kth-order antibunching effect for a new kind of excited even and odd q-coherent states

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Abstract A new kind of excited even q-coherent states $(a_q^{-1})^m\,|\alpha\rangle_q^{\mathrm{e}}$ and excited odd q-coherent states $(a_q^{-1})^m\,|\alpha\rangle_q^{\mathrm{o}}$ is constructed by acting with inverse boson operators on the even and odd q-coherent states. The m dependence of the kth-order antibunching effect is numerically studied for $k=2$, 3, 4, 5. It is shown that the kth-order antibunching effect enhances as m increases. The larger k, the quicker the antibunching effect enhances.

Key words quantum algebra, inverse boson operators, even and odd q-coherent state, kth-order antibunching effect

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1 Introduction

The coherent states of quantum algebra (q-coherent states) have attracted a lot of attention due to their possible applications in various branches of physics and mathematical physics. Since Biedenharn introduced the quantum group to the coherent states [1], q-coherent states (q-CSs) have been studied by many authors [2–6].

One of the most interesting subjects in quantum optics is to construct various quantum states based on the principles of quantum mechanics and to investigate their nonclassical properties. Since a method of generating new quantum states was introduced in 1991 by acting with boson creation operators a^+ on a coherent state [7], a series of excited states of light fields have been constructed and investigated [8–12]. In 2002, we introduced this method to the even and odd q-CSs, generated the excited even q-CSs$(a_q^+)^m\,|\alpha\rangle_q^{\mathrm{e}}$ and the excited odd q-CSs$(a_q^+)^m\,|\alpha\rangle_q^{\mathrm{o}}$, and investigated their nonclassical properties [13, 14].

Recently, one pays attention to another method of generating new quantum states by acting with inverse boson annihilation operators a^{-1} on some typical quantum states [15, 16]. In this paper, we apply the new method to even and odd q-CSs. The remaining part of the paper is organized as follows: first we generate a new kind of excited even q-CSs$(a_q^{-1})^m\,|\alpha\rangle_q^{\mathrm{e}}$ and excited odd q-CSs$(a_q^{-1})^m\,|\alpha\rangle_q^{\mathrm{o}}$; second we study the 2nd order antibunching effect and compare the numerical result with the old-style excited even q-CSs$(a_q^+)^m\,|\alpha\rangle_q^{\mathrm{e}}$ and excited odd q-CSs$(a_q^+)^m\,|\alpha\rangle_q^{\mathrm{o}}$ [13]; third we investigate the kth-order antibunching effect for $k=3$, 4, 5; and finally a simple discussion is given.

2 A new kind of excited even and odd q-coherent states

In the Fock representation the even and odd q-CSs can be expressed as

$$|\alpha\rangle_q^{\mathrm{e}}=\sum_{n=0}^{\infty}\frac{\alpha^{2n}}{\sqrt{[2n]_q!}}\,|2n\rangle_q\,, \tag{1}$$

$$|\alpha\rangle_q^{\mathrm{o}}=\sum_{n=0}^{\infty}\frac{\alpha^{2n+1}}{\sqrt{[2n+1]_q!}}\,|2n+1\rangle_q\,, \tag{2}$$

where $\alpha=r\mathrm{e}^{\mathrm{i}\theta}$ is a complex parameter, $[n]_q$ and $[n]_q!$ are defined by

$$[n]_q=(q^n-q^{-n})/(q-q^{-1}), \tag{3}$$

$$[n]_q!=[n]_q[n-1]_q\cdots[1]_q\,. \tag{4}$$

In the following we consider only states with $0<q<1$ (because $[n]_{q^{-1}}=[n]_q$). For $q\to 1$, Eq. (1) and

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Eq. (2) reduce to the ordinary even and odd coherent states.

Now we construct a new kind of excited even and odd q-CSs by acting repeatedly with an inverse boson q-annihilation operator a_q^{-1} on the even and odd q-CSs

$$|\alpha,m\rangle_q^{\mathrm{e}} = C_m^{\mathrm{e}}(a_q^{-1})^m|\alpha\rangle_q^{\mathrm{e}} = C_m^{\mathrm{e}}\sum_{n=0}^{\infty}\frac{\alpha^{2n}}{\sqrt{[2n+m]_q!}}|2n+m\rangle_q\ , \quad (5)$$

$$|\alpha,m\rangle_q^{\mathrm{o}} = C_m^{\mathrm{o}}(a_q^{-1})^m|\alpha\rangle_q^{\mathrm{o}} = C_m^{\mathrm{o}}\sum_{n=0}^{\infty}\frac{\alpha^{2n+1}}{\sqrt{[2n+1+m]_q!}}|2n+1+m\rangle_q\ . \quad (6)$$

The q-annihilation operator a_q, q-creation operator a_q^+ and q-number operator N_q satisfy the commutation relations

$$a_q a_q^+ - q a_q^+ a_q = q^{-N_q}\ , \quad (7)$$

$$[N_q,\, a_q] = -a_q, \quad [N_q,\, a_q^+] = a_q^+\ . \quad (8)$$

To keep the parity, in what follows we consider $m = 2, 4, 6, \cdots$. For $m = 0$, Eq. (5) and Eq. (6) reduce to Eq. (1) and Eq. (2). C_m^{e} and C_m^{o} are normalization constants

$$(C_m^{\mathrm{e}})^{-2} = \sum_{n=0}^{\infty}\frac{(r^2)^{2n}}{[2n+m]_q!}\ , \quad (9)$$

$$(C_m^{\mathrm{o}})^{-2} = \sum_{n=0}^{\infty}\frac{(r^2)^{2n+1}}{[2n+1+m]_q!}\ . \quad (10)$$

3 *k*th-order antibunching effect

3.1 Second order antibunching effect

The 2nd order correlation function of the q-light fields is defined by [3, 13]

$$g^{(2)}(0) \equiv {}_q\langle a_q^{+2}a_q^2\rangle_q \Big/ \left|{}_q\langle a_q^+ a_q\rangle_q\right|^2.$$

Whenever $g^{(2)}(0) < 1$, the q-light fields exhibit a 2nd order antibunching effect, called 'antibunching effect' for short.

For $|\alpha,m\rangle_q^{\mathrm{e}}$ and $|\alpha,m\rangle_q^{\mathrm{o}}$ we obtain the 2nd order correlation functions

$$g_{\mathrm{e}}^{(2)}(0) = {}_q^{\mathrm{e}}\langle\alpha,m|a_q^{+2}a_q^2|\alpha,m\rangle_q^{\mathrm{e}} \Big/ \langle N_{\mathrm{e}}\rangle^2, \quad (11)$$

$$g_{\mathrm{o}}^{(2)}(0) = {}_q^{\mathrm{o}}\langle\alpha,m|a_q^{+2}a_q^2|\alpha,m\rangle_q^{\mathrm{o}} \Big/ \langle N_{\mathrm{o}}\rangle^2, \quad (12)$$

where

$$\langle N_{\mathrm{e}}\rangle = {}_q^{\mathrm{e}}\langle\alpha,m|a_q^+ a_q|\alpha,m\rangle_q^{\mathrm{e}} = (C_m^{\mathrm{e}})^2\sum_{n=0}^{\infty}\frac{(r^2)^{2n}}{[2n+m]_q!}\times[2n+m]_q\ , \quad (13)$$

$$\langle N_{\mathrm{o}}\rangle = {}_q^{\mathrm{o}}\langle\alpha,m|a_q^+ a_q|\alpha,m\rangle_q^{\mathrm{o}} = (C_m^{\mathrm{o}})^2\sum_{n=0}^{\infty}\frac{(r^2)^{2n+1}}{[2n+1+m]_q!}\times[2n+1+m]_q\ , \quad (14)$$

$${}_q^{\mathrm{e}}\langle\alpha,m|a_q^{+2}a_q^2|\alpha,m\rangle_q^{\mathrm{e}} = (C_m^{\mathrm{e}})^2\sum_{n=0}^{\infty}\frac{(r^2)^{2n}}{[2n+m]_q!}\times [2n-1+m]_q\times[2n+m]_q\ , \quad (15)$$

$${}_q^{\mathrm{o}}\langle\alpha,m|a_q^{+2}a_q^2|\alpha,m\rangle_q^{\mathrm{o}} = (C_m^{\mathrm{o}})^2\sum_{n=0}^{\infty}\frac{(r^2)^{2n+1}}{[2n+1+m]_q!}\times [2n+m]_q\times[2n+1+m]_q\ . \quad (16)$$

From Eq. (3), Eq. (4), and Eqs. (9)–(16), we can obtain $g_{\mathrm{e}}^{(2)}(0)$ and $g_{\mathrm{o}}^{(2)}(0)$ as functions of $x(=|\alpha|^2 = r^2)$ for $m = 0, 2, 4, 6, 8, \cdots$ and $q = 0.1, 0.2, 0.3, 0.4, \cdots, 0.9$. Fig. 1 and Fig. 2 give the results of $q = 0.9$ and $m = 0, 2, 4$. Fig. 3 and Fig. 4 give the results of $q = 0.3$ and $m = 0, 2, 4$.

When the q is close to 1 (e.g. $q = 0.9$), as can be seen, for large $x(=|\alpha|^2 = r^2)$, the q-light fields don't exhibit an antibunching effect. For small x, the

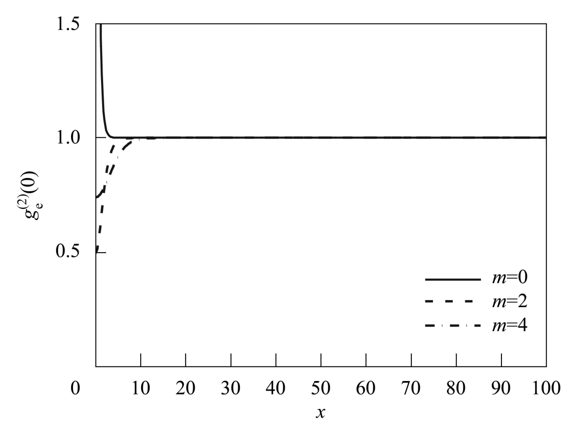

Fig. 1. $g_{\mathrm{e}}^{(2)}(0)$ versus m and $x(=r^2)$ for $q=0.9$.

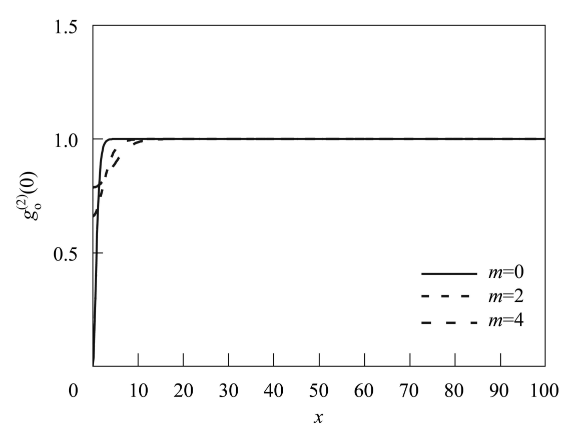

Fig. 2. $g_{\mathrm{o}}^{(2)}(0)$ versus m and $x(=r^2)$ for $q=0.9$.

excited even q-CSs ($m \neq 0$) exhibit an antibunching effect but the unexcited even q-CSs ($m = 0$) exhibit a strong bunching effect; the antibunching effect is weaker in the excited odd q-CSs ($m \neq 0$) than in the unexcited odd q-CSs ($m = 0$). This is analogous to the old-style excited even and odd q-CSs [13].

For small q, far from 1 (e.g. q=0.3), in a wide region of large x, the antibunching effect is greatly enhanced as m increases in the excited even q-CSs$(a_q^{-1})^m |\alpha\rangle_q^{\rm e}$ as well as in the excited odd q-CSs$(a_q^{-1})^m |\alpha\rangle_q^{\rm o}$ (see Fig. 3 and Fig. 4). This differs greatly from the old-style excited even q-CSs$(a_q^{+})^m |\alpha\rangle_q^{\rm e}$ and excited odd q-CSs$(a_q^{+})^m |\alpha\rangle_q^{\rm o}$ [13], in which the antibunching effect is independent of m for large x.

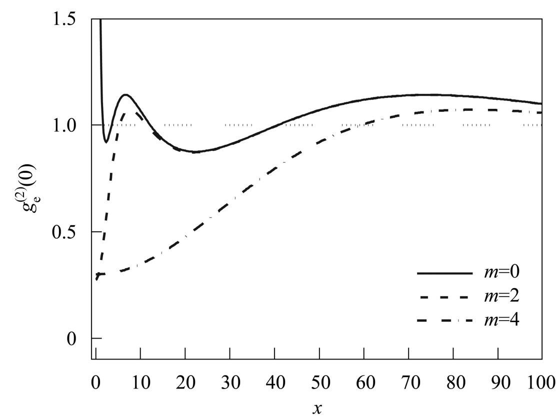

Fig. 3. $g_{\rm e}^{(2)}(0)$ versus m and $x(=r^2)$ for $q=0.3$.

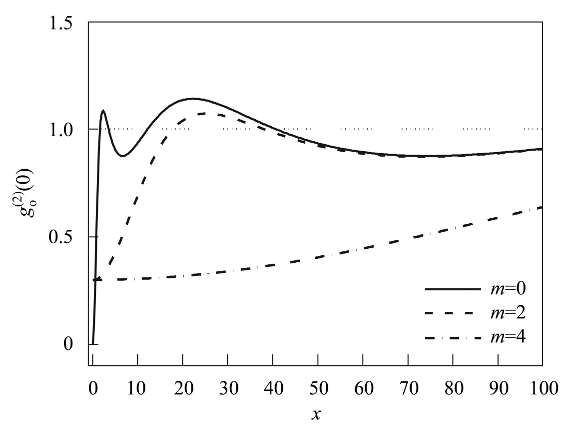

Fig. 4. $g_{\rm o}^{(2)}(0)$ versus m and $x(=r^2)$ for $q=0.3$.

3.2 Higher order antibunching effect

The kth-order correlation function of the q-light fields is defined by [4]

$$g^{(k)}(0) \equiv {}_q\langle a_q^{+k} a_q^k\rangle_q \Big/ \left|{}_q\langle a_q^{+} a_q\rangle_q\right|^k, \quad k=3,\,4,\,5,\cdots .$$

Whenever $g^{(k)}(0) < 1$ the q-light fields exhibit a kth-order antibunching effect.

For $|\alpha,m\rangle_q^{\rm e}$ and $|\alpha,m\rangle_q^{\rm o}$ we obtain the kth-order correlation functions as

$$g_{\rm e}^{(k)}(0) = {}_q^{\rm e}\langle\alpha,m|\, a_q^{+k} a_q^k \,|\alpha,m\rangle_q^{\rm e} \Big/ \langle N_{\rm e}\rangle^k, \qquad (17)$$

$$g_{\rm o}^{(k)}(0) = {}_q^{\rm o}\langle\alpha,m|\, a_q^{+k} a_q^k \,|\alpha,m\rangle_q^{\rm o} \Big/ \langle N_{\rm o}\rangle^k, \qquad (18)$$

where

$${}_q^{\rm e}\langle\alpha,m|\, a_q^{+k} a_q^k \,|\alpha,m\rangle_q^{\rm e} = (C_m^{\rm e})^2 \sum_{n=0}^{\infty} \frac{(r^2)^{2n}}{[2n+m]_q!} \prod_{j=1}^{k} [2n+m+1-j]_q, \qquad (19)$$

$${}_q^{\rm o}\langle\alpha,m|\, a_q^{+k} a_q^k \,|\alpha,m\rangle_q^{\rm o} = (C_m^{\rm o})^2 \sum_{n=0}^{\infty} \frac{(r^2)^{2n+1}}{[2n+1+m]_q!} \prod_{j=1}^{k} [2n+m+2-j]_q\,. \qquad (20)$$

From Eq. (3), Eq. (4), Eq. (9), Eq. (10), Eq. (13), Eq. (14), and Eq. (17)–Eq. (20), we obtain $g_{\rm e}^{(k)}(0)$ and $g_{\rm o}^{(k)}(0)$ as functions of $x(=|\alpha|^2=r^2)$ for $k=3,\,4,\,5,\,\cdots$, $m=0,\,2,\,4,\,6,\,\cdots$ and $q=0.1,\,0.2,\,0.3,\,0.4,\,\cdots,\,0.9$. Fig. 5 and Fig. 6 give the results for k=3, q=0.3 and m=0, 2, 4; Fig. 7 and Fig. 8 give the results for $k=4$, $q=0.3$ and $m=0$, 2, 4; Fig. 9 and Fig. 10 give the results for $k=5$, $q=0.3$ and $m=0$, 2, 4.

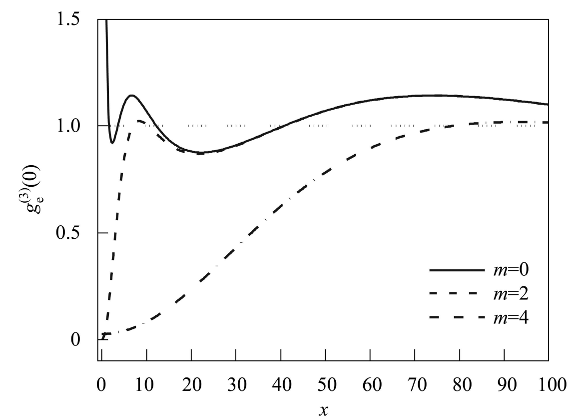

Fig. 5. $g_{\rm e}^{(3)}(0)$ versus m and $x(=r^2)$ for $q=0.3$.

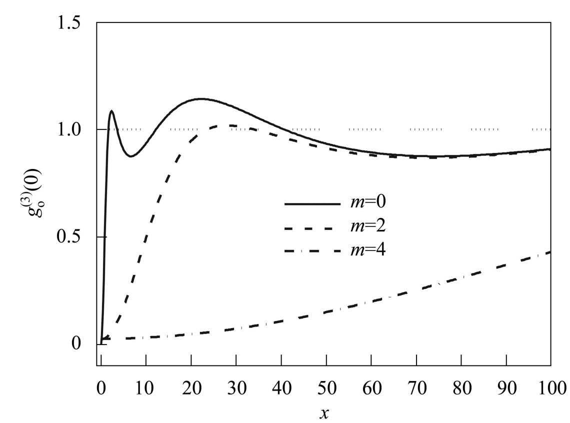

Fig. 6. $g_{\rm o}^{(3)}(0)$ versus m and $x(=r^2)$ for $q=0.3$.

For large q (e.g. $q=0.9$) the results are similar to those of Fig. 1 and Fig. 2.

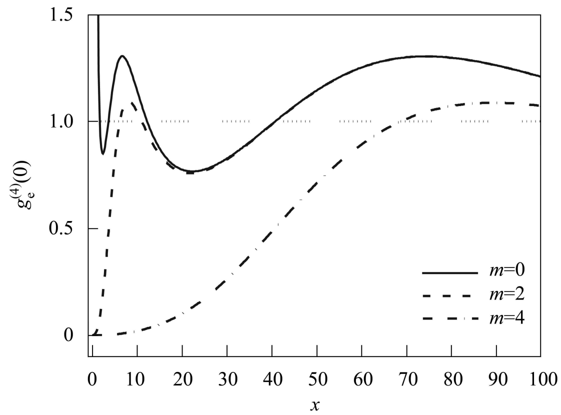

Fig. 7. $g_e^{(4)}(0)$ versus m and $x(=r^2)$ for $q=0.3$.

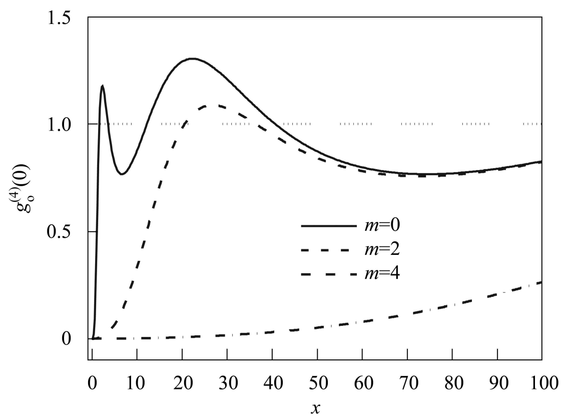

Fig. 8. $g_o^{(4)}(0)$ versus m and $x(=r^2)$ for $q=0.3$.

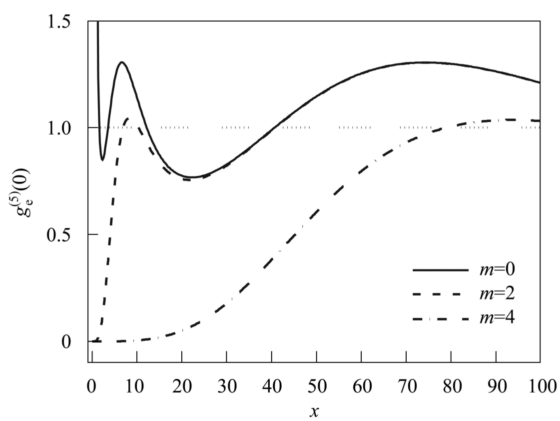

Fig. 9. $g_e^{(5)}(0)$ versus m and $x(=r^2)$ for $q=0.3$.

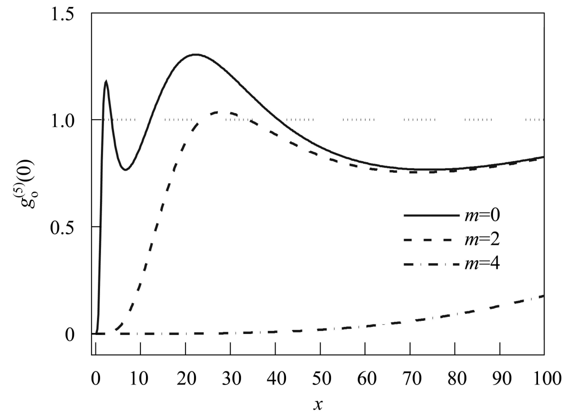

Fig. 10. $g_o^{(5)}(0)$ versus m and $x(=r^2)$ for $q=0.3$.

4 Conclusion

A new kind of excited even q-CSs$(a_q^{-1})^m|\alpha\rangle_q^e$ and excited odd q-CSs$(a_q^{-1})^m|\alpha\rangle_q^o$ is constructed by acting with the inverse boson operators on the even and odd q-CSs. The m dependence of the kth-order antibunching effect is numerically studied for $k=2$, 3, 4, 5. The main results obtained are summarized in the following.

(a) For a given q and $m=0$, $g_o^{(3)}(0)=g_o^{(2)}(0)$, $g_e^{(3)}(0)=g_e^{(2)}(0)$, $g_o^{(5)}(0)=g_o^{(4)}(0)$, $g_e^{(5)}(0)=g_e^{(4)}(0)$, $\cdots$; for $m\neq 0$, $g_o^{(3)}(0)\neq g_o^{(2)}(0)$, $g_e^{(3)}(0)\neq g_e^{(2)}(0)$, $g_o^{(5)}(0)\neq g_o^{(4)}(0)$, $g_e^{(5)}(0)\neq g_e^{(4)}(0)$, $\cdots$.

(b) For q close to 1 (e.g. $q=0.9$), the kth-order antibunching effect merely appears in a narrow region of small x, similar to the old-style excited even q-CSs$(a_q^+)^m|\alpha\rangle_q^e$ and excited odd q-CSs$(a_q^+)^m|\alpha\rangle_q^o$ [13].

(c) For small q far from 1 (e.g. $q=0.3$) and small x, the excited even q-CSs ($m\neq 0$) exhibit a kth-order antibunching effect but the unexcited even q-CSs ($m=0$) don't. The 2nd-order antibunching effect is weaker in the excited odd q-CSs ($m\neq 0$) than in the unexcited odd q-CSs ($m=0$), but the higher-order antibunching effect is not.

(d) For small q far from 1, in a wide region of large x (e.g. $x>0.6$ for $q=0.3$) in the excited even q-CSs$(a_q^{-1})^m|\alpha\rangle_q^e$ and the excited odd q-CSs$(a_q^{-1})^m|\alpha\rangle_q^o$, the kth-order antibunching effect ($k=2$, 3, 4, 5) is enhanced as m increases. The larger k is, the quicker the antibunching effect is enhanced.

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