

Electromagnetic processes in a χ EFT framework^{*}

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Abstract Recently, we have derived a two-nucleon potential and consistent nuclear electromagnetic currents in chiral effective field theory with pions and nucleons as explicit degrees of freedom. The calculation of the currents has been carried out to include N³LO corrections, consisting of two-pion exchange and contact contributions. The latter involve unknown low-energy constants (LECs), some of which have been fixed by fitting the np *S*- and *P*-wave phase shifts up to 100 MeV lab energies. The remaining LECs entering the current operator are determined so as to reproduce the experimental deuteron and trinucleon magnetic moments, as well as the np cross section. This electromagnetic current operator is utilized to study the nd and n³He radiative captures at thermal neutron energies. Here we discuss our results stressing on the important role played by the LECs in reproducing the experimental data.

Key words chiral effective field theory, nuclear electromagnetic currents

PACS 13.40.-f, 21.10.Ky, 25.40.Lw

Quantum chromodynamics (QCD) is the underlying theory of the strong interaction. On this basis, interactions among the relevant degrees of freedom of nuclear physics, such as pions, nucleons, and delta-isobars, are completely determined by the quark and gluon dynamic. At low energies though, the strong coupling constant becomes too large to allow for application of perturbative techniques to solve QCD. Consequently, we are still far from a quantitative understanding of the low-energy physics by ab initio calculations from QCD. Chiral effective field theory (χ EFT) exploits the symmetries exhibited by QCD in the low-energy regime, in particular chiral symmetry, to constrain the form of the interactions of the pions among themselves and with the other degrees of freedom [1]. The pion couples by powers of its momentum Q and the Lagrangians describing these interactions can be expanded in powers of Q/Λ_χ , where $\Lambda_\chi \sim 1$ GeV represents the chiral-symmetry breaking scale and characterizes the convergence of the expansion.

The effectiveness of the theory is then confined to kinematic regions where the constraint $Q \ll \Lambda_\chi$ is realized. The unknown coefficients of the chiral expansion, i.e. the low energy constant (LECs), need to be fixed by comparison with the experimental data. χ EFT provides an expansion of the Lagrangians in powers of a small momentum as opposed to an expansion in the strong coupling constant, restoring de facto the applicability of perturbative techniques also in the low-energy regime. Due to the chiral expansion it is possible, in principle, to evaluate an observable to any degree of desired accuracy and to know a priori the hierarchy of interactions contributing to the low energy process under study.

Since the pioneering work of Weinberg [2], this calculational scheme has been widely utilized in nuclear physics and nuclear χ EFT has developed into an intense field of research. Nuclear two- and three-body interactions [3], as well as interactions of electroweak probes with nuclei [4, 5] have been studied within the

Received 19 January 2010

^{*} DOE DE-AC05-06OR23177

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χ EFT approach.

Recently, we have derived the nuclear electromagnetic (EM) currents in χ EFT [6, 7], retaining, as degrees of freedom, pions and nucleons. The calculation has been carried out in time-ordered perturbation theory [6] with non-relativistic Hamiltonians derived from the chiral Lagrangians of Refs. [2, 8, 9]. The strong and electromagnetic interaction Hamiltonians required to evaluate the EM current operator up to N^3LO accuracy—that is eQ in the chiral expansion, Q denoting the low momentum scale, and e being the electric charge—are listed in Refs. [6, 7].

In Fig. 1 we show the contributions to the current operator up to N^2LO (eQ^0). The LO (eQ^{-2}) term is given by a one-body contribution, consisting of the standard convection and spin-magnetization nucleon currents, while pion-exchange currents occur at NLO (eQ^{-1}). The N^2LO term is due to $(Q/M)^2$ relativistic corrections—where M denotes the nucleon mass—to the LO one-body current.

In Fig. 2 we list the N^3LO contributions, which can be separated into three classes: i) one-loop two-pion exchange terms, represented by diagrams (a)-(i); ii) tree-level term involving the nuclear-electromagnetic Hamiltonian of order eQ^2 at the vertex illustrated by a full circle in diagram (j); and iii) contact currents of minimal and non-minimal nature, illustrated by diagram (k).

The last two contributions involve unknown LECs. In particular, the tree-level current of the type shown in panel (j), depends on three LECs, two of them multiply isovector structures and the remaining one multiplies an isoscalar structure. Incidentally, the

isovector part of this tree-level current has the same structure as the current involving the excitation of a delta-isobar [6]. This resonance saturation argument is exploited to infer the ratio between the two LECs multiplying the isovector terms in the current of diagram (j) (see below). Contact currents of non-minimal character, panel (k) in Fig. 1, depend on two additional unknown LECs, multiplying respectively an isoscalar and an isovector structure, while those obtained via minimal substitution are expressed in terms of LECs entering the contact two-nucleon chiral potential of order Q^2 (or N^2LO) [7]. The two-nucleon potential has been derived in Ref. [7] up to N^2LO and these LECs have been fixed by fitting the np S - and P -wave phase shifts up to 100 MeV laboratory energies [7]. Thus total number of unknown LECs to be determined is reduced to four.

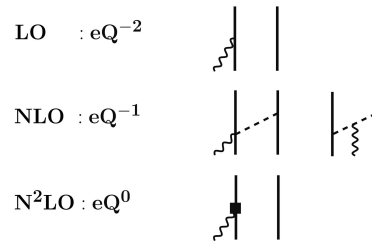


Fig. 1. Diagrams illustrating one- and two-body currents up to N^2LO (eQ^0). Nucleons, pions, and photons are denoted by solid, dashed, and wavy lines, respectively. The square represents the relativistic correction to the LO one-body current. Only one among the possible time orderings is shown for the NLO diagrams.

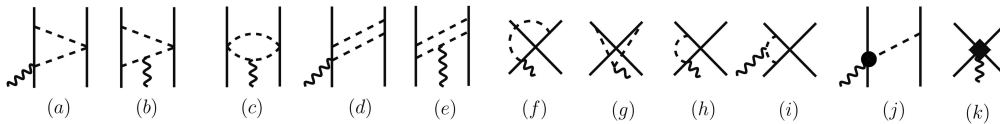


Fig. 2. Diagrams illustrating two-body currents entering at N^3LO (eQ), notation as in Fig. 1. Only one among the possible time orderings is shown for diagrams (a)-(j).

An important aspect of the derivation of the EM currents (and two-nucleon potential) is to retain both irreducible diagrams and recoil-corrected reducible ones [6]. The latter arise from expanding the energy denominators (in reducible diagrams) in powers of nucleon kinetic energy differences to pion energies (these ratios are of order Q). Partial cancellations occur between the irreducible and recoil-corrected reducible contributions both at N^2LO and N^3LO [6]. We also note that this approach leads to N^3LO EM

currents that satisfy the continuity equation with the corresponding N^2LO two-body potential [6]. The expressions for the two-pion-exchange N^3LO currents in panels (a)-(i) of Fig. 2 are in agreement with those obtained by Kölling et al. in Ref. [10] by the method of the unitary transformations. However, they are different from those derived by Park et al. in Ref. [5] in covariant perturbation theory, since these authors include irreducible contributions only.

We now present a study of the nd and $n^3\text{He}$ ra-

diative capture at thermal neutron energies within the hybrid approach, where the EM χ EFT current operator described above is used to evaluate transition matrix elements between nuclear wave functions obtained with realistic Hamiltonian with two- and three-body potentials. In order to study the model dependence of the calculated observables, we use two different combinations of two- and three-body potential, namely the Argonne v_{18} [11] with the Urbana-IX[12] three-nucleon potential (AV18/UIX), and the N^3LO [13] and N^2LO [14] chiral two- and three-nucleon potentials ($N3LO/N2LO$). We study the sensitivity of the observables to variations of the cutoff Λ , introduced to regularize the EM current operator via the momentum cutoff $C_\Lambda(k) = \exp(-k^4/\Lambda^4)$. In our study, Λ varies from 500 to 700 MeV which corresponds to “removing” short-range physics at distance scales less $1/(3m_\pi)$.

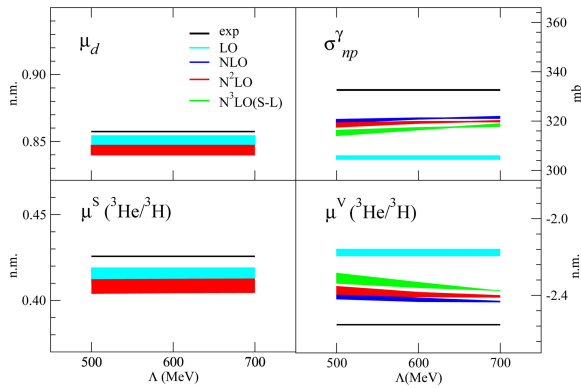


Fig. 3. (color online). Cumulative LO , NLO , N^2LO , and $N^3LO(S-L)$ contributions for the deuteron and trinucleon isoscalar and isovector magnetic moments, and np radiative capture.

Out of the four unknown LECs entering the EM current operator, two multiply isoscalar structures and two multiply isovector operator structures. We fix these LECs by reproducing the experimental values of two isoscalar observables, i.e. the deuteron $[\mu_d]$ and the isoscalar $[\mu^S(^3\text{He}/^3\text{H})]$ combination of the trinucleon magnetic moments, and two isovector observables, i. e. the isovector $[\mu^V(^3\text{He}/^3\text{H})]$ combination of the trinucleon magnetic moments and the np cross section $[\sigma_{np}^\gamma]$ at thermal neutron energies. The results are shown in Fig. 3 where the cumulative contributions at LO , NLO , N^2LO , and $N^3LO(S-L)$ are represented. The cumulative contribution $N^3LO(S-L)$ is given by the terms up to N^2LO plus the N^3LO contributions associated with pion loops (represented in panels (a)-(i) of Fig. 2), which depend on the (known) nucleon axial coupling constant, pion decay

amplitude, and pion mass, as well as with contact currents, which depend on the LECs obtained from the fits to the np phase shifts.

The LECs entering the complete current, denoted in what follows as $N^3LO(LECs)$, are fixed, for each value of the cutoff Λ , so as to reproduce the experimental values which in Fig. 3 are represented by the black band, including experimental errors. The sensitivity of the results to the two Hamiltonian models utilized (AV18/UIX and the $N3LO/N2LO$) is represented by the thickness of the color bands. We note that the sign of the N^2LO and $N^3LO(S-L)$ contributions is opposite to that of the LO and NLO contributions. This increases the discrepancy between theory and experiment.

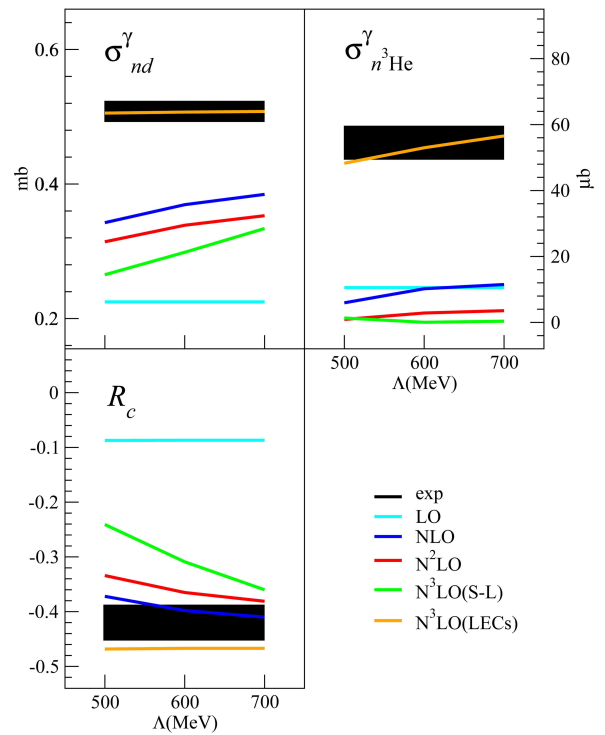


Fig. 4. (color online). Cumulative LO , NLO , N^2LO , $N^3LO(S-L)$, and $N^3LO(LECs)$ contributions to the nd (σ_{nd}^γ) and $n^3\text{He}$ ($\sigma_{n^3\text{He}}^\gamma$) cross sections (right and left top panel respectively), and circular polarization factor R_c .

Having fixed all the LECs, we are left with a completely determined EM current operator which can now be used to make predictions for the $n(d, \gamma)^3\text{H}$ and $n(^3\text{He}, \gamma)^4\text{He}$ reactions’ cross sections—denoted as σ_{nd}^γ and $\sigma_{n^3\text{He}}^\gamma$ respectively—and the circular polarization factor R_c associated with the capture of polarized neutrons on deuterons. In this calculation we have used the AV18/UIX ($N3LO/N2LO$) combination of two- and three-nucleon potentials for the $A=3$ ($A=4$) processes; calculations with the $N3LO/N2LO$

(AV18/UIX) potential models are in progress. The predictions are represented in Fig. 4 along with the experimental data, shown in black, which are from Ref. [15] for nd and Ref. [16] for $n^3\text{He}$. The complete $N^3\text{LO}$ (LECs) current is shown in Fig. 4 by the orange lines. The calculated nd cross section is in excellent agreement with the measured value and is weakly dependent on the cutoff. The cross section for the $n^3\text{He}$ reaction undergoes a 5% variation when the cutoff changes from 500 to 700 MeV, but is still within the experimental error band. These reactions are known to be dominated by many-body components of the current operator, which provide most of the calculated cross section [17]. This trend is confirmed here: the LO contribution to the cross sections is highly suppressed, and provides only about 46% (18%) of the total calculated nd ($n^3\text{He}$) value. What is more interesting though, is the large contribution associated with the $N^3\text{LO}$ (LECs) currents in both these reactions. These currents are crucial for bringing theory into agreement with experiment.

We are presently in the process of extending these hybrid studies to different realistic Hamiltonian mod-

els, with the goal of quantifying the sensitivity of the cross sections to the wave functions employed in the calculations. Obviously, our ultimate objective is to perform a fully consistent χ EFT calculation, using the $N^2\text{LO}$ potential derived in Ref. [7], along with the EM currents we presented here. In Ref. [7] we show the deuteron wave functions obtained with the $N^2\text{LO}$ chiral potential and compare them with those corresponding to the AV18. The two sets of wave functions display a different behavior at short range, in particular the $N^2\text{LO}$ D-wave component is significantly smaller than the AV18. From this perspective, it will be interesting to establish whether these chiral potential and currents lead to a satisfactory description of the nd and $n^3\text{He}$ captures.

We thank J.L. Goity and R.B. Wiringa for useful discussions. The work of R.S. is supported by the U.S. Department of Energy, Office of Nuclear Physics, under contracts DE-AC05-06OR23177. Some of the calculations were made possible by grants of computing time from the National Energy Research Supercomputer Center.

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