

What do we learn from the ρ - π puzzle^{*}

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Abstract The experimental observation indicates that the branching ratio of $\psi' \rightarrow \rho\pi$ is very small while the ρ - π channel is a main one in J/ψ decays. To understand the puzzle, various interpretations have been proposed. Meanwhile according to the hadronic helicity selection rule, this decay mode should be suppressed. Numerical calculations are needed to determine how it is suppressed. We calculate the branching ratios of $J/\psi \rightarrow \rho\pi$ and $\pi\pi$ in the framework of QCD. The results show that the branching ratios are proportional to $\left(\frac{m_u + m_d}{M_{J/\psi}}\right)^2$ for the $\rho\pi$ mode and $\left(\frac{m_u - m_d}{M_{J/\psi}}\right)^2$ for the $\pi\pi$ mode which is isospin violated. The theoretical prediction of the ratio of $J/\psi \rightarrow \rho\pi$ is smaller than data, but not too small to invoke a completely new mechanism. Thus the puzzle is still standing even though we learn much knowledge towards the puzzle and this will help to finally interpret the puzzle and then gain a deeper insight to the heavy quarkonia.

Key words ρ - π puzzle, hadronic helicity conservation, final state interaction

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1 Introduction

The ρ - π puzzle has been standing for years. The puzzle is phrased as $J/\psi \rightarrow \rho\pi$ is a main channel in J/ψ decays, while the branching ratio of $\psi' \rightarrow \rho\pi$ is very small. It seems to contradict to the general understanding of the charmonia family.

In the regular theoretical framework, there should be a relation

$$R = \frac{BR(\psi' \rightarrow ggg)}{BR(J/\psi \rightarrow ggg)} = \frac{\Gamma(\psi' \rightarrow e^+e^-)}{\Gamma(J/\psi \rightarrow e^+e^-)} \cdot \frac{\Gamma_t(J/\psi)}{\Gamma_t(\psi')},$$

where Γ_t is the total width. This estimate on the ratio originates from the fact that if both J/ψ and ψ' are c - \bar{c} bound states, as commonly conjectured, in the hadronic decays, c and \bar{c} annihilate into three gluons which then convert into hadrons, whereas in the leptonic decays, c and \bar{c} annihilate into a virtual photon which turns into a lepton-pair. In this picture, the amplitudes of the hadronic decay which occurs via a three-gluon intermediate state, and the leptonic decay, which occurs via a virtual photon intermediate state are proportional to the wavefunction at the origin $\psi(0)$. If everything worked well, the ratio should

be close to 12%–14%, which is called as the 14% rule (now, it is sometimes called as 12% rule, anyhow it is a sizable number in contrast to the data.). However the data tell us that this ratio is much smaller than this value.

Some theoretical interpretations have been proposed. Rosner et al. [1] suggested that the quantum number of the observed ψ' may be not a pure $2S$ state which is the first radial excited state of the $c\bar{c}$ system, but a mixture of $2S$ and $1D$ states. The amplitudes are then

$$\langle \rho\pi | \psi' \rangle = \langle \rho\pi | 2^3S_1 \rangle \cos\phi - \langle \rho\pi | 1^3D_1 \rangle \sin\phi \sim 0,$$

$$\langle \rho\pi | \psi'' \rangle = \langle \rho\pi | 2^3S_1 \rangle \sin\phi + \langle \rho\pi | 1^3D_1 \rangle \cos\phi \sim \langle \rho\pi | 2^3S_1 \rangle / \sin\phi,$$

where ϕ is fixed to -27° or 12° by fitting data. By the destructive interference between the contributions of the two components to the amplitude of $\psi' \rightarrow \rho\pi$, the smallness is explained. Suzuki [2] alternatively suggested that the relative phase between the one-photon and gluonic decay amplitudes or the hadronic excess in ψ' decay may result in the small branching ratio.

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The final state interactions may also give a reasonable explanation [3]. The first proposal can be tested in the decays of $\psi'' \rightarrow \rho\pi$ which has not been well measured yet. In Ref. [2], the author suggested that the one-photon amplitude is sizable and it can be tested in some other modes, for example $\psi \rightarrow \pi\pi$ if the process is dominated by the electromagnetic interaction. The hadronic excess can also be tested in the decays of other higher excited states of the ψ - family and even the Υ - family. The final state interaction may play an important role in D and B decays, and also in decays of ψ mesons as suggested in the literature. The difficulties are how to properly evaluate such effects. The final state interaction process is induced by the strong interaction at lower energy region, thus it is governed by the non-perturbative QCD, which is not yet fully understood in the present theoretical framework yet. People need to invoke some phenomenological models to carry out the calculations. We will give a more detailed discussion on the estimation of final state interactions somewhere else. Here we only use our results to discuss the puzzle.

In our work [4], we simultaneously consider the FSI and the direct decay of J/ψ into a vector and a pseudoscalar mesons and conclude that both of them contribute to the widths and their interference should be destructive to explain data. This observation indicates that even though the OZI-forbidden process is sizable, it cannot be consistent with data. The result implies that the hadronic helicity conservation

indeed greatly suppresses the process of $J/\psi \rightarrow \rho\pi$ and as the data demand an explanation, one should consider what is the origin of the problem.

2 Some relevant formulae and numerical results

In a straightforward calculation based on the SM, we estimate the decay width of the OZI forbidden process $J/\psi \rightarrow \rho\pi$ [5], and find that the width is indeed proportional to $(m_q/m_{J/\psi})^2$ which is coming from the hadronic helicity suppression factor. Numerically the branching ratio of $J/\psi \rightarrow \rho\pi$ should be smaller than 0.1%. The same situation appears for $\psi' \rightarrow \rho\pi$. It was qualitatively discussed by Brodsky et al [6]. As aforementioned above, to testify the calculation, we recalculate the subprocess $J/\psi \rightarrow 3g \rightarrow \pi\pi$, which is an isospin violating reaction and usually is supposed to be dominated by the subprocess $J/\psi \rightarrow \gamma\pi\pi$ because the EM interaction violates isospin as is well known. Our result indicates that in the OZI forbidden subprocess the transition amplitude is proportional to $(m_u - m_d)/m_{J/\psi}$, i.e. the mass difference results in the isospin violation instead. Our numerical result is of the same order as the contribution from $J/\psi \rightarrow \gamma\pi\pi$. All the results are consistent with our physics picture and qualitatively reasonable. Therefore we can trust our calculations for the process $J/\psi \rightarrow \rho\pi$. Our numerical results are listed in Table 1.

Table 1. Decay widths (Γ) of $J/\psi \rightarrow \pi^+\rho^- + \pi^-\rho^+$ based on the three distribution functions, ϕ_1 , ϕ_2 and ϕ_3 , respectively [5].

m_u/MeV	m_d/MeV	$\Gamma(\phi_1)/\text{MeV}$	$\Gamma(\phi_2)/\text{MeV}$	$\Gamma(\phi_3)/\text{MeV}$	exp/MeV
2	2	1.04×10^{-4}	7.21×10^{-5}	5.11×10^{-4}	
3	3	2.36×10^{-4}	1.6×10^{-4}	1.17×10^{-3}	
4	4	4.12×10^{-4}	2.9×10^{-4}	2.08×10^{-3}	$(1.06 \pm 0.08) \times 10^{-3}$
5	5	6.69×10^{-4}	4.54×10^{-4}	3.38×10^{-3}	
6	6	9.75×10^{-4}	6.68×10^{-4}	4.88×10^{-3}	

As indicated in Ref. [6], the structure of J/ψ may be not a pure $c\bar{c}$ charmonium, but consists of other components, such as the hybrids $c\bar{c}g$, $c\bar{c}q\bar{q}$ and etc.

To understand the smallness of the ratio R , one can expect that either there is a problem with ψ' as Rosner et al. [1] do, or there is something obscure in the J/ψ structure as Brodsky and many others indicated. Our above numerical results show that even though the hadronic selection rule works in the cases of $J/\psi \rightarrow \rho\pi$ and $\psi' \rightarrow \rho\pi$, the suppression is not too serious and the theoretical prediction is only one

order smaller than the data.

3 Discussion and our conclusion

Therefore a tentative conclusion may be drawn that the $\rho\pi$ puzzle may be not due to the mixing structures of ψ' and ψ'' , but neither to an anomalous structure of J/ψ itself. It seems that both of the proposals cannot independently explain the ‘‘puzzle’’, more complicated mechanisms may be needed.

This is a great challenge to our understanding be-

cause the $c\bar{c}$ structure of J/ψ has been recognized almost from the very beginning of its discovery. If it is not a pure $c\bar{c}$, all the previous works in terms of the potential models, where many parameters are fixed by fitting data, should be re-considered. There may be some other mechanisms which were not taken into account, or may exist contributions from new physics beyond the standard model (SM). However, the latter seems not very promising because the concerned energy range is rather low and the SM works perfectly well to explain the data for most states and processes. Thus one should be obviously inclined to the first proposal that J/ψ is not a pure S-wave bound state of $c\bar{c}$.

It is noted that the calculated results unless for the distribution ϕ_3 , are one order smaller than the data. The same situation happens to the ψ' , but it is still hard to draw a conclusion that the 14% rule is due to existence of higher Fock states in J/ψ or other mechanisms which further suppress the reaction of $\psi' \rightarrow \rho\pi$, may be both. It forms an intriguing challenge to our understanding of the hadron structures. This whole picture also applies to the Υ family, therefore the future experiments would provide hints to finally solve the puzzle.

There have been some theoretical explanations besides those we discussed above. In the work of Mo, Yuan and Wang [7], the authors described the recent status of the theoretical research as well as the experimental measurements on this interesting subject.

Interestingly, there exists also an alternative opinion towards the subject, Suzuki [2], Zhao [8] deny it as a “puzzle”, because they considers that the electromagnetic interaction may play an important role in J/ψ decays where $c\bar{c}$ annihilate into a virtual photon which later fragment into hadrons. In this picture it is supposed that a destructive interference between the contribution of three-gluon and single-photon processes would suppress $\psi' \rightarrow \rho\pi$. Since its amplitude can be roughly estimated in terms of the measured rate of the J/ψ leptonic decay, there should be a strong constraint on the proposal. Moreover, if it is true, the interference would also appear to other decay modes and the picture should be further investigated and tested by more accurate experimental data available in the future, especially from the BESIII.

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