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Single-bunch emittance dilution in the perfect ILC main linac*

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Abstract: In the ILC (International Linear Collider) main linac, low emittance preservation is the most important issue for beam dynamics study. As the main sources of emittance dilution, the dispersive and wakefield effects were studied in this paper. The theoretical calculations and numerical simulations of these two effects on single-bunch emittance dilution, without any misalignment errors, are presented in detail.

Key words: ILC, main linac, single-bunch emittance dilution

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1 Introduction

It is well known that ultra-low vertical emittance is one of the key parameters for the future of linear colliders. The main components related to the emittance are: damping rings to generate the very low emittance for electron and positron beams, bunch compressors to decrease the length of the particle bunches, main linacs to accelerate the beams and final focus regions to demagnify the beams to very small sizes.

After the beams are generated in the damping rings, they need to be accelerated in the linacs to the desired final energy while preserving a small beam emittance. As the longest linear part, the main linac is a significant subsystem concerning the issue of emittance preservation and much work has been dedicated to the topic [1–8].

The ILC main linac is composed of superconducting cavities and FODO cells where the normalized focusing strength, the cell length and the quadrupole length are kept constant [9]. Such a lattice has a constant phase advance per cell and a constant beta function, which is different from most room-temperture linacs. In this paper we will study the natural emittance dilutions for the ILC main linac, assuming all the elements are well aligned. Natural dilution means that, once it occurs, it can not be recovered by any correction method. So we aim to limit emittance dilution by considering the perfect linac. We will break the natural dilutions into dispersive effect and wakefield effect, which are the two dominant sources of transverse emittance dilution.

2 Theoretical analysis of single-bunch emittance dilution

In high energy linear accelerators, the longitudinal position is relatively fixed. Since the transverse motion does not affect the longitudinal one, we can parameterize the transverse motion with the longitudinal coordinates. Thus, the equation of motion in linear accelerator can be expressed as:

$$\frac{1}{\gamma(s)}\frac{\mathrm{d}}{\mathrm{d}s}\gamma(s)\frac{\mathrm{d}}{\mathrm{d}s}y(s;z,\delta)-(1-\delta)K_1[y(s;z,\delta)-y_{\mathrm{q}}]$$
$$=(1-\delta)G_y+\frac{(1-\delta)Ne^2}{m_0\gamma(s)}\int_z^{\infty}\mathrm{d}z'\int_{-\infty}^{\infty}\mathrm{d}\delta'\rho(z',\delta')W_{\perp 1}$$
$$\times(s;z'-z)[y(s;z',\delta')-y_{\mathrm{a}}], \quad (1)$$

where K_1 and G_y are the normalized focusing and bending strengths, and $W_{\perp 1}$ is the transverse dipole wakefield. The short-range dipole wakefield for ILC superconducting cavities is shown in Fig. 1 [10]. In addition, y_{q} and y_{a} are the misalignments of the quadrupoles and the accelerating structures, and m_0 is the static energy of electrons.

We have to point out that the energy spread consists of an uncorrelated energy spread and a correla-

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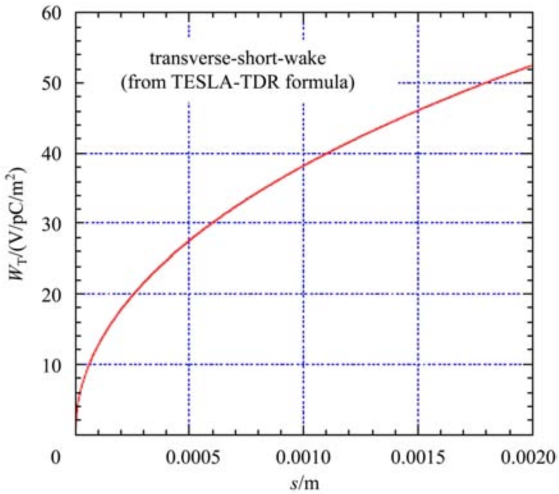

Fig. 1. The dipole short-range transverse wakefield for ILC superconducting cavities.

ted energy spread. We will only consider the uncorrelated energy spread, which is inevitable for natural dilution while neglecting the correlated energy spread.

2.1 Dispersive effect

Before the calculation of dispersive emittance growth, it is worth estimating the degree of filamentation in the ILC main linac.

$$\delta\nu = \frac{1}{4\pi}\int_0^L \delta K_1\beta \mathrm{d}s = \frac{1}{4\pi}\int_0^L \frac{\delta_0\gamma_0}{\gamma(s)}K_1\beta \mathrm{d}s$$

$$= \frac{1}{4\pi}\delta_0 |K_1L_{\rm q}|(\hat{\beta}-\check{\beta})\sum_{n=0}^{N_{\rm cell}-1}\frac{\gamma_0}{\gamma_{\rm f}}. \quad (2)$$

Using the approximation for the FODO cell [11],

$$|K_1L_{\rm q}|(\hat{\beta}-\check{\beta}) = 4{\rm tg}\frac{\varphi_{\rm c}}{2}, \quad (3)$$

where $N_{\rm cell}$ is the number of FODO cells in the linac and $\varphi_{\rm c}$ is the phase advance per cell.

We can get

$$\delta\nu \approx \frac{1}{4\pi}\delta_0 4{\rm tg}\frac{\varphi_{\rm c}}{2}\frac{\gamma_0}{L_{\rm cell}}\int_0^L\frac{1}{\gamma(s)}\mathrm{d}s$$

$$= \frac{\delta_0}{\pi}{\rm tg}\frac{\varphi_{\rm c}}{2}\frac{\gamma_0}{L_{\rm cell}}\frac{L}{\gamma_{\rm f}-\gamma_0}\ln\frac{\gamma_{\rm f}}{\gamma_0}$$

$$\simeq \frac{\delta_0}{\pi}{\rm tg}\frac{\varphi_{\rm c}}{2}N_{\rm cell}\frac{\gamma_0}{\gamma_{\rm f}}\ln\frac{\gamma_{\rm f}}{\gamma_0}. \quad (4)$$

For the ILC main linac, the beam is accelerated from 15 GeV to 250 GeV, $\varphi_{\rm c}$=60°, δ_0=1.3%, L=11500 m, $L_{\rm cell}$=76 m. We find $\delta\nu$=0.06 and thus we conclude that the ILC main linac is in the small filamentation regime.

Then, we will use two micro-particle models to discuss the dispersive dilution. We assume that $y_{\rm c}$ is the offset of a particle with design energy and y_δ has an energy deviation equal to the uncorrelated energy spread. According to (1), the equations for $y_{\rm c}$ and y_δ are:

$$\begin{cases}\dfrac{1}{\gamma(s)}\dfrac{\mathrm{d}}{\mathrm{d}s}\gamma(s)\dfrac{\mathrm{d}}{\mathrm{d}s}y_{\rm c}-K_1(y_{\rm c}-y_{\rm q})=G_y \\ \dfrac{1}{\gamma(s)}\dfrac{\mathrm{d}}{\mathrm{d}s}\gamma(s)\dfrac{\mathrm{d}}{\mathrm{d}s}y_\delta-(1-\delta)K_1(y_\delta-y_{\rm q})=(1-\delta)G_y\end{cases}, \quad (5)$$

where we have neglected the wakefield contribution.

For $\Delta y_{\rm d}=y_{\rm c}-y_\delta$

$$\frac{1}{\gamma(s)}\frac{\mathrm{d}}{\mathrm{d}s}\gamma(s)\frac{\mathrm{d}}{\mathrm{d}s}\Delta y_{\rm d}-(1-\delta)K_1\Delta y_{\rm d}$$

$$= \delta G_y-\delta K_1y_{\rm q}+\delta K_1y_{\rm c}. \quad (6)$$

The solution to this equation can be expressed by the Green function with convolution method. The R_{12} element of the transport matrix is the Green function for the linear accelerator [5]. This yields:

$$\Delta y_{\rm d}(s) = \int_0^s \mathrm{d}s'\delta(s')(G_y-K_1y_{\rm q})R_{12}(\delta;s',s)$$

$$+\int_0^s \mathrm{d}s'\delta(s')K_1y_{\rm c}R_{12}(\delta;s',s). \quad (7)$$

The first item of the right side is proportional to the kick of the kickers and the misalignment of the quadrupoles, which are not considered. The second contribution is proportional to the centroid trajectory due to betatron oscillation down the linac. We will just consider this item. Using the R_{11} matrix element, the central oscillation can be written [5]:

$$y_{\rm c}=y_0\sqrt{\frac{\beta(s)}{\beta_0}}\sqrt{\frac{\gamma_0}{\gamma(s)}}\cos(\varphi(s)-\varphi_0), \quad (8)$$

where we have assumed the starting point is symmetry α=0. Then Eq. (7) can be written:

$$\langle\Delta y_{\rm d}^2\rangle = \left\langle\left\{\sum_{n=0}^{N_{\rm q}-1}\delta_n(K_1L_{\rm q})_n y_0\sqrt{\frac{\beta_n}{\beta_0}}\sqrt{\frac{\gamma_0}{\gamma_n}}\cos\varphi_n\sqrt{\beta_n\beta_{\rm f}}\sqrt{\frac{\gamma_n}{\gamma_{\rm f}}}\sin\varphi_{n{\rm f}}(\delta)\right\}^2\right\rangle$$

$$\simeq \left\langle\left\{\sum_{n=0}^{N_{\rm q}-1}\frac{\delta_0\gamma_0}{\gamma_n}|K_1L_{\rm q}|(\hat{\beta}-\check{\beta})y_0\sqrt{\frac{\beta_{\rm f}}{\beta_0}}\sqrt{\frac{\gamma_0}{\gamma_{\rm f}}}\cos\varphi_n\sin\varphi_{n{\rm f}}(0)\right\}^2\right\rangle$$

$$\simeq \delta_0^2\left(4\mathrm{tg}\frac{\varphi_\mathrm{c}}{2}\right)^2\langle y_0^2\rangle\frac{\beta_\mathrm{f}}{\beta_0}\frac{\gamma_0}{\gamma_\mathrm{f}}\left(\frac{1}{2}\frac{1}{2}\right)^2\left(\sum_{n=0}^{N_\mathrm{cell}-1}\frac{\gamma_0}{\gamma_n}\right)^2$$

$$\simeq \delta_0^2\left(4\mathrm{tg}\frac{\varphi_\mathrm{c}}{2}\right)^2\langle y_0^2\rangle\frac{\beta_\mathrm{f}}{\beta_0}\frac{\gamma_0}{\gamma_\mathrm{f}}\left(\frac{1}{2}\frac{1}{2}\right)^2\left(\frac{\gamma_0}{L_\mathrm{cell}}\right)^2\left(\int_0^L\frac{1}{\gamma(s)}\mathrm{d}s\right)^2$$

$$= \delta_0^2\mathrm{tg}^2\frac{\varphi_\mathrm{c}}{2}\langle y_0^2\rangle\frac{\beta_\mathrm{f}}{\beta_0}\frac{\gamma_0}{\gamma_\mathrm{f}}\left(\frac{\gamma_0}{L_\mathrm{cell}}\frac{L}{\gamma_\mathrm{f}-\gamma_0}\right)^2\ln^2\frac{\gamma_\mathrm{f}}{\gamma_0}, \quad (9)$$

where δ_n, β_n and γ_n are the energy spread, the beta function and the relative energy factor at the nth quadrupole, L_q is the quadrupole length, L_cell is the length per FODO cell, L is the total length of the main linac, φ_n is the phase advance from the starting point of the nth quadrupole and $\varphi_{n\mathrm{f}}(\delta)$ is the phase advance from the nth quadrupole to the end for the off-momentum particle. Additionally, the index 0 and f stand for the initial point and the end of the linac, respectively.

At this point, we need to know the relation between the emittance and the dispersion (or Δy_d). We can use an approximation [6]:

$$\varepsilon = \varepsilon_0\sqrt{1+\frac{2\mathit{\Gamma}}{\varepsilon_0}}\approx\varepsilon_0\left(1+\frac{\mathit{\Gamma}}{\varepsilon_0}\right)=\varepsilon_0+\mathit{\Gamma}(2\mathit{\Gamma}$$

$$= \gamma\Delta y^2+2\alpha\Delta y\Delta y'+\beta\Delta y'^2). \quad (10)$$

So,

$$\Delta\varepsilon = \mathit{\Gamma}=\frac{\langle\Delta y_\mathrm{d}^2\rangle}{\beta_\mathrm{f}}$$

$$= \frac{\langle y_0^2\rangle}{\beta_0}\delta_0^2\mathrm{tg}^2\frac{\varphi_\mathrm{c}}{2}\left(\frac{\gamma_0}{\gamma_\mathrm{f}}\right)^3N_\mathrm{cell}^2\ln^2\frac{\gamma_\mathrm{f}}{\gamma_0}. \quad (11)$$

Now we know that natural emittance growth due to dispersive effect is proportional to the square of the initial uncorrelated energy spread and the total number of FODO cells. In addition, natural dilution is related to the phase advance per cell so that it is related to the focusing strength of the lattice. We can draw the conclusion that a smaller initial energy spread and a weaker focusing lattice will ease this dispersive effect.

If we neglect the initial transverse momentum and assume the initial oscillation amplitude is equal to the initial beam size ($y_0=\sigma_0$), the normalized emittance dilution will be:

$$\frac{\gamma_\mathrm{f}\Delta\varepsilon}{\gamma_0\varepsilon_0}=\delta_0^2\mathrm{tg}^2\frac{\varphi_\mathrm{c}}{2}\left(\frac{\gamma_0}{\gamma_\mathrm{f}}\right)^2N_\mathrm{cell}^2\ln^2\frac{\gamma_\mathrm{f}}{\gamma_0}\approx 3\%. \quad (12)$$

2.2 Wakefield effect

We will use a similar method to discuss the wakefield effect. Two micro-particles are located at the head of the bunch $z=\sigma_z$ and the tail $z=-\sigma_z$, respectively. The equations for these two particles are:

$$\begin{cases}\dfrac{1}{\gamma(s)}\dfrac{\mathrm{d}}{\mathrm{d}s}\gamma(s)\dfrac{\mathrm{d}}{\mathrm{d}s}y_+-K_1(y_+-y_\mathrm{q})=G_y\\ \dfrac{1}{\gamma(s)}\dfrac{\mathrm{d}}{\mathrm{d}s}\gamma(s)\dfrac{\mathrm{d}}{\mathrm{d}s}y_--K_1(y_--y_\mathrm{q})\\ =G_y+\dfrac{Ne^2}{2m_0\gamma(s)}W_{\perp1}(2\sigma_z)(y_+-y_\mathrm{a})\end{cases}, \quad (13)$$

where y_+ and y_- are the vertical coordinates for the head particle and the tail particle, and $W_{\perp1}(2\sigma_z)$ is the short-range wakefield at the location $2\sigma_z$ behind the head particle.

Now, the orbit difference between these two particles is found from (13):

$$\frac{1}{\gamma(s)}\frac{\mathrm{d}}{\mathrm{d}s}\gamma(s)\frac{\mathrm{d}}{\mathrm{d}s}\Delta y_w-K_1\Delta y_w$$

$$= -\frac{Ne^2}{2m_0\gamma(s)}W_{\perp1}(2\sigma_z)y_++\frac{Ne^2}{2m_0\gamma(s)}W_{\perp1}(2\sigma_z)y_\mathrm{a}$$

$$\times(\Delta y_w=y(z=\sigma_z)-y(z=-\sigma_z)). \quad (14)$$

Neglecting the misalignment of the cavities, the solution for Δy_w can be expressed by the convolution of the right side of (14) and the R_{12} element of the transport matrix (As we have said, the correlated energy spread was not considered here):

$$\Delta y_w(s)=\int_0^s\mathrm{d}s'\frac{Ne^2}{2m_0\gamma(s)}W_{\perp1}(2\sigma_z)y_+R_{12}(0;s',s), \quad (15)$$

$$\langle\Delta y_w^2\rangle = \left\langle\left\{\sum_{n=0}^{N_\mathrm{cavity}-1}\frac{Ne^2L_\mathrm{c}}{2m_0\gamma_n}W_{\perp1}(2\sigma_z)y_0\sqrt{\frac{\beta_n}{\beta_0}}\sqrt{\frac{\gamma_0}{\gamma_n}}\cos\varphi_n\sqrt{\beta_n\beta_\mathrm{f}}\sqrt{\frac{\gamma_n}{\gamma_\mathrm{f}}}\sin\varphi_{n\mathrm{f}}\right\}^2\right\rangle$$

$$= \left\langle \left\{ \sum_{n=0}^{N_{\text{cavity}}-1} \frac{Ne^2 L_{\text{c}}}{2m_0\gamma_n} W_{\perp 1}(2\sigma_z) y_0 \beta_n \sqrt{\frac{\beta_{\text{f}}}{\beta_0}} \sqrt{\frac{\gamma_0}{\gamma_{\text{f}}}} \cos\varphi_n \sin\varphi_{n\text{f}} \right\}^2 \right\rangle$$

$$\simeq \beta_{\text{f}} \frac{\langle y_0^2 \rangle}{\beta_0} \left(\frac{Ne^2 W_{\perp 1}(2\sigma_z)}{2m_0} \right)^2 \frac{\gamma_0}{\gamma_{\text{f}}} \frac{\overline{\beta}^2}{16} \left(\sum_{n=0}^{N_{\text{cavity}}-1} \frac{L_{\text{c}}}{\gamma_n} \right)^2$$

$$\simeq \beta_{\text{f}} \frac{\langle y_0^2 \rangle}{\beta_0} \left(\frac{Ne^2 W_{\perp 1}(2\sigma_z)}{2m_0} \right)^2 \frac{\gamma_0}{\gamma_{\text{f}}} \frac{\overline{\beta}^2}{16} \left(\int_0^L \frac{1}{\gamma(s)} \text{d}s \right)^2, \qquad (16)$$

where L_{c} is the cavity length. To get (16), we have used the average beta function as an approximation for the focusing quadrupoles and defocusing quadrupoles.

$$\Delta\varepsilon = \Gamma = \frac{\langle \Delta y_w^2 \rangle}{\beta_{\text{f}}} = \frac{\langle y_0^2 \rangle}{\beta_0} \times \left(\frac{Ne^2 W_{\perp 1}(2\sigma_z)}{2m_0} \right)^2 \frac{\gamma_0}{\gamma_{\text{f}}} \frac{\overline{\beta}^2}{16} \left(\frac{L}{\gamma_{\text{f}} - \gamma_0} \right)^2 \ln^2 \frac{\gamma_{\text{f}}}{\gamma_0}. \qquad (17)$$

For the ILC main linac, $N = 2\times10^{10}$, $\overline{\beta} = 80$ m, $W_{\perp 1}(2\sigma_z)$=2.8×10^{13} V/C/m^2 and the accelerating gradient is

$$g = \frac{\gamma_{\text{f}} - \gamma_0}{L} = 59 \text{ m}^{-1}.$$

So, the normalized emittance growth is:

$$\frac{\gamma_{\text{f}} \Delta\varepsilon}{\gamma_0 \varepsilon_0} \approx \left(\frac{Ne^2 W_{\perp 1}(2\sigma_z)}{2m_0} \right)^2 \frac{\overline{\beta}^2}{16} \frac{1}{g^2} \ln^2 \frac{\gamma_{\text{f}}}{\gamma_0} \approx 0.7\%. \qquad (18)$$

Thus, we can see that emittance growth is proportional to the square of the short-range transverse wakefield and the average beta function while inproportional to the square of accelerating gradient. We can draw a further conclusion that the wakefield effect acheives a smaller wakefield, stronger focusing and higher accelerating gradient. Also we can see that the wakefield effect in the ILC main linac is very small when compared with the dispersive effect. This result is reasonable because the large iris of superconducting cavity results in a small wakefield.

3 Simulation results

The significance of the theoretical analysis is the predictive scaling of the emitance growth with the structure parameters rather than giving the numerical result. In order to get the real final emittance growth, we have to rely on simulations. Here, we use the SLEPT [12] code to perform the simulations with the dispersive effect and wakefield effect separately, and also with both effects. The results are shown in Fig. 2 where we have set the injection error to be the initial beam size.

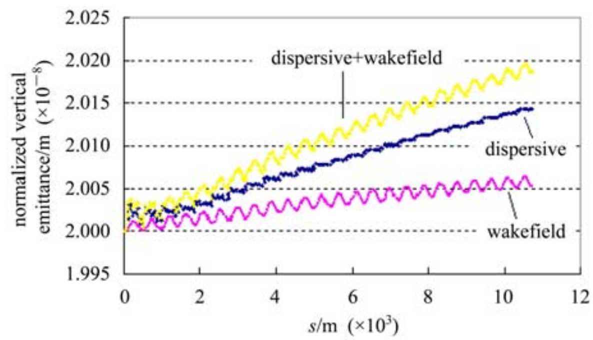

Fig. 2. Simulations of the natural emittance growth in the ILC main linac; the injection offset is set to be the initial beam size.

Our simulations give a 0.25% and 0.7% emittance growth for the wakefield effect and dispersive effect, respectively. Including both effects, we get 0.9% dilution. If we want to limit the emittance dilution to 6%, which refers to 3% luminosity reduction, the tolerance for the injection error is about $2.4\sigma_0$ (about 10 μm). In practice, this condition is not difficult to achieve.

We need to point out that the simulation results are more accurate than the theoretical estimate (12) and (18) because the simulation has considered the longitudinal Gauss distribution of particles while the theoretical method just uses two particle models. Another reason for the difference in these two methods is that we replace $\cos\varphi_n$ and $\sin\varphi_{n\text{f}}$ both in (9) and (16) by 1/2 in order to obtain an easy estimation. This is such a rough approximation of the real case that the theoretical results (12) and (18) provide larger coefficients.

Meanwhile, we can see that there are oscillations in the lines with the wakefield effect. Here, the oscillations come from the curved linac. Strictly speaking, the beam will not go through the center of the cavities because the main linac is aligned with the Earth's curvature. It is just like a systematic cavity offset error. This kind of offset field, combined with betatron oscillation, will tilt the bunch longitudinal with the period of betatron wavelength $2\pi\beta$ [13] (about 500 m for the ILC main linac). Anyway, the troughs of the oscillated lines show the net emittance growth along the linac.

4 Conclusion

In this paper, the single-bunch emittance dilution in the perfect ILC main linac resulting from the dispersive effect and wakefield effect is studied through both theoretical method and numerical method. We obtained the theoretical formulae that can clearly show the relations between the single-bunch emittance growth and the machine parameters and these formulae have been checked by our simulation results. From our study, the wakefield effect is weak for the ILC when compared with the dispersive effects. According to our simulation results, the total emittance growth is 0.9%, which gives the limit of single-bunch emittance growth in the ILC main linac. Since natural dilution is only a small part of the real emittance growth, we will includ misalignment errors and correction algorithms in future studies.

References

1 Chao A, Richter B, YAO C Y. Nucl. Instrum. Methods A, 1980, **178**: 1
2 Ruth R D. Emittance Preservation in Linear Colliders. Proc. of 1966 US/CERN Part. Acc. School, 1986
3 Ruth R D. Beam Dynamics in Linear Colliders . Proc. of 1990 Linear Acc. Conf., 1990
4 Seeman J T. Effects of RF Deflections on Beam Dynamics in Linear Colliders. SLAC-PUB-5069, 1989
5 Raubenhenmer Tor O. The Generation and Acceleration of Low Emittance Flat Beams for Future Linear Colliders. SLAC-387, 1992
6 Raubenhenmer Tor O, Ruth R D. Nucl. Instrum. Methods A, 1991, **302**: 191
7 GAO J. Nucl. Instrum. Methods A, 2000, **441**: 314
8 Amatuni G A, Khachatryan V, Tsakanov V M, Brinkmann R. On the Single Bunch Emittance Preservation in TESLA. TESLA 2001-02, 2001
9 ILC GDE. ILC Reference Design Report- Accelerator. ILC-Report-2007-001, 2007
10 Kubo K. Lecture for the First ILC School, 2006
11 Wiedemann H. Scaling of FODO Cell Parameters. SLAC-PEP-Note-39, 1973
12 http://lcdev.kek.jp/~kkubo/reports/MainLinac-simulation/SLEPT/SLEPT-index.html
13 Yakovlev V, Solyak N et al. Simple Estimation of the Emittance Dilution Caused by the Transverse Kick in the ILC Main Linac. TD-SRF TD Note, 2008