Non-commutative phase space and its space-time symmetry*

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Abstract First a description of 2+1 dimensional non-commutative (NC) phase space is presented, and then we find that in this formulation the generalized Bopp's shift has a symmetric representation and one can easily and straightforwardly define the star product on NC phase space. Then we define non-commutative Lorentz transformations both on NC space and NC phase space. We also discuss the Poincare symmetry. Finally we point out that our NC phase space formulation and the NC Lorentz transformations are applicable to any even dimensional NC space and NC phase space.

Key words non-commutative phase space, space-time symmetry, Lorentz transformation

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1 Introduction

In recent years there has been an increasing interest in the study of physics on non-commutative space, because the effects of the space non-commutativity may become significant in extreme situations such as at the string scale or at the TeV and even higher energy levels. There are many papers devoted to the study of various aspects of the quantum field theory and quantum mechanics on NC space, where space-space is non-commuting, but the momentum-momentum is commuting, or on NC phase space, where both space-space and momentum-momentum are non-commuting, for references see Refs. [1–5]. Although quantum theories on NC space and NC phase space have been extensively studied in the literature, the description of NC phase space is far from complete, for example, it is not easy to define the star product on NC phase space in the formulations of Refs. [6, 7]. Other important issues we want to discuss are the space-time symmetries of the NC space and NC phase space. As we know, the Lorentz symmetry plays a central role in any realistic quantum field theory. There are different approaches in the formulations of Lorentz and Poincare symmetries on NC space, for references see Refs. [8–10]. Ref. [8] studied Lorentz transformation on NC space and claimed that the NC gauge theories are invariant under the NC Lorentz transformations. Because of the singularity of matrix θ_{ij} the NC Lorentz transformation in Ref. [8] may not be applicable for the 3+1 dimensional NC space. Also there are no discussions about the Lorentz transformation on NC phase space in the literature.

In this paper, we will give a description of NC phase space in 2+1 dimensions, where the star product can be easily defined. On 2+1 dimensional space-time we extend the results of Ref. [8] to the NC phase space, and we find that our formulation is applicable to any even space dimensions.

This paper is organized as follows: in Section 2 we present a description of NC phase space on $2+1$ dimensions. In Section 3 we discuss NC Lorentz and Poincare transformations both on NC space and NC phase space. Conclusion remarks are given in the last section.

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2 The description of NC phase space

On NC space, the NC algebra is,

$$[\hat{x}_\mu,\hat{x}_\nu]=\mathrm{i}\theta_{\mu\nu}\,,\quad [\hat{x}_\mu,\hat{p}_\nu]=\mathrm{i}\hbar\delta_{\mu\nu}\,,\quad [\hat{p}_\mu,\hat{p}_\nu]=0\,,\quad (1)$$

where the Greek indices μ and ν run from 0 to 2. In order not to spoil unitarity [11, 12] and physical casuality [13], one needs to set $\theta^{0\mu}=0$; in two dimensions the constant anti-symmetric matrix element θ_{ij} can be written as $\theta_{ij}=\epsilon_{ij}\theta$, and θ is related to the space-space non-commutativity.

Non-commutative field theory is constructed from commutative field theory by replacing, in the action, the usual multiplication product among fields with the star product among fields. The star product between two fields is defined as

$$(f*g)(x) = f(x)\mathrm{e}^{\frac{\mathrm{i}}{2}\overleftarrow{\partial_\mu}\theta^{\mu\nu}\overrightarrow{\partial_\nu}}g(x) = f(x)g(x)+\frac{\mathrm{i}}{2}\theta^{\mu\nu}\partial_\mu f\,\partial_\nu g+\mathcal{O}(\theta^2).\quad (2)$$

This star product can be replaced by a shift which is called Bopp's shift

$$\hat{x}_\mu=x_\mu-\frac{1}{2}\theta_{\mu\nu}p^\nu\,,\qquad \hat{p}_\mu=p_\mu\,,\quad (3)$$

where x_μ and p_ν are coordinates and momenta on commuting space-time. After applying this shift, the effect caused by space-space non-commutativity can be calculated in the commuting space.

The Bose-Einstein statistics on non-commutative quantum mechanics requires both space-space and momentum-momentum non-commutativity [7]. In the following we present our formulation to NC phase space. On NC phase space the NC algebra reads,

$$[\hat{x}_\mu,\hat{x}_\nu]=\mathrm{i}\theta_{\mu\nu}\,,\quad [\hat{x}_\mu,\hat{p}_\mu]=\mathrm{i}\hbar\delta_{\mu\nu}\,,\quad [\hat{p}_\mu,\hat{p}_\nu]=\mathrm{i}\bar{\theta}_{\mu\nu}\,.\quad (4)$$

On NC phase space we set also $\bar{\theta}^{0\mu}=0$, $\bar{\theta}_{\mu\nu}$ is a very small anti-symmetric matrix element, it reflects the non-commutativity of the momenta, and $\bar{\theta}_{ij}=\epsilon_{ij}\bar{\theta}$.

From Eq. (4) we obtain a generalized Bopp's shift [6] as

$$\hat{x}_\mu=\alpha x_\mu-\frac{1}{2\hbar\alpha}\theta_{\mu\nu}p^\nu,\quad (5)$$

$$\hat{p}_\mu=\alpha p_\mu+\frac{1}{2\hbar\alpha}\bar{\theta}_{\mu\nu}x^\nu,\quad (6)$$

where $\alpha=1-\dfrac{\theta\bar{\theta}}{8\hbar^2}=1+\mathcal{O}(\theta^2)$. Hereafter we choose $\alpha=1$. It is easy to check that the above generalized Bopp's shift is consistent with the algebraic relation (4). Then the star product on NC phase space can be easily defined as

$$(f*g)(x) = f(x,p)\mathrm{e}^{\frac{\mathrm{i}}{2}\overleftarrow{\partial^x_\mu}\theta^{\mu\nu}\overrightarrow{\partial^x_\nu}+\frac{\mathrm{i}}{2}\overleftarrow{\partial^p_\mu}\bar{\theta}^{\mu\nu}\overrightarrow{\partial^p_\nu}}g(x,p) = f(x,p)g(x,p)+\frac{\mathrm{i}}{2}\theta^{\mu\nu}\partial^x_\mu f(x,p)\partial^x_\nu g(x,p)+\frac{\mathrm{i}}{2}\bar{\theta}^{\mu\nu}\partial^p_\mu f(x,p)\partial^p_\nu g(x,p)+\mathcal{O}(\theta^2).\quad (7)$$

In NC quantum mechanics and NC quantum field theory, the star product between two fields on NC phase space can be replaced by the generalized Bopp's shift (5) for coordinates and (6) for momenta.

We would like to stress that our formulation above is well defined on 2-dimensional NC phase space, and it can be generalized only to any even dimensional case.

3 Lorentz transformation on non-commutative phase space

Let's first discuss the Lorentz transformation on NC space-time [8]. In an analogous way as in commutative space-time one could introduce, on NC space, a NC Lorentz transformation as $\hat{x}_\mu=\Lambda^\nu_\mu\hat{x}_\nu$. But this kind of definition of the Lorentz transformation is not consistent with the algebra (1), since it would require that $\theta_{\mu\nu}$ transforms as $\Lambda^\alpha_\mu\Lambda^\beta_\nu\delta_{\alpha\beta}$. This makes little sense, because $\theta_{\mu\nu}$ is a constant and does not change under Lorentz transformation.

From the Bopp's shift (3) on NC space one finds that

$$x_\mu=\hat{x}_\mu+\frac{1}{2\hbar}\theta_{\mu\nu}\hat{p}^\nu\,,\quad p_\mu=\hat{p}_\mu\,.\quad (8)$$

On commuting space-time we can define a Lorentz transformation as follows

$$x'_\mu=\Lambda^\nu_\mu x_\nu\,,\quad (9)$$

which leaves the interval

$$s^2=\eta_{\mu\nu}x^\mu x^\nu\quad (10)$$

invariant if $\eta_{\mu\nu}\Lambda^\mu_\alpha\Lambda^\nu_\beta=\eta_{\alpha\beta}$. Under the Lorentz transformation (9) the momentum p_μ transforms as a Lorentz vector

$$p'_\mu=\Lambda^\nu_\mu p_\nu\,.\quad (11)$$

From the Bopp's shift (3) we obtain the following Lorentz transformation on NC space-time which is induced by the Lorentz transformation (9) and (11)

$$\hat{x}'_\mu = x'_\mu-\frac{1}{2\hbar}\theta_{\mu\nu}p'^\nu=\Lambda^\nu_\mu x_\nu-\frac{1}{2\hbar}\theta_{\mu\nu}\Lambda^\nu_\rho p^\rho = \Lambda^\nu_\mu\hat{x}_\nu+\frac{1}{2\hbar}\Lambda^\nu_\mu\theta_{\nu\rho}\hat{p}_\rho-\frac{1}{2\hbar}\theta_{\mu\nu}\Lambda^\nu_\rho\hat{p}^\rho\,.\quad (12)$$

The above equation defines the non-commutative Lorentz transformation on NC space-time. From this transformation one may note that rather than $\theta^{\mu\nu}$ transforms as a Lorentz tensor, $\theta^{\mu\nu}p_\nu$ transforms as a Lorentz vector. So it is easy to check that the commutation relation (1) on NC space-time is invariant under this transformation. Also, obviously, when $\theta^{\mu\nu}\to 0$, the NC Lorentz transformation above becomes a usual Lorentz transformation on commuting space-time. From the shift (8) and the Lorentz invariant interval (10) one finds the square of the NC length

$$s^2_{\rm ncs}=\hat{x}^\mu\hat{x}_\mu+\frac{1}{\hbar}\theta_{\mu\nu}\hat{x}^\mu\hat{p}^\nu+\frac{1}{4\hbar^2}\theta^{\mu\alpha}\theta_{\mu\beta}\hat{p}_\alpha\hat{p}^\beta. \qquad (13)$$

Straight forward calculation shows that $s^2_{\rm nc}$ is invariant under the NC Lorentz transformation (12). So we have defined a non-commutative Lorentz transformation which leaves $s^2_{\rm nc}$ invariant.

Now we are in a position to generalize the Lorentz transformation on NC space to NC phase space. From the generalized Bopp's shift (5) and (6) on NC phase space, we obtain it's inverse transformations

$$x_\mu = \gamma\left(\hat{x}_\mu+\frac{1}{2\hbar}\theta_{\mu\nu}\hat{p}^\nu\right), \qquad (14)$$

$$p_\mu = \gamma\left(\hat{p}_\mu-\frac{1}{2\hbar}\bar{\theta}_{\mu\nu}\hat{x}^\nu\right), \qquad (15)$$

where $\gamma=4\hbar^2/(4\hbar^2-\theta\bar{\theta})$. The Lorentz transformations (9) and (11) induce the following transformations on NC phase space

$$\hat{x}'_\mu = \Lambda^\nu_\mu x_\nu-\frac{1}{2\hbar}\theta_{\mu\nu}\Lambda^\nu_\rho p^\rho, \qquad (16)$$

$$\hat{p}'_\mu = \Lambda^\nu_\mu p_\nu+\frac{1}{2\hbar}\bar{\theta}_{\mu\nu}\Lambda^\nu_\rho x^\rho. \qquad (17)$$

Inserting Eqs. (14) and (15) into Eqs. (16) and (17), one obtains

$$\begin{aligned}\hat{x}'_\mu &= \gamma(\Lambda^\nu_\mu\hat{x}_\nu+\frac{1}{2\hbar}\Lambda^\mu_\nu\theta_{\nu\lambda}\hat{p}^\lambda-\frac{1}{2\hbar}\theta_{\mu\nu}\Lambda^\nu_\lambda\hat{p}^\lambda+\\ &\quad \frac{1}{4\hbar^2}\theta_{\mu\nu}\bar{\theta}^{\lambda\alpha}\Lambda^\nu_\lambda\hat{x}_\alpha), \qquad (18)\end{aligned}$$

$$\begin{aligned}\hat{p}'_\mu &= \gamma(\Lambda^\nu_\mu\hat{p}_\nu-\frac{1}{2\hbar}\Lambda^\mu_\nu\bar{\theta}_{\nu\lambda}\hat{x}^\lambda-\frac{1}{2\hbar}\bar{\theta}_{\mu\nu}\Lambda^\nu_\lambda\hat{x}^\lambda-\\ &\quad \frac{1}{4\hbar^2}\bar{\theta}_{\mu\nu}\theta^{\lambda\alpha}\Lambda^\nu_\lambda\hat{p}_\alpha). \qquad (19)\end{aligned}$$

Eqs. (18) and (19) define the non-commutative Lorentz transformations on NC phase space, the $\theta_{\mu\nu}p^\nu$ and $\bar{\theta}_{\mu\nu}x^\nu$ transform as Lorentz vectors on NC phase space. When $\bar{\theta}\to 0$, the Lorentz transformations (18) and (19) on NC phase space reduce to the Lorentz transformations on NC space. Using the inverse of the generalized Bopp's shift (14) and (15), one finds that the square of the non-commutative length on NC phase space is given by

$$s^2_{\rm ncps}=\gamma^2\left(\hat{x}^\mu\hat{x}_\mu+\frac{1}{\hbar}\theta_{\mu\nu}\hat{x}^\mu\hat{p}^\nu+\frac{1}{4\hbar^2}\theta^{\mu\alpha}\theta_{\mu\beta}\hat{p}_\alpha\hat{p}^\beta\right). \qquad (20)$$

One can check that $s^2_{\rm ncps}$ is left invariant by the NC Lorentz transformations on NC phase space.

It is straightforward to extend our results above to a Poincare transformation, since a shift by a constant of the non-commutative coordinates is compatible with the algebraic relations Eq. (1) on NC space and Eq. (4) on NC phase space. An infinitesimal non-commutative Poincare transformation $\Lambda^\mu_\nu=\delta^\mu_\nu+\omega^\mu_\nu$, $a^\mu=\epsilon^\mu$ is implemented by the operator

$$U(1+\omega,\epsilon)=1+\frac{\mathrm{i}}{2}\omega_{\mu\nu}J^{\mu\nu}-\mathrm{i}\epsilon_\mu p^\mu+\cdots \qquad (21)$$

with $J_{\mu\nu}=x_\mu p_\nu-x_\nu p_\mu$. The operator is un-deformed, because the NC Poincare transformation is induced by the Poincare transformation on commuting space. So the Lie algebra of the Lorentz group is also undeformed,

$$\begin{aligned}[J_{\mu\nu},J_{\rho\sigma}] &= -\mathrm{i}(\eta_{\nu\rho}J_{\mu\sigma}-\eta_{\mu\rho}J_{\nu\sigma}-\eta_{\sigma\mu}J_{\rho\nu}+\eta_{\sigma\nu}J_{\rho\mu}),\\ [p_\mu,J_{\rho\sigma}] &= -\mathrm{i}(\eta_{\mu\rho}p_\sigma-\eta_{\mu\sigma}p_\rho),\\ [p_\mu,p_\nu] &= 0. \end{aligned}\qquad (22)$$

One may apply our formulation to field theory. To do so, derivatives have to be defined. From Ref. [14] the derivative on NC space can be defined as

$$\begin{aligned}\hat{\partial}^x_\mu f(x,p) &= -\mathrm{i}\theta^{-1}_{\mu\nu}[\hat{x}^\nu,f(x,p)]=\\ &\quad -\mathrm{i}\theta^{-1}_{\mu\nu}\left[x^\nu-\frac{1}{2\hbar}\theta^{\nu\alpha}p_\alpha,f(x,p)\right]. \end{aligned}\qquad (23)$$

Similarly, on NC phase space we define the derivative of momentum as follows

$$\begin{aligned}\hat{\partial}^p_\mu f(x,p) &= -\mathrm{i}\bar{\theta}^{-1}_{\mu\nu}[\hat{p}^\nu,f(x,p)]=\\ &\quad -\mathrm{i}\bar{\theta}^{-1}_{\mu\nu}\left[p^\nu+\frac{1}{2\hbar}\bar{\theta}^{\nu\alpha}x_\alpha,f(x,p)\right]. \end{aligned}\qquad (24)$$

Under NC Lorentz transformations, the derivatives transform as

$$\begin{aligned}\hat{\partial}'^x_\mu &= \theta^{-1}_{\mu\alpha}\Lambda^\alpha_\beta\theta^{\beta\sigma}\hat{\partial}^x_\sigma,\\ \hat{\partial}'^p_\mu &= \bar{\theta}^{-1}_{\mu\alpha}\Lambda^\alpha_\beta\bar{\theta}^{\beta\sigma}\hat{\partial}^p_\sigma. \end{aligned}\qquad (25)$$

Now let's consider a non-commutative action for a Dirac fermion coupled to a Yang-Mills gauge field. The action in $2+1$ dimension is given by

$$S=\int\mathrm{d}^3\bar{\hat{\Psi}}(\hat{x})(\mathrm{i}\hat{\not{D}}-m)\hat{\Psi}(\hat{x})-\frac{1}{2}\int\mathrm{d}^3x\mathrm{Tr}\hat{F}_{\mu\nu}(\hat{x})\hat{F}^{\mu\nu}(\hat{x}). \qquad (26)$$

Under the NC Lorentz transformations, Ref. [8] found that the NC Yang-Mills potential transforms as

$$\hat{A}'_\mu = \theta^{-1}_{\mu\alpha}\Lambda^\alpha_\beta\theta^{\beta\sigma}\hat{A}_\sigma \ , \tag{27}$$

the NC covariant derivative transforms as

$$\hat{D}'_\mu = \theta^{-1}_{\mu\alpha}\Lambda^\alpha_\sigma\theta^{\sigma\alpha}\hat{D}_\sigma \ , \tag{28}$$

and the field strength tensor transforms as

$$\hat{F}'_{\mu\nu} = \theta^{-1}_{\mu\rho}\Lambda^\rho_\sigma\theta^{\sigma\alpha}\theta^{-1}_{\nu\zeta}\Lambda^\zeta_\eta\theta^{\eta\beta}\hat{F}_{\alpha\beta} \ , \tag{29}$$

and the NC spinor field transforms as

$$\hat{\Psi}' = \exp\left(-\frac{\mathrm{i}}{2}\omega^{\alpha\beta}S_{\alpha\beta}\right)\hat{\Psi} \ , \tag{30}$$

with $S_{\mu\nu} = \frac{\mathrm{i}}{4}[\gamma_\mu,\gamma_\nu]$. If the fields are taken in the enveloping algebra, the classical field, namely the leading order of the Seiberg-Witten map, also transforms according to Eqs. (27)–(30). Though Eqs. (27)–(30) have the same form both for $2+1$ dimensional NC space time and the $3+1$ dimensional NC space time in Ref. [8], because of the singularity of the matrix $\Theta = (\theta_{ij})$ in $3+1$ dimensional NC space-time, Eqs. (27)–(30) can not be applicable to $3+1$ dimensional NC space time. Detailed discussions will be given in the conclusion section.

To replace the noncommutative argument of the function by a commutative one, we need to introduce a star product, $f(\hat{x})g(\hat{x}) = f(x)\star g(x)$. The star product here is given in Eq. (2) for NC space and defined in Eq. (7) for NC phase space. It is easy to verify that these star product is invariant under NC Lorentz transformations. After replacing NC coordinates by the commutative ones through the star product and expanding the fields in the enveloping algebra by using Seiberg-Witten Map, Eq. (26), to first order of θ, reads

$$\begin{aligned} S = \int \mathrm{d}^3x \Big[& \bar{\psi}(\mathrm{i}\not{D} - m)\psi - \frac{1}{4}\theta^{\mu\nu}\bar{\psi}F_{\mu\nu}(i\not{D} - m)\psi - \\ & \frac{1}{2}\theta^{\mu\nu}\bar{\psi}\gamma^\rho F_{\rho\mu}\,\mathrm{i}D_\nu\psi - \frac{1}{2}\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} + \\ & \frac{1}{4}\theta^{\mu\nu}\,\mathrm{Tr}\,F_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} - \theta^{\mu\nu}\,\mathrm{Tr}\,F_{\mu\rho}F_{\nu\sigma}F^{\rho\sigma}\Big] + \mathcal{O}(\theta^2) \ , \end{aligned} \tag{31}$$

which is invariant under noncommutative Lorentz transformation.

4 Conclusions

In this paper we give a description of NC phase space, where the star product can be easily defined (see Eq. (7)). Following the Ref. [8], we define a noncommutative Lorentz transformation both on NC space and NC phase space in $2+1$ dimensions. The basic idea is to define the Lorentz transformations for commutative coordinates (9) and momenta (11) and then to feed back these transformations to the noncommutative sectors via the variable transformations (14) and (15). The algebraic relations both on NC space and NC phase space are invariant under NC Lorentz transformations, and the θ-expanded gauge field action is also invariant under NC Lorentz transformations.

In this paper though Greek indices are introduced, we assume that μ,ν run as i,j and take values from 1 to 2, because we have set $\theta_{0\mu} = 0$. Why do we consider a 2-dimensional NC space and NC phase space instead of the 3-dimensional situation. The main reason is that on 3-dimensional NC space and NC phase space the non-commutative parameters θ_{ij} and $\bar{\theta}_{ij}$ can be considered as anti-symmetric matrices, $\Theta = (\theta_{ij}), \bar{\Theta} = (\bar{\theta}_{ij})$, the elements

$$\theta_{ij} = \epsilon_{ijk}\theta_k, \quad \bar{\theta}_{ij} = \epsilon_{ijk}\bar{\theta}_k, \tag{32}$$

i.e.

$$\Theta = \begin{pmatrix} 0 & \theta_3 & -\theta_2 \\ -\theta_3 & 0 & \theta_1 \\ \theta_2 & -\theta_1 & 0 \end{pmatrix},$$

$$\bar{\Theta} = \begin{pmatrix} 0 & \bar{\theta}_3 & -\bar{\theta}_2 \\ -\bar{\theta}_3 & 0 & \bar{\theta}_1 \\ \bar{\theta}_2 & -\bar{\theta}_1 & 0 \end{pmatrix}. \tag{33}$$

It is easy to find that these two anti-symmetric matrices are singular matrices, because $\det\Theta = \det\bar{\Theta} = 0$. For these reasons we can not define the inverse of these matrices, and terms related to θ^{-1}_{ij} or $\bar{\theta}^{-1}_{ij}$ will make no sense on 3-dimensional NC space and NC phase space. This problem also exists in Ref. [8], for example, the Eqs. (17)–(21) of the Ref. [8] make little sense in $3+1$ dimensions. The NC phase space formulation and the NC Lorentz transformation of this paper can be easily extended to any even dimensional space, however, for any odd dimensional case, the anti-symmetric matrices Θ and $\bar{\Theta}$ are singular, the method here would not be applicable, which will be discussed in our forthcoming studies.

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