Laser Compton scattering for a linearly polarized laser*

LI Dong-Guo(李冬国)[1)]

Free Electron Laser Group, Shanghai Institute of Applied Physics, CAS, Shanghai 201800, China

Abstract: Laser Compton light sources are potential candidates for the next generation of high-brightness X or γ-ray sources. When increasing the laser power to obtain intense X-ray laser, nonlinear Compton scattering happens. Nonlinear Compton scattering of linearly polarized laser beam is discussed in this paper. A complete transition probability formula is introduced and the polarization properties of final photons are discussed for different conditions.

Key words: laser, X-ray, Compton scattering, nonlinear quantum electrodynamics

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1 Introduction

In the interaction process between electron and laser beams, if the laser intensity parameter η increases to a high value, nonlinear Compton scattering (NLCS) happens, i.e. an electron absorbs multi-photons from the laser field and radiates a single photon. Through the Compton scattering of a polarized laser and a relativistic electron beam, high energy and polarized X (or γ)-rays can be obtained [1].

This new light emitted from linear or nonlinear Compton scattering has lots of excellent properties: ultrashort and tunable wavelength, wide energy spectrum, highly collimated, ultrahigh brightness, ultrashort pulses and it is highly polarized [$\sim$100%]. Due to these excellent properties, laser Compton X-rays can be used in wide research fields: providing a new insight into natural (structural biology) and life sciences; studying structural and electronic properties of matter on an atomic scale; analysis of chemical reactions at ultrafast time resolution; application in high energy and particle physics (such as generation of polarized positron sources, e^+e^- pair creation, $\gamma\gamma$ collider, CP violation, etc. [2–4]).

The rest of this paper is organized as follows: In Sec. 2, an analytical formula of the transition probability and the Monte-Carlo simulation of NLCS scattered photons for a linearly polarized laser are given, the energy and angle distributions are reported, then the characteristics of final photon polarization are discussed. Finally, the conclusions are given in Sec. 3.

2 Strong laser Compton scattering

2.1 Compton scattering process

The intensity of a laser field is characterized by a dimensionless invariant parameter η [5–7], which is defined by the 4-vector-potential A_μ of the laser field

$$\eta\equiv\frac{e\sqrt{-\langle A_\mu A^\mu\rangle}}{mc^2}=\frac{\lambda_{\rm L}}{2\sigma_{\rm L}}\sqrt{P_{\rm L}[GW]/43}\,, \tag{1}$$

where e and m are the charge and rest mass of the electron, c is the light speed, $P_{\rm L}$, $\lambda_{\rm L}$ and $\sigma_{\rm L}$ are the power, wavelength and minimum rms spot size of the laser pulse, the brackets represent the local average over the laser phase. Laser intensity parameter η characterizes the nonlinear effect (nonlinear quantum effect) in the Compton scattering process. When the laser parameter η is small, an electron absorbs only one photon, this is normal Compton scattering, or linear Compton scattering, which can be represented as

$$e+\gamma_{\rm L}\rightarrow e'+\gamma\,, \tag{2}$$

where $\gamma_{\rm L}$ represents the photon from the initial laser, γ is the emitted high-energy photon, e(e′) is the initial

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1) E-mail: leedongguo2000@yahoo.com

(scattered) electron, as shown in Fig. 1(a). When increasing the laser intensity η to obtain intense X-ray, the nonlinear effect happens: an electron has a chance to absorb multiple photons from the laser field and radiate a single high energy photon, which is called NLCS, i.e.

$$e+n\gamma_{L}\rightarrow e'+\gamma, \tag{3}$$

where $n\geqslant 1$ indicates the number of absorbed photons, as shown in Fig. 1(b).

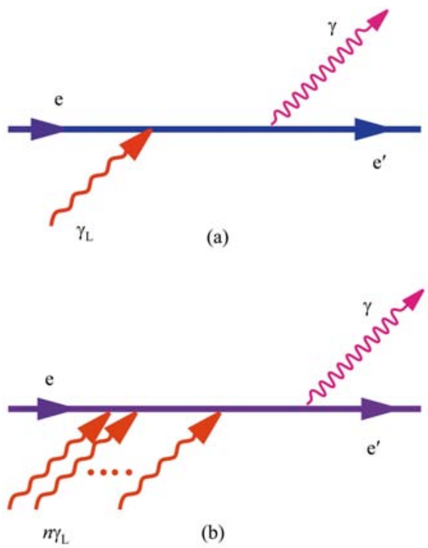

Fig. 1. (color online) Processes for linear Compton scattering (a) and nonlinear Compton scattering (b).

2.2 Kinematics

When an electron moves in a strong laser field, its kinematics process is affected by the laser field, and its average momentum (which is called 'quasi-momentum') is given by [8, 9]

$$q^{\mu}\equiv\langle\psi|\hat{p}^{\mu}|\psi\rangle=p^{\mu}+\frac{m^2\eta^2}{2(kp)}k^{\mu}, \tag{4}$$

where p^{μ}, $\hat{p}^{\mu}$ and ψ are the 4-momentum, 4-momentum operator and wavefunction of the electron, and k^{μ} is the 4-wavenumber of the photon. The conservation of energy and momentum of the electron in a strong laser field is substituted by their relevant 4-quasi-momenta q and q':

$$q+nk=q'+k'. \tag{5}$$

The quantum effect in a Compton process can be characterized by an invariant parameter $\lambda=2k{\cdot}p/m^2\approx 4\omega E/m^2$, where $\omega(E)$ is the energy of the laser photon (initial electron), and '$\approx$' means the approximation for relativistic incident electron beam.

The final photon energy ω' is expressed by a dimensionless Lorentz invariant parameter x:

$$x=\frac{k{\cdot}k'}{k{\cdot}p}=\frac{n\lambda}{1+\eta^2+n\lambda+u^2}\approx\frac{\omega'}{E}, \tag{6}$$

where $u\approx\gamma\theta$ and θ is the scattering polar angle.

As all colliding cases can be changed into 'head-on' colliding through Lorentz transformation, we will adopt 'head-on' framework throughout the present paper for the simplicity of defining the azimuthal scattering angle ϕ, which is shown in Fig. 2. For a relativistic electron beam, the scattered photons are limited in a narrow cone with a half-opening angle of $1/\gamma$. From Eq. (6), one can obtain the energy of scattered photon for the head-on scattering case,

$$\omega'_n=\frac{n(1+\beta)\omega}{1-\beta\cos\theta+\left[\dfrac{n\hbar\omega}{\gamma mc^2}+\dfrac{\eta^2}{2(1+\beta)\gamma^2}\right](1+\cos\theta)}, \tag{7}$$

where β is the normalized velocity of the initial electron, and θ is the backscattering polar angle of scattered photons. The frequency ω'_n corresponds to the nth harmonic of ω. The maximum energy of scattered photons emitted from backward direction (i.e. $\theta=0$) is given by

$$x_{n,\max}=n\lambda/(1+\eta^2+n\lambda). \tag{8}$$

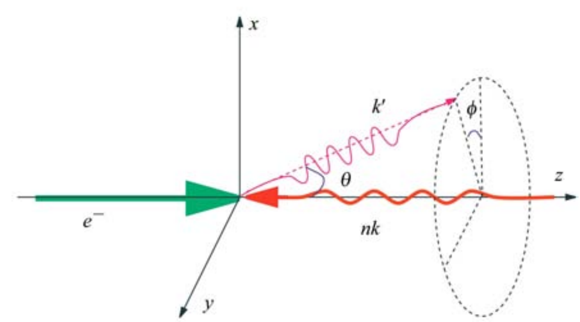

Fig. 2. (color online) Head-on frame for scattering process.

2.3 Transition probability

A collision process can be treated by the following superposition [9]

$$\sum_f|f\rangle\langle f|S|i\rangle, \tag{9}$$

where $|i\rangle$ denotes the initial states and $|f\rangle$ the final states. The coefficient $S_{fi}=\langle f|S|i\rangle$ is called scattering matrix or S-matrix. The S-matrix element for a transition of the electron from the initial state $\psi_{\rm p}$ to the final state $\psi_{\rm p'}$ with emission of a high energy photon having 4-momentum $k'^{\mu}=(\omega',k')$ and polarization 4-vector ϵ' is expressed as [9]

$$S_{fi}=-\mathrm{i}e\int\bar{\psi}_{p'}\,\gamma\cdot\epsilon'^{*}\,\psi_p\frac{\mathrm{e}^{\mathrm{i}k'x}}{\sqrt{2\omega'}}\mathrm{d}^4x\,. \quad (10)$$

Square $|S_{fi}|^2$ gives the transition probability (interaction rate) per unit time by an electron of the scattering from an initial state $|i\rangle$ into a final state $|f\rangle$, i.e.

$$W|_{i\to f}=|S_{fi}|^2=\sum_{n=1}^{\infty}W_n\,, \quad (11)$$

where W_n is the transition probability for n photons absorbtion(nth harmonic). For a certain energy x and azimuthal angle ϕ, W_n can be expressed by

$$W_n=\int W_n(x,\phi)\frac{\mathrm{d}x\mathrm{d}\phi}{2\pi}. \quad (12)$$

After using Volkov's solution and the polarization density matrix of scattered photons, we finally obtain the differential transition probability formula of nth harmonic NLCS for linearly polarized lasers as [5–7]

$$W_n(x,\phi)=\frac{\alpha m^2\eta^2}{2q_0}\left[f_{0n}+f_{1n}\xi_1'+f_{3n}\xi_3'\right], \quad (13)$$

where α is the fine structure constant, Stokes parameter ξ' represents the polarization component to be measured by the detector and functions $f(x,\phi)$ are given by [5–7]

$$\begin{aligned} f_{0n}=&-\frac{|A_n^{(0)}|^2}{\eta^2}+\left(1-x+\frac{1}{1-x}\right)\\ &\times\left[|A_n^{(1)}|^2-A_n^{(0)}A_n^{(2)}\right],\\ f_{1n}=&\ \frac{|A_n^{(0)}|^2}{\eta^2}u^2\sin2\phi+2\sqrt{2}\frac{u}{\eta}\sin\phi A_n^{(0)}A_n^{(1)}, \quad (14)\\ f_{3n}=&-(1+2u^2\sin^2\phi)\frac{|A_n^{(0)}|^2}{\eta^2}\\ &+2\left[|A_n^{(1)}|^2-A_n^{(0)}A_n^{(2)}\right]. \end{aligned}$$

Uing the exact polarization definition, a new polarization term f_{1n} appears, f_{3n} was given by Ref. [5] and function $A_n^{(s)}$ is defined by

$$A_n^{(s)}=\oint\frac{\mathrm{d}\phi}{2\pi}\cos^{s}\phi\mathrm{e}^{\mathrm{i}[n\phi-\alpha_1\sin\phi+(\alpha_2/2)\sin(2\phi)]},\ s=0,1,2.$$

with arguments

$$\alpha_1=-2\sqrt{2}\frac{\eta}{\lambda}\frac{xu}{1-x}\cos\phi\,,\quad\text{and}\quad\alpha_2=\frac{\eta^2}{\lambda}\frac{x}{1-x}\,.$$

2.4 Energy and angle distributions

Using the transition probability formula Eq. (13), one can give the energy spectrum of the laser Compton scattering X-ray, as shown in Fig. 3(a) for $\lambda=1.0\times10^{-4}$ and $\eta=0.8$. Sharp peaks are seen at the high energy edge only for the odd harmonics. Fig. 3(b) shows the energy spectrum of scattered photons whose polarization is parallel or perpendicular to the laser polarization, where the upper (lower) lines for parallel (perpendicular) for each harmonic.

Figure 4 shows the correlation between the energy and polar angle of the laser Compton X-ray(the parameters of 2nd CO_2 laser power stage in the BNL-ATF Compton scattering experiment are used here

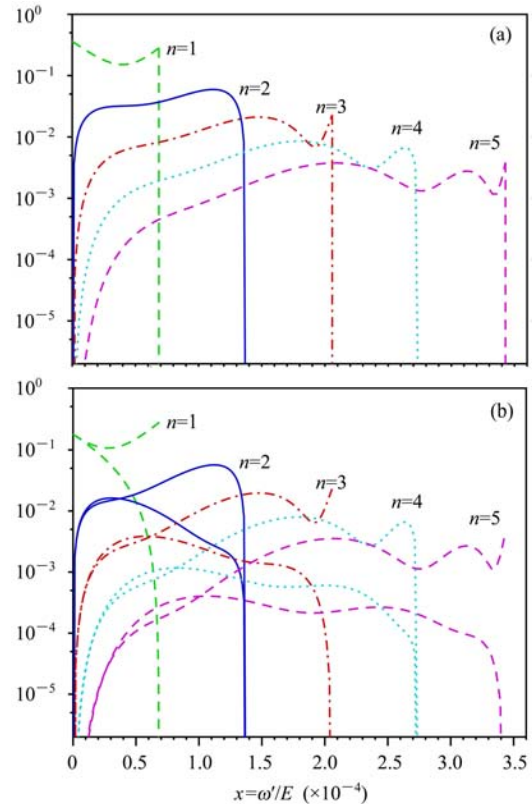

Fig. 3. (color online) Energy spectra for the first 5th harmonic, where W_n is in units of $\alpha m^2/E$. (b) shows spectrum of scattered photons with polarization.

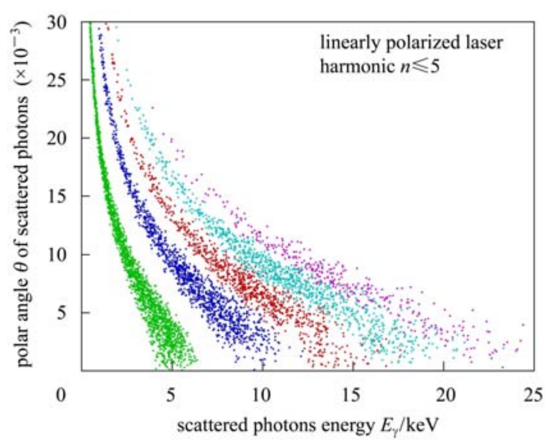

Fig. 4. (color online) Energy via polar-angle distribution.

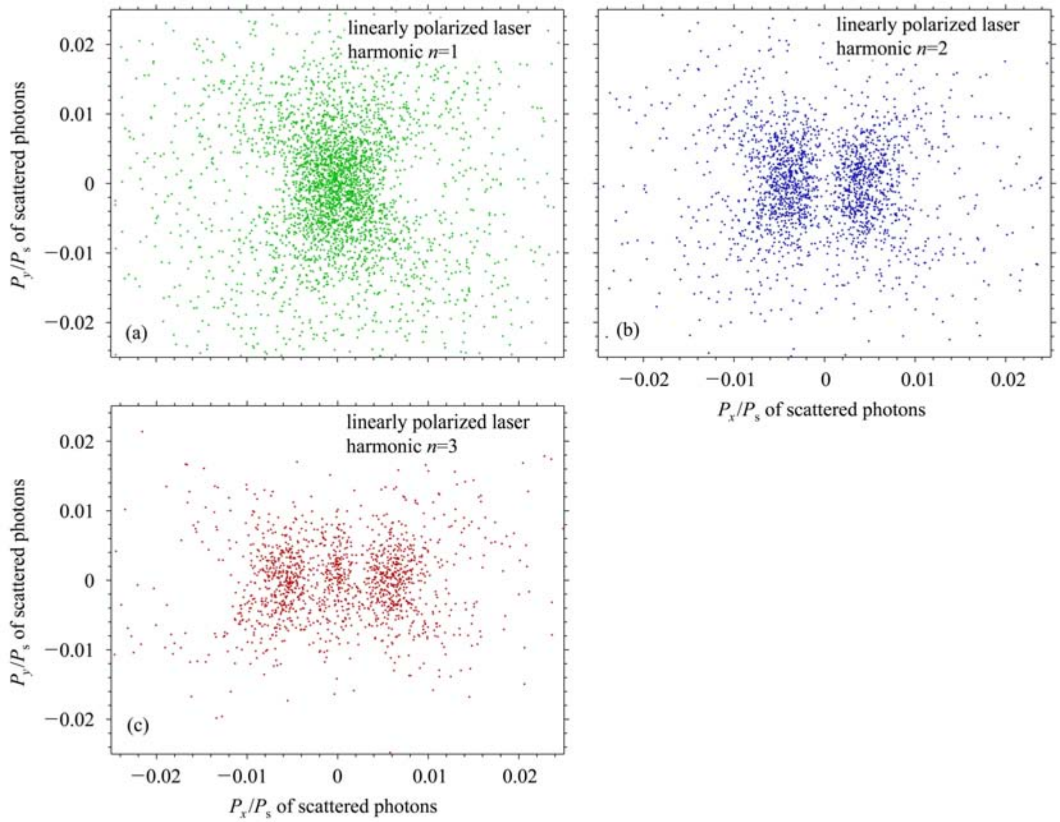

Fig. 5. (color online) Transverse profile of the first three harmonic scattered photons for linearly polarized laser case, where P_x, P_y and P_s are the momentums of scattered photons.

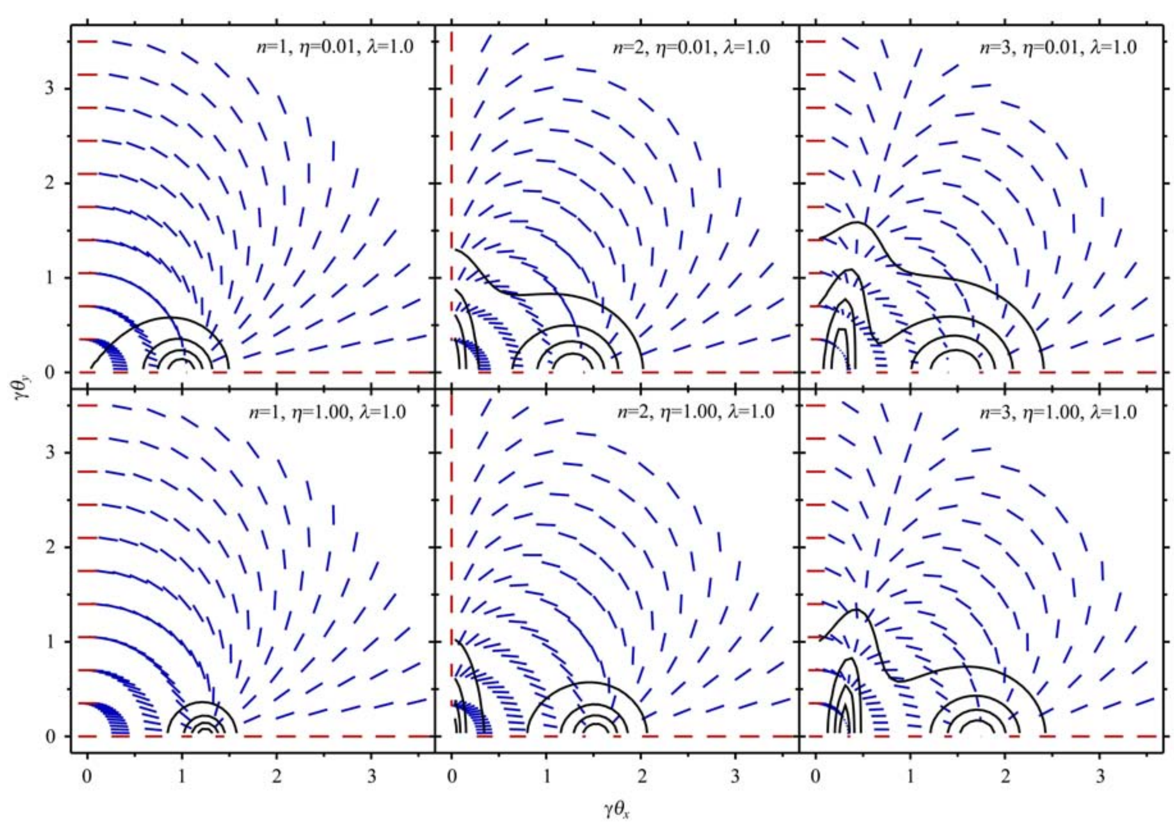

Fig. 6. Final polarization in $(\gamma\theta_x, \gamma\theta_y)$ plane.

and the following simulations). Scattered photons can be found in the backscattering region $\theta \approx 0$ for the odd harmonic case. Fig. 5(a)–(c) show the transverse profile of the laser Compton X-ray of the first three harmonics. The pattern for the first harmonic has one peak and a dumbbell form, which is the same as the patterns of dipole radiation in the classical theory. The patterns for the second and third harmonics have two and three peaks.

2.5 Final photon polarization

The final photons are like an ensemble, to explain the collective character, a local average polarization $\xi(x,\phi)$ is used, which is the average polarization value of lots of final photons at position (x,ϕ). In experiment, the measured value is ξ rather than ξ', and its value covers the whole region $[-1,+1]$ rather being a discrete value. Linear polarization of photons (measured value) for the nth harmonic at (x,ϕ) can be characterized by $\xi_3 = f_{3n}/f_{0n}$, $\xi_1 = f_{1n}/f_{0n}$ [6, 7]. The degree of linear polarization $\xi_{\rm L}$ and the angle $\phi_{\rm L}$ of polarization plane (measured from the polarization plane of the laser) are given by $\xi_{\rm L} = \sqrt{\xi_1^2+\xi_3^2}$, $\xi_3 = \xi_{\rm L}\cos 2\phi_{\rm L}$ and $\xi_1 = \xi_{\rm L}\sin 2\phi_{\rm L}$. Final polarization in the $(\gamma\theta_x, \gamma\theta_y)$ plane is plotted in Fig. 6 for the first three harmonics, where $\lambda = 1$, $\eta = 0.01, 1.0$. The length and direction of the short lines indicate the degree and direction of the polarization. The polarization degree is also shown by contours (the uppermost contour means 80% and the lowermost 20%).

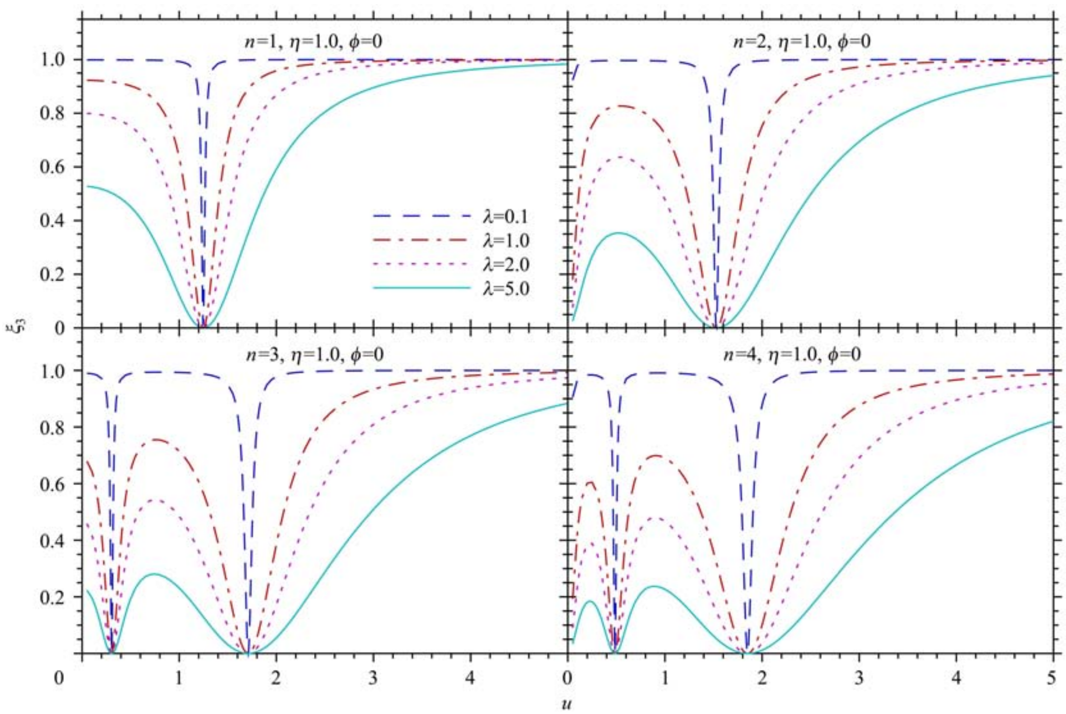

Fig. 7. (color online) $\xi_{\rm L}$ vs polar angle $u(\approx\gamma\theta)$.

Final polarization ξ_3 vs polar angle $u \approx \gamma\theta$ is plotted in Fig. 7, for the first four harmonics under conditions $\phi = 0°$ (x-axis, $\xi_1 \equiv 0$ on this plane), where $\eta = 1$ and $\lambda = 0.1, 1, 2, 5$. One finds $\xi_3 \geqslant 0$ anywhere on the x-axis and there are $[(n+1)/2]$ zero points. The location of the zeroes is independent of λ as a function of u (note that it depends on λ as a function of x). The term of f_{1n} in Eq. (13) is necessary for calculating the final photon polarization, as ξ_1 equals zero only exactly on the $\phi = 0°$ and $90°$ planes. Fig. 8 shows ξ_1 on $\phi = 45°$ plane. One finds the following facts:

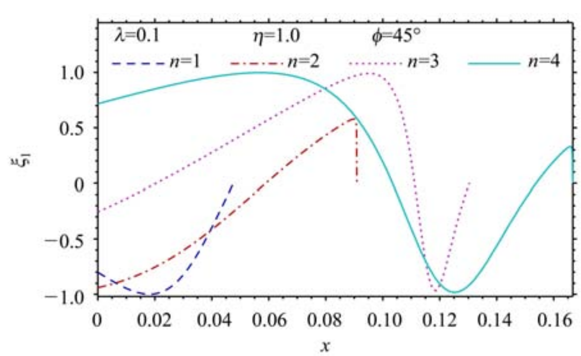

Fig. 8. (color online) ξ_1 vs energy x on $\phi = 45°$ plane.

1) No final circular polarization appears.

2) When $\lambda \ll 1$, the degree of polarization is nearly 100% for any harmonic n and laser strength η, except in the vicinity of a few points on the x-axis.

3) Final polarization is parallel to the laser polarization on the x-axis ($\phi = 0°$), but it is parallel for odd harmonic case and perpendicular for even harmonic case on the y-axis ($\phi = 90°$).

4) For a large polar angle θ, the polarization degree is almost 100%.

5) For a small polar angle, the polarization decreases as λ increases, but increases as η increases.

3 Conclusions

NLCS for a linearly polarized laser is discussed and a complete transition probability formula Eq. (13) is introduced. Using both f_{1n} and f_{3n}, we can exactly describe the degree and plane direction of final polarization for laser Compton X-rays in different polar and azimuthal scattering angles. Finally, the polarization properties of laser Compton X-rays are discussed.

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