Constraints of gravitational baryo/leptogenesis on Cardassian expansion

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Abstract: In this paper, proceeding from the relation between the Cardassian model and the accelerated expansion of the universe, adopting a parametric method which does not depend on a precise mechanism for gravitational baryo/leptogenesis and using the model parameter of CPT-violating interaction, we study the role of the modified Friedmann equation which plays a role in the matter asymmetry of the early epoch and the accelerated expansion of the present universe. Thus the appropriate Cardassian component in the radiation-dominated era or in the matter-dominated universe can be obtained. The results indicate that early CPT-violation is included in the Cardassian term. In the same way, the present Cardassian term that belongs to a quintessence-like model can drive the universe towards a flat, matter-dominated and accelerating expansion.

Key words: baryo/leptogenesis, cardassian, parametric method

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1 Introduction

The measurement of baryon-to-entropy ratio with $n_{\rm B}/s \sim 10^{-10}$ has revealed the mystery of baryon asymmetry in the universe. Generally, this is considered to be generated from an initial symmetric phase as long as the three conditions Sakharov argued [1] are satisfied: (1) baryon number non-conserving interactions; (2) C and CP violations; (3) out of thermal equilibrium. In an expanding universe, however, when the CP/CPT-violating interaction $(\partial_\mu Q)\,J^\mu$ is non-zero (where Q denotes the scalar field, J^μ can be any current which yields a net B-L charge in thermal equilibrium, and B and L stand for baryon and lepton number, respectively), the asymmetry can be generated while maintaining thermal equilibrium. For example, the gravitational interaction is

$$\frac{1}{M_*^2}\int {\rm d}^4x\sqrt{-g}\,(\partial_\mu R)\,J^\mu \tag{1}$$

which gives a baryon asymmetry [2]

$$\frac{n_{\rm B}}{s} \sim \left.\frac{\dot{R}}{M_*^2 T}\right|_{T_{\rm D}}, \tag{2}$$

where R is the Ricci scalar curvature, M_* is the cut-off scale of the effective theory, $T_{\rm D}$ denotes the temperature at which B-violation decouples, and a dot means time derivative. For a constant equation of state ω in $\dot{R} = \sqrt{3}\,(1-3\omega)\,(1+\omega)\,(8\pi G\rho)^{3/2}$ which shows that in the radiation-dominated era following inflation $\omega \approx 1/3$ seems not to be suitable for baryo/leptogenesis as the right-hand side of Eq. (2) would vanish (where ω denotes the ratio of the pressure P to the energy density ρ). Some possibilities for a non-vanishing $\dot{R}$ during that period have been proposed in Refs. [2–11].

On the other hand, in order to explain the nature of the present accelerated expansion of the universe, dark energy has been one of the most active fields in modern cosmology [12]. Unlike dark matter, dark energy is supposed to be distributed smoothly, exhibits a repulsive effect, and becomes significant on a very large scale [13]. The models of dark energy in the literature can be classified into three groups: (1) some new form of particle; (2) some form of modified gravity; (3) a departure from the cosmological principle. As is well known, the most plausible dark energy candidate is vacuum energy [14, 15]; however, the cosmological constant, acting as a dark energy density, suffers from the cosmological constant problem and the “coincidence problem”. Rather than dealing directly with the vacuum energy, a wide variety of

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scalar field models of dark energy have been exploited to achieve late-time acceleration, such as quintessence [16, 17], k-essence [18,19], phantom [20], chameleon [21] and chaplygin gas [22] etc. Meanwhile, as an alternative approach to explain the acceleration, Freese and Lewis [23] proposed a model of modification to the Friedmann equation which invokes no vacuum energy or any other dynamical components, the so-called Cardassian expansion. In their model, the universe has a flat geometry (k=0) and consists only of matter and radiation. Pure matter (or radiation) alone can drive an accelerated expansion if the Friedmann equation is modified from its usual form ($\Lambda=0$)

$$H^2=\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\rho \tag{3}$$

to

$$H^2=\frac{8\pi G}{3}g(\rho), \tag{4}$$

where H is the Hubble constant, $a(t)$ is the scale factor of the universe, and the energy density ρ contains only ordinary matter and radiation. Also, g is as a function of ρ which returns to ρ at early epochs but takes a different form that gives rise to an accelerated expansion in the recent past of the universe at $z<O(1)$. In Cardassian theory [24], the total energy density of the matter and radiation $\tilde{\rho}$ is

$$\tilde{\rho}=g(\rho). \tag{5}$$

The energy conservation equation in the FRW model is

$$\dot{\tilde{\rho}}=-3H\left(\tilde{\rho}+\tilde{P}\right), \tag{6}$$

where $\tilde{P}$ denotes the pressure of the universe. The time evolution of the Hubble parameter is given by

$$\dot{H}=-4\pi G\left(\tilde{\rho}+\tilde{P}\right), \tag{7}$$

where the pressure term

$$\tilde{P}=-\tilde{\rho}+\rho_{\mathrm{m}}\frac{\partial\tilde{\rho}}{\partial\rho_{\mathrm{m}}}+\frac{4}{3}\rho_{\gamma}\frac{\partial\tilde{\rho}}{\partial\rho_{\gamma}}, \tag{8}$$

where ρ_{γ} is the energy density of the radiation, and ρ_{m} is the energy density of ordinary matter. Eq. (4) can be written as

$$H^2=\frac{8\pi G}{3}\left[\rho+f(\rho)\right]=x. \tag{9}$$

The extra term $f(\rho)\equiv g(\rho)-\rho$ denotes the Cardassian component. Based on the above equations, there have been several Cardassian models such as the power law Cardassian, the modified polytropic model, and the exponential Cardassian universe in the literature [23–26]. For any suitable Cardassian model, there exist at least three requirements that should be satisfied [25]. First, the function $g(\rho)$ should return to the usual form ρ at early epochs in order to recover the thermal history of the standard cosmological model and the scenario for the formation of large-scale structure. Second, $g(\rho)$ should take a different form at a later time $z\sim O(1)$ in order to drive an accelerated expansion as indicated by the observations. Finally, the classical solution of the expansion should be stable. i.e., the sound speed c_{s}^2 of the classical perturbations of the total cosmological fluid around homogeneous FRW solutions cannot be negative. The required new term may arise as a consequence of our observable universe existing as a three-dimensional brane in a higher-dimensional universe, or due to a new fluid with negative pressure contributions [24, 26].

Although there have been many theoretical possibilities for gravitational baryo/leptogenesis in the early universe, the precise mechanism of obtaining a non-vanishing $\dot{R}$ is still uncertain. This makes it even harder to understand the origin of matter asymmetry. To produce a non-zero $\dot{R}$ in the radiation-dominated era, one has to turn to a modified gravity. In this vein, it is interesting to adopt a parametric method of the CPT-violating interaction in the Cardassian expansion, which does not refer to a detailed mechanism of $\dot{R}\neq 0$. For a given model parameter, for different epochs one may find the corresponding Cardassian component, in which the gravitational baryo/leptogenesis and the current expansion may both be described by a Cardassian model.

In this paper, we study a Cardassian modification that not only causes the current accelerated expansion, but also produces the required matter asymmetry by breaking CPT in the early universe. Using a parametric method in which the CPT-violating interaction appears as a model parameter at different epochs, where the corresponding Cardassian term may come to be spontaneous, it then represents some constraints on the Cardassian expansion.

2 Parametric method of *CPT*-violation in Cardassian model

In this section we start from the fundamental equation (9), and we can obtain the differential equation that contains CPT-violation where $\dot{R}$ is observationally characteristic of a different era based upon the FRW metric. Conveniently, the first derivative of $x=H^2$ with respect to the cosmic time t is

$$\dot{x}=2H\dot{H}, \tag{10}$$

and

$$\dot{H}=\frac{\ddot{a}}{a}-H^2. \tag{11}$$

Then

$$\frac{\ddot{a}}{a}=\frac{\dot{x}}{2\sqrt{x}}+x. \tag{12}$$

Thus the Ricci scalar curvature is

$$\begin{aligned}R &= -6\frac{\ddot{a}}{a}-6\left(\frac{\dot{a}}{a}\right)^2\\ &= \frac{-3\dot{x}}{\sqrt{x}}-12x.\end{aligned} \tag{13}$$

It is now easy to write down the time derivative of R:

$$\dot{R}=\frac{3}{2}x^{-3/2}\left(\dot{x}\right)^2-3x^{-1/2}\ddot{x}-12\dot{x}=K. \tag{14}$$

This is the gravitational CPT-violating interaction. Here the parameter K in our model is given as follows: (1) In the radiation-dominated era K is non-zero, therefore the interaction in Eq. (1) gives opposite-sign energy contributions that differ for particles and antiparticles, and therefore dynamically violates CPT [2]; (2) at present, experimental CPT results indicate that the CPT-violation effect in laboratory experiments is sufficiently tiny [27], thus the CPT-violating interaction becomes $K\simeq 0$ [7]. The key problem lies in solving Eq. (14). For the purpose of baryo/leptogenesis in the Cardassian expansion, let

$$u=\dot{x}, \tag{15}$$

and

$$\dot{u}=u\frac{\mathrm{d}u}{\mathrm{d}x}. \tag{16}$$

So Eq. (14) becomes

$$\frac{\mathrm{d}u}{\mathrm{d}x}-\frac{u}{2x}+\frac{K\sqrt{x}}{3u}+4\sqrt{x}=0 \tag{17}$$

or

$$\frac{\mathrm{d}u^2}{\mathrm{d}x}-\frac{u^2}{x}+8\sqrt{u^2x}+\frac{2K}{3}\sqrt{x}=0. \tag{18}$$

First, the solution of

$$\frac{\mathrm{d}\left(u^{(0)}\right)^2}{\mathrm{d}x}-\frac{\left(u^{(0)}\right)^2}{x}+8\sqrt{\left(u^{(0)}\right)^2x}=0 \tag{19}$$

is

$$\left(u^{(0)}\right)^2=\left(-4x+\mathrm{Con}\right)^2x, \tag{20}$$

where $u^{(0)}$ is connected to the standard FRW cosmology ($K=0$),

$$u^{(0)}=\dot{x}^{(0)} \tag{21}$$

and

$$\dot{x}=\dot{x}^{(0)}+\dot{x}'=u^{(0)}+u', \tag{22}$$

where u' comes from the Cardassian term x', Con is the constant of the integral. Eq. (17) then gives

$$\frac{\mathrm{d}u'}{\mathrm{d}x}-\frac{u'}{2x}+\frac{K\sqrt{x}}{3\left(u^{(0)}+u'\right)}=0. \tag{23}$$

One can find the above solution simply by using the constant-change method, i.e.,

$$u'=m(x)\sqrt{x}, \tag{24}$$

where $m(x)$ satisfies

$$\frac{\mathrm{d}m}{\mathrm{d}x}+\frac{K}{3\left(u^{(0)}+u'\right)}=0, \tag{25}$$

which shows that the time evolution of the Cardassian component x' is closely related to the CPT-violating interaction K. Hence, on the basis of Eq. (25) we may find the appropriate Cardassian term u' in the modified Friedmann equation which is observationally required both for the early universe and the present time.

3 Cardassian component in different epochs

(1) In the early universe, the class of first-order differential equation of Eq. (25) can be simplified, provided that the CPT-violating interaction K is non-zero due to the Cardassian component f, when

$$x^{(0)}=\frac{8\pi G}{3}\rho_\gamma. \tag{26}$$

In the standard FRW framework, we have

$$\dot{x}^{(0)}=-\frac{1}{2}t^{-3}=-4\left(x^{(0)}\right)^{3/2}, \tag{27}$$

which means Con=0 in Eq. (20). Generally, that is

$$u^{(0)}=-4x^{3/2}. \tag{28}$$

Since $x^{(0)}\gg x'$ we may assume $\left|u^{(0)}\right|\gg|u'|$, and then Eq. (25) becomes to be a simpler form:

$$\frac{\mathrm{d}m}{\mathrm{d}x}\simeq\frac{K}{12}x^{-3/2}, \tag{29}$$

which leads to

$$u'\simeq-\frac{K}{6}+C_1\sqrt{x}. \tag{30}$$

During early inflation, where $K=0$ we should have $u'=0$ and $C_1=0$. The time evolution of the Hubble parameter is then

$$\frac{\mathrm{d}x}{\mathrm{d}t}\simeq-\frac{K}{6}-4x^{3/2}, \tag{31}$$

$$\dot{H}\simeq-\frac{K}{12H}-2H^2, \tag{32}$$

and

$$R=-6\dot{H}-12H^2\simeq\frac{K}{2H}. \tag{33}$$

The Cardassian contribution is

$$x'=-\frac{K}{6}t, \tag{34}$$

where

$$\omega = \frac{1}{3} + \frac{K}{18H^2}, \tag{35}$$

where R and ω are not precisely equal to 0 and 1/3 when the CPT-violation comes to be a Cardassian component in the expanding universe. Note that the Cardassian term x' of Eq. (34) remains small relative to $x^{(0)}$ of Eq. (26). In other words, it does not dominate cosmic expansion during the early epoch; the thermal history of the standard cosmological model and the scenario for the formation of large-scale structure are almost unchanged. That does not conflict with the first requirement of the Cardassian model in Ref. [25].

(2) In the same way, for the present time one has

$$x^{(0)} = \frac{8\pi G}{3}\rho_{\mathrm{m}}. \tag{36}$$

So

$$\dot{x}^{(0)} = -\frac{8}{9}t^{-3} = \left(-4x^{(0)} + \mathrm{Con}\right)\sqrt{x^{(0)}}, \tag{37}$$

which means $\mathrm{Con} = x^{(0)}$ in Eq. (20). At this time,

$$u' = C_2\sqrt{x}. \tag{38}$$

Similarly, we can write

$$u = C_2 x^{1/2} + (-4x + \mathrm{Con})\, x^{1/2}, \tag{39}$$

$$\dot{H} = -2\left(H^2 - \frac{C_2 + \mathrm{Con}}{4}\right), \tag{40}$$

$$\omega = \frac{1}{3} - \frac{C_2 + \mathrm{Con}}{3H^2}. \tag{41}$$

According to $-1.38 < \omega < -0.82$ [28] one gets $3.46 < (C_2 + \mathrm{Con})/H^2 < 5.14$ from Eq. (41), and then

$$H = \sqrt{C_3}\frac{1 + C\exp\left(-4\sqrt{C_3}t\right)}{1 - C\exp\left(-4\sqrt{C_3}t\right)}, \tag{42}$$

where $C_3 = (C_2 + \mathrm{Con})/4$. Moreover, when $C_2 = 0$ we would have $\sqrt{C_3} = H_0^{(0)}/2$ and Eq. (42) should return to $H = H_0^{(0)}$. Here, taking $t_0 = 2/\left(3H_0^{(0)}\right)$ within the standard framework [12], one gets $C = \exp(4/3)/3$. Now from Eq. (42) we obtain

$$\tilde{\rho}(t) = \frac{3C_3}{8\pi G}\left[\frac{1 + C\exp\left(-4\sqrt{C_3}t\right)}{1 - C\exp\left(-4\sqrt{C_3}t\right)}\right]^2. \tag{43}$$

This means that if $C_2 \neq 0$ the relation between time and $g(\rho_{\mathrm{m}})$ includes $\dot{\tilde{\rho}} < 0$ and $\ddot{\tilde{\rho}} > 0$. Where the function $g(\rho) = \tilde{\rho}$ takes a different form at $z \sim O(1)$ at present, it conforms to the second requirement in Ref. [25]. In such a case, note that the speed of sound is given by [24]:

$$c_{\mathrm{s}}^2 = \left(\frac{\partial \tilde{P}}{\partial \tilde{\rho}}\right)_{\mathrm{s}} = \rho_{\mathrm{m}}\left(\frac{\partial^2 \tilde{\rho}}{\partial \rho_{\mathrm{m}}^2}\right)_{\mathrm{s}} \Big/ \left(\frac{\partial \tilde{\rho}}{\partial \rho_{\mathrm{m}}}\right)_{\mathrm{s}}, \tag{44}$$

where the pressure term

$$\tilde{P} = -\tilde{\rho} + \rho_{\mathrm{m}}\frac{\partial \tilde{\rho}}{\partial \rho_{\mathrm{m}}}. \tag{45}$$

Hence,

$$\frac{\partial \tilde{\rho}}{\partial \rho_{\mathrm{m}}} = \frac{\mathrm{d}\tilde{\rho}}{\mathrm{d}t}\cdot\frac{\mathrm{d}t}{\mathrm{d}\rho_{\mathrm{m}}} > 0 \tag{46}$$

and

$$\frac{\partial^2 \tilde{\rho}}{\partial \rho_{\mathrm{m}}^2} = \frac{\mathrm{d}^2\tilde{\rho}}{\mathrm{d}t^2}\cdot\left(\frac{\mathrm{d}t}{\mathrm{d}\rho_{\mathrm{m}}}\right)^2 > 0. \tag{47}$$

This is the speed of sound in this model which is positive $c_{\mathrm{s}}^2 > 0$ on a large scale ($C_2 \neq 0$), thus the third requirement in Ref. [25] is satisfied. Finally, from the result of Eq. (42) we can find another constraint on the Cardassian expansion. When H is treated within the limit of

$$H^2 \sim C_3, \tag{48}$$

Eq. (41) gives

$$\omega \sim -1, \tag{49}$$

and in general terms of H we have

$$\omega > -1, \tag{50}$$

which shows that our Cardassian model based on gravitational CPT-violation can be regarded as a quintessence-like model [29].

4 Conclusions

We propose in this paper a framework for gravitational baryo/leptogenesis where CPT-violation lies in the Cardassian modification, using a parametric method in which the model parameter represents the CPT-violating interaction. With the development of cosmic structure the violating interaction becomes frozen in an expanding universe following the radiation-dominated era; the Cardassian component can then give a flat, matter-dominated and accelerating expansion that does not conflict with any of the observational requirements. On the basis of the results of Eq. (34) and Eq. (42) we find two phenomenological constraints on the Cardassian model: (1) in the early universe, the ratio of the Cardassian term and the cosmic time is closely related to the required CPT-violation, hence baryon asymmetry; (2) at the present time, the accelerated expansion of the universe is driven by a qualified Cardassian component that belongs to a quintessence-like model.

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