Unruh-Verlinde temperature and energy of (2+1)-dimensional matter coupled black hole via entropic force*

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Abstract: Verlinde's recent work, which shows that gravity may be explained as an entropic force caused by the changes in information associated with the positions of material bodies, is extended to study the Unruh-Verlinde temperature and energy of a static spherically symmetric charged black hole. The results indicate that the Unruh-Verlinde temperature is equal to the Hawking temperature at the event horizon. The energy is dependent on the radius of the screen, which is also a consequence of the Gauss' laws of gravity and electrostatics.

Key words: black hole, Unruh-Verlinde temperature, energy, entropic force

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1 Introduction

Over the past few decades, Hawking radiation of black holes has been a hot spot of theoretical physics. Hawking radiation is one of the most important productions of quantum field theory in curved space time. It provides a link between general relativity, statistical physics, thermodynamics, and quantum field theory, and it is generally believed that the deep investigation of Hawking radiation may shed light on the setup of a satisfactory quantum gravity theory. Many valuable models, such as Kraus-Parikh-Wilczek's quantum tunnel method [1–3] and Robinson-Wilczek's gravitational anomalies method [4], which tried to explain the action of the particles tunnel across the event horizon, are put forward [5–13]. Their assumptions are directly motivated by Hawking's original thought experiment from which he discovered that quantum particle creation effects result in an effective emission of particles from a black hole with a blackbody spectrum at temperature [14, 15]

$$T=\hbar\kappa/2\pi. \tag{1}$$

A line element of a general static spherically symmetric black hole can be expressed as

$$\mathrm{d}s^2=-f(r)\mathrm{d}t^2+g(r)\mathrm{d}r^2+r^2\mathrm{d}\Omega^2. \tag{2}$$

The surface gravity of the event horizon is easily evaluated as

$$\kappa=\frac{1}{2}\partial_r f(r)\sqrt{f(r)g(r)}\Big|_{r_{\mathrm{H}}}. \tag{3}$$

An important thermodynamical quantity corresponding to a black hole is the Hawking temperature T_{H}, which is given by

$$T_{\mathrm{H}}=\frac{\hbar}{4\pi}\partial_r f(r)\sqrt{f(r)g(r)}\Big|_{r_{\mathrm{H}}}. \tag{4}$$

Recently, Verlinde [16] presented a new idea that gravity can be explained as an entropic force caused by the information changes when a material body moves away from the holographic screen. With the holographic principle and the equipartition theorem, Verlinde showed that Newton's law of gravitation can arise naturally and unavoidably in a theory in which space is emergent through a holographic scenario, and a relativistic generalization leads to the Einstein equations. Subsequently, with the idea of entropic force, the Newtonian gravity in loop quantum gravi-

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ty [17, 18], and the holographic dark energy [19] were derived from the entropic force formula. The Friedmann equations and the modified Friedmann equations for the Friedmann-Robertson-Walker universe in Einstein gravity [20] were also discussed from the viewpoint of entropic force. These similar ideas were also applied to the construction of holographic actions from black hole entropy [21]. Ref. [22] shows that gravity has a quantum informational origin. Likewise, for a general static spherically symmetric black hole, we can define its Unruh-Verlinde temperature.

In this paper, we have extended Verlinde's idea to a (2+1)-dimensional matter coupled black hole and calculated the Unruh-Verlinde temperature and the energy associated with holographic screens. Throughout this paper, the units ($G \equiv c \equiv 1$) are used.

2 Unruh-Verlinde temperature and the energy of a (2+1)-dimensional BTZ black hole coupled with nonlinear electrodynamics

The action describing the (2+1)-dimensional Einstein theory coupled with nonlinear electrodynamics is given by [23]

$$S=\int\sqrt{g}\left(\frac{1}{16\pi}(R-\Lambda)+L(F)\right)\mathrm{d}^3x. \tag{5}$$

The field equations via variational principle read as

$$G_{\mathrm{ab}}+\Lambda g_{\mathrm{ab}}=8\pi T_{\mathrm{ab}}, \tag{6}$$

$$T_{\mathrm{ab}}=g_{\mathrm{ab}}L(F)-F_{\mathrm{ac}}F^{\mathrm{c}}_{\mathrm{b}}L_{,\mathrm{F}}, \tag{7}$$

$$\nabla_{\mathrm{a}}(F^{\mathrm{ab}}L_{,\mathrm{F}})=0, \tag{8}$$

in which $L_{,\mathrm{F}}$ stands for the derivative of $L(F)$ with respect to $F=(F_{\mathrm{ab}}F^{\mathrm{ab}})/4$. The nonlinear field is chosen such that the energy momentum tensor (8) has a vanishing trace. The trace of the tensor gives

$$T=T_{\mathrm{ab}}g^{\mathrm{ab}}=3L(F)-4FL_{,\mathrm{F}}. \tag{9}$$

Hence, to have a vanishing trace, the electromagnetic Lagrangian is obtained as

$$L=c|F|^{3/4}, \tag{10}$$

where c is an integration constant. With reference to the paper [23], the complete solution to the above action is given by the metric

$$\mathrm{d}s^2=-f(r)\mathrm{d}t^2+f(r)^{-1}\mathrm{d}r^2+r^2\mathrm{d}\theta^2, \tag{11}$$

where the metric function $f(r)$ is given by

$$f(r)=-m+\frac{r^2}{l^2}+\frac{4q^2}{3r}. \tag{12}$$

Here m is the mass, $l^2=\Lambda^{-1}$ the case $\Lambda>0(\Lambda<0)$, corresponds to an asymptotically de-Sitter (anti de-Sitter) space-time, and q is the electric charge. This metric represents the BTZ black hole in nonlinear electrodynamics.

Based on Verlinde's idea [16], the holographic screens locate at equipotential surfaces, where the potential ϕ is defined by a time-like Killing vector ξ^{α}

$$\phi=\frac{1}{2}\lg(-\xi^{\alpha}\xi_{\alpha}), \tag{13}$$

and ξ^{α} satisfies the Killing equation

$$\nabla_{\alpha}\xi_{\beta}+\nabla_{\beta}\nabla_{\alpha}=0. \tag{14}$$

Its exponent e^{ϕ} represents the redshift factor that relates the local time coordinate to that at a reference point with $\phi=0$, which will be taken to be infinity. The potential ϕ is used to define a foliation of space, and the holographic screens are put at surfaces of constant redshift. So the entire screen has the same time coordinate. Then the local temperature on a screen can be defined by the acceleration of a particle that is located very close to the screen. The energy on the screen is calculated by the holographic principle and the equipartition rule of energy with the bit density on the screen.

The four-velocity u^{α} of the particle and its acceleration $a^{\beta}=u^{\alpha}\nabla_{\alpha}u^{\beta}$ can be expressed in terms of the Killing vector ξ^{β} as

$$u^{\beta}=\mathrm{e}^{-\phi}\xi^{\beta},\quad a^{\beta}=\mathrm{e}^{-2\phi}\xi^{\alpha}\nabla_{\alpha}\xi^{\beta}=-g^{\alpha\beta}\nabla_{\beta}\phi. \tag{15}$$

Note that just like in the non-relativistic situation, the acceleration is perpendicular to screen $\Im$. So we can turn it into a scalar quantity by contracting it with a unit outward pointing vector n^{β} normal to the screen $\Im$ and to ξ^{β}.

The local temperature T on the screen is now in analogy with the non-relativistic situation defined by

$$T=\frac{\hbar}{2\pi}\mathrm{e}^{\phi}n^{\beta}\nabla_{\beta}\phi, \tag{16}$$

where a redshift factor e^{ϕ} is inserted because the temperature T is measured with respect to the reference point at infinity. We will call the temperature defined in (16) as Unruh-Verlinde temperature.

Assuming that the change in entropy at the screen is 2π for a displacement by one Compton wavelength normal to the screen, we have

$$\nabla_{\alpha}S=-2\pi\frac{m}{\hbar}n_{\alpha}. \tag{17}$$

For a fixed particle near the screen, the entropic force now follows from

$$F_{\alpha}=T\nabla_{\alpha}S=-m\mathrm{e}^{\phi}\nabla_{\alpha}\phi, \tag{18}$$

where the additional factor e^{ϕ} arises from the redshift, and $\nabla_{\alpha}\phi$ is the relativistic analogue of Newton's acceleration a.

Consider a holographic screen on a closed surface of constant redshift ϕ, the number of bit N of the screen is assumed to be proportional to the area of the screen and is given by

$$\mathrm{d}N = \mathrm{d}A/\hbar. \tag{19}$$

Now, by assuming that each bit on the holographic screen contributes energy $T/2$ to the system, and by using the equipartition law of energy, one has

$$E = \frac{1}{2}\int_{\Im} T\mathrm{d}N. \tag{20}$$

Inserting the identifications for T and $\mathrm{d}N$, we can rewrite Eq. (20) as

$$E = \frac{1}{4\pi}\int_{\Im} e^{\phi}\nabla_{\phi}\mathrm{d}A. \tag{21}$$

By using the Killing equation,

$$\partial_{\alpha}\xi_{\beta} + \partial_{\beta}\xi_{\alpha} - 2\Gamma^{\lambda}_{\alpha\beta} = 0, \tag{22}$$

and the static and spherically symmetric properties of the black hole,

$$\partial_{t}\xi_{\alpha} = 0, \tag{23}$$

we can solve the time-like Killing vector field $\xi_{\alpha}(x)$ with the condition $\xi_{\alpha}\xi^{\alpha} = -1$ at infinity, which can be expressed as

$$\xi_{\alpha} = \left(m - \frac{r^2}{l^2} - \frac{4q^2}{3r}, 0, 0, 0\right). \tag{24}$$

Obviously, for the (2+1)-dimensional BTZ black hole, the Killing vector field ξ_{α} is a zero vector at the horizon.

The acceleration and Unruh-Verlinde temperature associated with the screen are calculated, respectively, as

$$a^{\alpha} = \left(0, -\frac{2r}{l^2} + \frac{4q^2}{3r^2}, 0, 0\right), \tag{25}$$

$$T = \frac{\hbar}{2\pi r^2}\left(-\frac{r^3}{l^2} + \frac{2q^2}{3}\right). \tag{26}$$

The Unruh-Verlinde temperature can be expressed in terms of the event horizons as

$$T|_{r=r_{\mathrm{h}}} = \frac{\hbar}{2\pi r_{\mathrm{h}}}\left(-\frac{r_{\mathrm{h}}^3}{l^2} + \frac{2q^2}{3}\right), \tag{27}$$

which is equal to the Hawking temperature T_{h}. Considering a zero cosmological constant $\Lambda = 0$, we find that a (2+1)-dimension BTZ black hole has just one event horizon at

$$r_{\mathrm{h}} = -\frac{4q^2}{3m}. \tag{28}$$

Substituting Eq. (28) into Eq. (27), we can obtain the Hawking temperature at the event horizon as

$$T_{\mathrm{h}} = \frac{3\hbar}{16\pi}\frac{m^2}{q^2}. \tag{29}$$

This result in Ref. [23] is consistent. From Eq. (21), this energy is dependent on the radius of the screen, which is also a consequence of the Gauss' laws of gravity and electrostatics. To the 2+1-dimension BTZ black hole, the energy on the event horizon $r = r_{\mathrm{h}}$ is

$$E|_{r=r_{\mathrm{h}}} = \frac{32q^6}{27m^3l^2} + \frac{q^2}{3}. \tag{30}$$

3 Unruh-Verlinde temperature and the energy of a (2+1)-dimensional BTZ black hole with linear electrodynamics

The metric for the charged BTZ black hole in linear electrodynamics is given by [24]

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2, \tag{31}$$

with the metric function

$$f(r) = -m + \frac{r^2}{l^2} - 2q^2\ln\left(\frac{r}{l}\right), \tag{32}$$

where q is the electric charge, m is the mass and $l^2 = \Lambda^{-1}$. By using the Killing equation (22) and $\partial_{t}\xi_{\mu} = 0$, $\xi_{\alpha}\xi^{\alpha} = -1$ at infinity for the Killing vector of the charged BTZ black hole, we can also solve the time-like Killing vector ξ_{μ}. The result is of the form

$$\xi_{\alpha} = \left(m - \frac{r^2}{l^2} + 2q^2\ln\left(\frac{r}{l}\right), 0, 0, 0\right), \tag{33}$$

which is also zero at the horizon. The acceleration and Unruh-Verlinde temperature are read as

$$a^{\alpha} = \left(0, \frac{2r}{l^2} + \frac{2q^2}{r}, 0, 0\right), \tag{34}$$

$$T = \frac{\hbar}{2\pi r}\left(\frac{r^2}{l^2} + q^2\right). \tag{35}$$

The Unruh-Verlinde temperature associated with the event horizon $r = r_{\mathrm{h}}$ is just the Hawking temperature T_{h},

$$T|_{r=r_{\mathrm{h}}} = \frac{\hbar}{2\pi r_{\mathrm{h}}}\left(\frac{r_{\mathrm{h}}^2}{l^2} + q^2\right). \tag{36}$$

The energy on the screen is

$$E = \frac{r^3}{l^2} + q^2 r. \tag{37}$$

The energy on the event horizon $r = r_{\mathrm{h}}$ is

$$E|_{r=r_{\mathrm{h}}} = \frac{r_{\mathrm{h}}^3}{l^2} + q^2 r_{\mathrm{h}}. \tag{38}$$

4 Conclusion

Verlinde's theory states that the space-time can be described as an information device made of holographic surfaces (screens) on which the information about the physical systems can be stored. The relevant information about the physical dynamics can be recovered by analyzing the variation in the information on the screens and it is independent of the details of the particular theory used to describe the physical system. In this paper, with the holographic principle and the equipartition theorem, we investigate the Unruh-Verlinde temperatures and energies on a holographic screen from the (2+1)-dimensional BTZ black hole and the charged BTZ black hole. The results show that the Unruh-Verlinde temperature is equal to the Hawking temperature on the event horizon of these black holes. The energy is dependent on the radius of the screen, which is also a consequence of the Gauss' laws of gravity and electrostatics. Our result supports Verlinde's theory.

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