(0)}_{\bar{\nu}_\mathrm{x}}, \quad (7)\end{aligned}$$

for the normal hierarchy ($\Delta m^2_{31}>0$), and

$$\begin{aligned}F^{\mathrm{D(I)}}_{\nu_\mathrm{e}} &= \begin{cases}P_{2\mathrm{e}}F^{(0)}_{\nu_\mathrm{e}}+(1-P_{2\mathrm{e}})F^{(0)}_{\nu_\mathrm{x}}, & (E<E_\mathrm{C})\\ F^{(0)}_{\nu_\mathrm{x}}, & (E>E_\mathrm{C})\end{cases}\\ F^{\mathrm{D(I)}}_{\bar{\nu}_\mathrm{e}} &= \bar{P}_\mathrm{H}(1-\bar{P}_{2\mathrm{e}})F^{(0)}_{\bar{\nu}_\mathrm{e}}+(1+\bar{P}_{2\mathrm{e}}\bar{P}_\mathrm{H}-\bar{P}_\mathrm{H})F^{(0)}_{\bar{\nu}_\mathrm{x}},\\ 2F^{\mathrm{D(I)}}_{\nu_\mathrm{x}} &= \begin{cases}(1-P_{2\mathrm{e}})F^{(0)}_{\nu_\mathrm{e}}+(1+P_{2\mathrm{e}})F^{(0)}_{\nu_\mathrm{x}}, & (E<E_\mathrm{C})\\ F^{(0)}_{\nu_\mathrm{e}}+F^{(0)}_{\nu_\mathrm{x}}, & (E>E_\mathrm{C})\end{cases}\\ 2F^{\mathrm{D(I)}}_{\bar{\nu}_\mathrm{x}} &= (1+P_{2\mathrm{e}}P_\mathrm{H}-P_\mathrm{H})F^{(0)}_{\bar{\nu}_\mathrm{e}}+\\ &\quad (1+P_\mathrm{H}-P_{2\mathrm{e}}P_\mathrm{H})F^{(0)}_{\bar{\nu}_\mathrm{x}}, \quad (8)\end{aligned}$$

for the inverted hierarchy ($\varDelta m^2_{31}<0$). In Eqs. (7) and (8), $F^{(0)}_{\nu_\alpha}$ is the time-integrated neutrino energy spectrum of flavor α in vacuum which can be described by the Fermi-Dirac distribution

$$F^{(0)}_\alpha(E)=\frac{L^{(0)}_\alpha}{F_{\alpha 3}T^4_\alpha}\frac{E^2}{\exp(E/T_\alpha-\eta_\alpha)+1}, \quad (9)$$

where T_α is the temperature of the neutrino [32, 33]

$$\begin{aligned}T_{\nu_\mathrm{e}} &= 3-4\ \mathrm{MeV}, \quad T_{\bar{\nu}_\mathrm{e}}=5-6\ \mathrm{MeV},\\ T_{\nu_\mathrm{x}} &= 7-9\ \mathrm{MeV}, \quad (10)\end{aligned}$$

η_α is the pinching parameter of the spectra to represent the deviation from being exactly thermal [32, 33]

$$\eta_{\nu_\mathrm{e}}\approx 3-5, \quad \eta_{\bar{\nu}_\mathrm{e}}\approx 2.0-2.5, \quad \eta_{\nu_\mathrm{x}}\approx 0-2, \quad (11)$$

$L^{(0)}_\alpha$ is the the luminosity and $F_{\alpha j}$ is defined by

$$F_{\alpha j}=\int_0^\infty\frac{x^j}{\exp(x-\eta_\alpha)+1}\mathrm{d}x.$$

In the next section, using the relation between the event number of SN neutrinos detected at Daya Bay, N and the mixing angle θ_{13}, we will propose a possible method to acquire information about θ_{13} being smaller than 1.5°.

3 Acquire information about θ_{13} being smaller than 1.5° at Daya Bay

It can be seen from the above section that, using Eqs. (1), (3), (5), (6), (7), and (8), we can obtain a relation between the event number of different flavor SN neutrinos, N, and the mixing angle θ_{13}. Letting R be the ratio of the event number of ν_e over that of

$\bar{\nu}_e$, one can obtain R as a function of θ_{13}. Therefore, we can propose a possible method to acquire some information about the mixing angle θ_{13} by detecting SN neutrinos. In this section we will just consider the process of the neutrino-carbon reactions, since our method is not suitable for the inverse beta-decay and the neutrino-electron reactions in the Daya Bay experiment. The details are discussed at length in Ref. [19].

The Daya Bay Collaboration uses Linear Alkyl Benzene (LAB) as the main part of the liquid scintillator and the total detector mass is about 300 tons. LAB has a chemical composition containing C and H with a ratio of the numbers of C and H atoms, N_C/N_H, of about 0.6. Therefore, the total numbers of target protons $N_T^{(C)} = 1.32\times10^{31}$. For the neutrino-neutrino reactions, the effective cross sections are as follows [34]:

$$\langle\sigma(^{12}\mathrm{C}(\nu_e, e^-)^{12}\mathrm{N})\rangle = 1.85\times10^{-43}\ \mathrm{cm}^2,$$
$$\langle\sigma(^{12}\mathrm{C}(\bar{\nu}_e, e^+)^{12}\mathrm{B})\rangle = 1.87\times10^{-42}\ \mathrm{cm}^2, \quad (12)$$

for the charged-current capture, and

$$\langle\sigma(\nu_e\,^{12}\mathrm{C})\rangle = 1.33\times10^{-43}\ \mathrm{cm}^2,$$
$$\langle\sigma(\bar{\nu}_e\,^{12}\mathrm{C})\rangle = 6.88\times10^{-43}\ \mathrm{cm}^2, \quad (13)$$
$$\langle\sigma(\nu_x(\bar{\nu}_x)^{12}\mathrm{C})\rangle = 3.73\times10^{-42}\ \mathrm{cm}^2, \quad x=\mu,\tau,$$

for the neutral-current capture. Using Eqs. (1), (3), (5), (6), (7), (8), (12) and (13), we plot the ratio R of the event number of ν_e over that of $\bar{\nu}_e$ which can be detected at Daya Bay as a function of the mixing angle θ_{13}. The result is shown in Fig. 2.

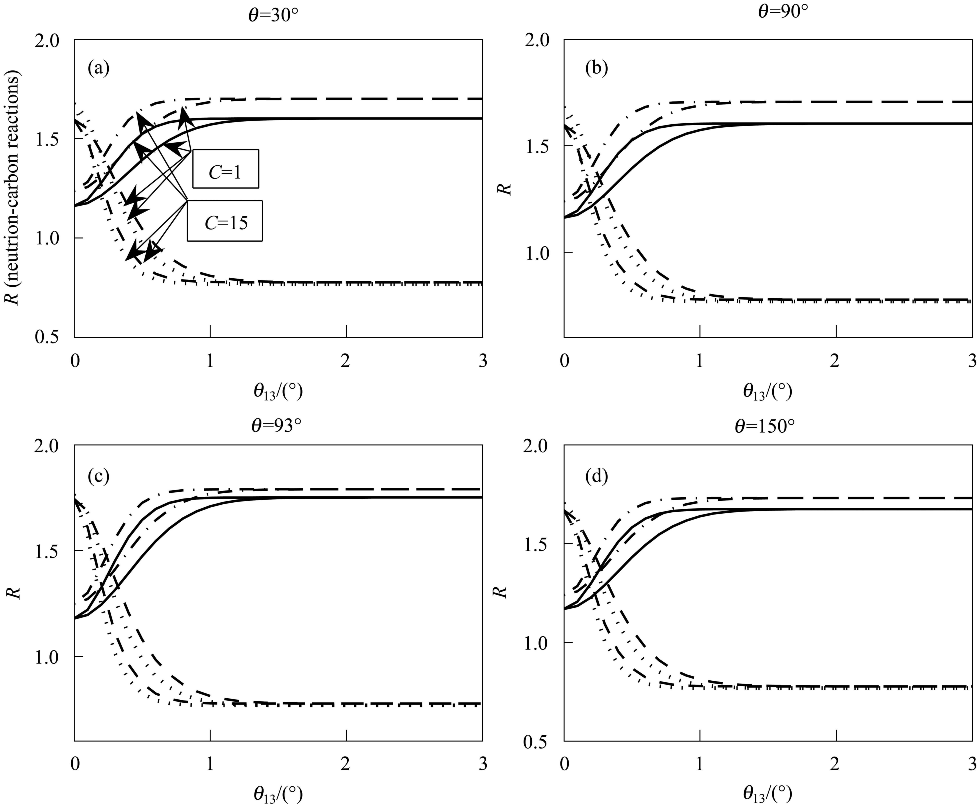

Fig. 2. The ratio of the event number of ν_e to that of $\bar{\nu}_e$, R, as a function of the mixing angle θ_{13} in the channel of neutrino-carbon reactions at the Daya Bay experiment. The incident angle is (a) $\theta = 30°$; (b) $\theta = 90°$; (c) $\theta = 93°$; (d) $\theta = 150°$. The solid curves correspond to the normal hierarchy (max), the dashed curves correspond to the inverted hierarchy (max), the dot-dashed curves correspond to the normal hierarchy (min), the dotted curves correspond to the inverted hierarchy (min), where "max" ("min") corresponds to the maximum (minimum) values of T_α and η_α.

It can be seen from Fig. 2 that the uncertainties of R due to T_α and η_α are not large. For $\theta_{13} \leqslant 1.5°$, R is very sensitive to θ_{13}. However, for $\theta_{13} > 1.5°$, R is nearly independent of θ_{13}. Therefore, when θ_{13} is smaller than 1.5°, we may restrict the mixing angle θ_{13} to a small range and get information on the mass hierarchy by detecting the ratio of event numbers of SN neutrinos, this even though there are still some uncertainties due to the incident angle θ, the mass hierarchy Δm^2_{31} and the structure coefficient C of the SN density function.

The sensitivity to $\sin^2 2\theta_{13}$ of the Daya Bay experiment will reach 0.01, i.e., allow to determine θ_{13} down to about 3°. Therefore, if the actual value of θ_{13} is smaller than 3°, the Daya Bay experiment can only provide an upper limit for θ_{13}. However, if an SN explosion takes place during the operation of Daya Bay, roughly within the cosmic distance considered here, it is possible to reach a much smaller value of θ_{13} through the ratio of the event numbers of different flavor SN neutrinos in the channel of neutrino-carbon reactions as discussed above. It is interesting to note that because of the multi detectors set up, experiments such as the Daya Bay have an internal coincidence check for SN neutrino events.

4 Summary and discussion

In this paper we first discussed the detection of SN neutrinos on earth. Since neutrino flavor conversions inside the SN depend on the neutrino mixing angle θ_{13}, we gave a possible method to acquire information on θ_{13} being smaller than 1.5° by detecting SN neutrinos at Daya Bay.

We let the parameters in the neutrino energy spectra (the temperatures and the pinching parameters) vary in some reasonable ranges. In fact, the simulations from the two leading groups, the Livermore group [35] and the Garching group [36, 37], led to parameters which agree within about 20%–30%. However, their central values of the SN parameters are different.

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