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Determination of the strong coupling constant using available experimental data

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Abstract We determine the strong coupling constant α_s up to 4-loop in perturbative QCD. Testing QCD requires the measurement of α_s over ranges of energy scales. In this analysis, the value of α_s is determined from the unpolarized structure functions data points by minimizing the χ^2 function between the theory result and experimental data. Using perturbative QCD calculations from threshold corrections, we obtain $\alpha_s(M_Z^2)=0.1139\pm0.0020$ at N^3LO which is in good agreement with the very recently results from the inclusive jet cross section in p$\bar{\text{p}}$ collisions at $\sqrt{s}$=1.96 TeV.

Key words strong coupling, DIS data, N^3LO

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1 Introduction

DIS processes have played and still play a very important role for our understanding of QCD and of nucleon structure [1]. In fact, DIS structure functions have been the subject of detailed theoretical and experimental investigations. Today, with high-precision data from the electronproton collider HERA and in view of the outstanding importance of hard scattering processes at proton(anti)proton colliders like the TEVATRON and the forthcoming Large Hadron Collider (LHC) at CERN, a quantitative understanding of deep-inelastic processes is indispensable. The strong coupling α_s is a fundamental parameter of the Standard Model. QCD dose not predict the actual value of α_s, however it definitely predicts the functional form of energy dependence α_s. The value of α_s, at given energy or momentum transfer scale Q, must be obtained from experiment. Determining α_s at a specific energy scale Q is therefore a fundamental measurement, to be compared with measurements of the electromagnetic coupling α_s, of the elementary electric charge, or of the gravitational constant. Testing QCD, however, requires the measurement of α_s over ranges of energy scales. In this analysis, the value of α_s is determined from the unpolarized structure functions data points by minimizing the χ^2 function between the theory result and experimental data using the Jacobi polynomials [2] and MINUIT [3] program.

2 The running coupling constant

The strong coupling constant α_s plays a more central role in the QCD analysis of parton densities in the moment space. We employ the following normalization for the strong coupling constant

$$a_s(Q^2)\equiv\alpha_s(Q^2)/4\pi. \qquad (1)$$

At N^{m}LO the scale dependence of a_s is given by

$$Q^2\frac{\partial a_s(Q^2)}{\partial Q^2}=\beta_{N^m\mathrm{LO}}(a_s)=-\sum_{k=0}^{m}\beta_k a_s^{k+2}, \qquad (2)$$

The perturbative expansion of the β function is calculated to complete up to k=3, i.e. N^3LO

$$\begin{aligned}\beta_0 &= 11-2/3f, \quad \beta_1=102-38/3f,\\ \beta_2 &= 2857/2-5033/18f+325/54f^2,\\ \beta_3 &= 29243.0-6946.30f+405.089f^2+1093/729f^3,\end{aligned} \qquad (3)$$

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where f stands for the number of effectively massless quark flavors at the energy scale Q. In complete 4-loop approximation and using the Λ-parametrization, the running coupling is given by:

$$a_s(Q^2) = \frac{1}{\beta_0 L_\Lambda} - \frac{1}{(\beta_0 L_\Lambda)^2} b_1 \ln L_\Lambda + \frac{1}{(\beta_0 L_\Lambda)^3}\left[b_1^2\left(\ln^2 L_\Lambda - \ln L_\Lambda - 1\right) + b_2\right] + \frac{1}{(\beta_0 L_\Lambda)^4}\left[b_1^3\left(-\ln^3 L_\Lambda + \frac{5}{2}\ln^2 L_\Lambda + 2\ln L_\Lambda - \frac{1}{2}\right) - 3b_1 b_2 \ln L_\Lambda + \frac{b_3}{2}\right], \quad (4)$$

where $L_\Lambda \equiv \ln(Q^2/\Lambda^2)$, $b_k \equiv \beta_k/\beta_0$, and Λ is the QCD scale parameter. The first line of Eq. (4) includes the 1- and the 2-loop coefficients, the second line is the 3-loop and the third and fourth lines denote the 4-loop correction, respectively [4]. If we be able to extract the Λ from the QCD fits, then it is possible to determine the $\alpha_s(Q^2)$. The strong coupling $\alpha_s(Q^2)$ in 4-loop and for different values of Λ=0.123, 0.223, 0.323 GeV is presented in Fig. 1.

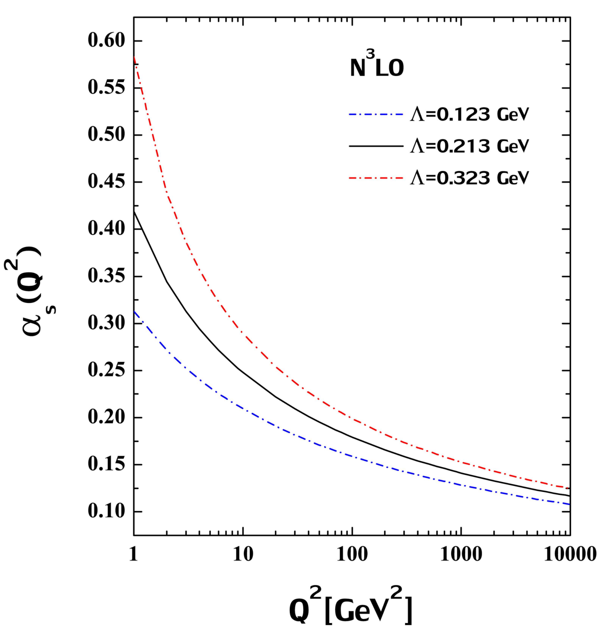

Fig. 1. (color online). The strong coupling $\alpha_s(Q)$ in 4-loop and for different values of Λ.

3 Method of the QCD fits

One of the simplest and fastest possibilities in the structure function reconstruction from the QCD predictions for its Mellin moments is Jacobi polynomials expansion [5, 6]. The Jacobi polynomials are especially suitable for this purpose since they allow one to factor out an essential part of the x-dependence of structure function into the weight function [7]. According to this method, one can relate the F_2 structure function with its Mellin moments

$$F_2^{k,N_{\max}}(x,Q^2) = x^\beta(1-x)^\alpha \sum_{n=0}^{N_{\max}} \Theta_n^{\alpha,\beta}(x) \times \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) F_2^k(j+2,Q^2), \quad (5)$$

where $N_{\max}$ is the number of polynomials, k denotes the three cases, i.e. $k = p,\ d,\ NS$. Jacobi polynomials of order n, $\Theta_n^{\alpha,\beta}(x)$, satisfy the orthogonality condition with the weight function $w^{\alpha\beta} = x^\beta(1-x)^\alpha$

$$\int_0^1 \mathrm{d}x\, w^{\alpha\beta} \Theta_k^{\alpha,\beta}(x) \Theta_l^{\alpha,\beta}(x) = \delta_{k,l}\ . \quad (6)$$

In above, $c_j^{(n)}(\alpha,\beta)$ are the coefficients expressed through $\Gamma-$ functions and satisfy the orthogonality relation in Eq. (6) and $F_2(j+2,Q^2)$ are the moments of non-singlet structure functions. $N_{\max}$, α and β have to be chosen so as to achieve the fastest convergence of the series on the right-hand side of Eq. (5) and to reconstruct F_2 with the required accuracy. In our analysis we use $N_{\max} = 9$, $\alpha = 3.0$ and $\beta = 0.5$. The same method has been applied to calculate the non-singlet structure function xF_3 from their moments [8–12], for non-singlet structure function F_2 [13–15] and for polarized structure function xg_1 [16–20]. According to this method the Q^2-dependence of the structure function is defined by the Q^2-dependence of the moments. By QCD fits of the world data for $F_2^{p,d}$, we can extract both valence quark densities and Λ using the Jacobi polynomials method [2].

For the non-singlet QCD analysis we use the structure function data measured in charged lepton proton and deuteron deep-inelastic scattering. The experiments contributing to the statistics are BCDMS [21], SLAC [22], NMC [23], H1 [24], and ZEUS [25]. In our QCD analysis we use three data samples: $F_2^p(x,Q^2)$, $F_2^d(x,Q^2)$ in the non-singlet regime and the valence quark region $x \geqslant 0.3$ and $F_2^{NS} = 2(F_2^p - F_2^d)$ in the region $x < 0.3$. In Fig. 2 the proton, deuteron and non-singlet data for $F_2^p(x,Q^2)$, $F_2^d(x,Q^2)$ and $F_2^{NS}(x,Q^2)$ are shown in the non-singlet regime and the valence quark region $x \geqslant 0.3$. The solid lines correspond to the N^3LO QCD fit. In the non-singlet QCD analysis we extract the strong coupling constant α_s in terms of four massless flavors determining $\Lambda_{\rm QCD}$. Up to N^3LO results fitting the data, are

$$\Lambda_{\rm QCD}^{(4)} = 213.2 \pm 28\ \text{MeV},\ \text{LO},$$

$$\begin{aligned}
\Lambda_{\rm QCD}^{(4)} &= 263.8 \pm 30\ {\rm MeV},\ N{\rm LO}, \\
\Lambda_{\rm QCD}^{(4)} &= 239.9 \pm 27\ {\rm MeV},\ N^2{\rm LO}, \\
\Lambda_{\rm QCD}^{(4)} &= 241.4 \pm 29\ {\rm MeV},\ N^3{\rm LO}. \qquad (7)
\end{aligned}$$

These results can be expressed in terms of $\alpha_{\rm s}(M_{\rm Z}^2)$:

$$\begin{aligned}
\alpha_{\rm s}(M_{\rm Z}^2) &= 0.1281 \pm 0.0028,\ {\rm LO}, \\
\alpha_{\rm s}(M_{\rm Z}^2) &= 0.1149 \pm 0.0021,\ N{\rm LO}, \\
\alpha_{\rm s}(M_{\rm Z}^2) &= 0.1131 \pm 0.0019,\ N^2{\rm LO}, \\
\alpha_{\rm s}(M_{\rm Z}^2) &= 0.1139 \pm 0.0020,\ N^3{\rm LO}. \qquad (8)
\end{aligned}$$

Note that in above results we use the matching between n_f and n_{f+1} flavor couplings [26]. To be capable to compare with other measurement of $\Lambda_{\rm QCD}$ we adopt this prescription. The N^3LO values of $\alpha_{\rm s}(M_{\rm Z}^2)$ can also be compared with results from other QCD analysis $\alpha_{\rm s}(M_{\rm Z}^2) = 0.1134^{+0.0019}_{-0.0021}$ [27] and with the value of the world average 0.1189 ± 0.0010 [28], while the world average of $\alpha_{\rm s}(M_{\rm Z}^2) = 0.1184 \pm 0.0007$ has been reported very recently [4].

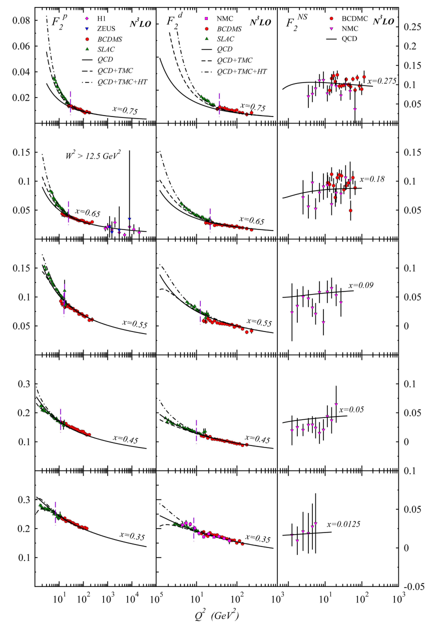

Fig. 2. (color online). The structure functions F_2^p, F_2^d, and F_2^{NS} as a function of Q^2 in intervals of x. The vertical dashed lines indicate the regions with $W^2 > 12.5$ GeV^2 [2].

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