Exploring nuclear symmetry energy with isospin dependence in neutron skin thickness of nuclei*

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Abstract: We propose an alternative way to constrain the density dependence of the symmetry energy from the neutron skin thickness of nuclei which shows a linear relation to both the isospin asymmetry and the nuclear charge with a form of $Z^{2/3}$. The relation of the neutron skin thickness to the nuclear charge and isospin asymmetry is systematically studied with the data from antiprotonic atom measurement, and with the extended Thomas-Fermi approach incorporating the Skyrme energy density functional. An obviously linear relationship between the slope parameter L of the nuclear symmetry energy and the isospin asymmetry dependent parameter of the neutron skin thickness can be found, by adopting 70 Skyrme interactions in the calculations. Combining the available experimental data, the constraint of -20 MeV $\lesssim L \lesssim 82$ MeV on the slope parameter of the symmetry energy is obtained. The Skyrme interactions satisfying the constraint are selected.

Key words: nuclear symmetry energy, neutron skin thickness, isospin asymmetry, nuclear charge

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1 Introduction

The nuclear symmetry energy $S(\rho)$ is the difference in energy per nucleon between pure neutron matter and symmetric nuclear matter, which is the key ingredient of the nuclear equation of state (EoS) for asymmetric nuclear matter. It governs the important properties of nuclei and neutron stars. It also plays a significant role in nuclear reaction dynamics and the stability of the phases within the neutron star and the interior cooling process. It is well known that the density dependence of $S(\rho)$ for cold nuclear matter predicted from different models is extremely variable. Acquiring more accurate knowledge of the density dependence of the symmetry energy has become one of the main goals in nuclear physics at present and has stimulated many theoretical and experimental studies [1–5]. To characterize the density dependence of the symmetry energy, $S(\rho)$ is expanded near saturation density (ρ_0) as

$$S(\rho)=S(\rho_0)+\frac{L}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right)+\frac{K_{\rm sym}}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2+..., \quad (1)$$

with the slope parameter $L=3\rho_0\frac{{\rm d}S(\rho)}{{\rm d}\rho}\Big|_{\rho_0}$ and the curvature parameter $K_{\rm sym}=9\rho_0^2\left(\frac{{\rm d}^2S(\rho)}{{\rm d}\rho^2}\right)\Big|_{\rho_0}$.

The calibration of neutron skin thickness of nuclei defined by $\Delta R_{\rm np}=\langle r_{\rm n}^2\rangle^{\frac{1}{2}}-\langle r_{\rm p}^2\rangle^{\frac{1}{2}}$ has attracted a lot of attention in recent years because of the sensitivity of $\Delta R_{\rm np}$ to the density dependence of the symmetry energy. The calculations in either non-relativistic or relativistic mean-field models show a well-defined linear correlation between the $\Delta R_{\rm np}$ of heavy nuclei and the slope parameter L of the symmetry energy

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at the saturation density [6–8]. Thus, $\Delta R_{\rm np}$ of nuclei can be used as a powerful observable to constrain the density dependence of the symmetry energy at ρ_0 and lower densities. The difficulty in the calibration of neutron skin thickness of nuclei stems from the difficulty in the measurement of neutron distribution. The main methods to measure the neutron distribution or neutron skin thickness include hadron scattering [9–11], π^- elastic scattering [12], antiprotonic atoms [13–15], excitation of giant dipole [16, 17] and spin-dipole resonances [18, 19] on inelastic alpha scattering. Unfortunately, the obtained values of $\Delta R_{\rm np}$ from different experimental methods depend on the used analysis model and sometimes are not totally consistent with each other. It is also hard to judge the model dependence of the systematic error in different experimental methods. The parity-violating electron scattering will be a hopeful option to measure the neutron distribution with unprecedented precision of 1% in a model independent way. However, it is still not available. In this case, it will be difficult to accurately and consistently constrain the symmetry energy by directly using the data for neutron skin thickness. We note that the $\Delta R_{\rm np}$ of 26 stable nuclei all over the periodic table (from ^{40}Ca to ^{238}U) have been accumulated from antiprotonic atom measurement. In Ref. [13], the dependence of $\Delta R_{\rm np}$ on the isospin asymmetry $I=(N-Z)/A$ for these 26 nuclei was extracted from experimental data of antiprotonic atom measurement, which reads

$$\Delta R_{\rm np}=(-0.03\pm0.02)+(0.90\pm0.15)I. \qquad (2)$$

Recently, Warda et al. represented this relationship in a droplet model with surface width dependence [20]. However, it is already known that the antiprotons are only sensitive to the tail of the neutron distribution. An assumed shape for the neutron density is needed to extract the rms radius. Therefore the uncertainty in the value of $\Delta R_{\rm np}$ is unavoidable in this approach.

Because the neutron skin thickness is the difference between the neutron and proton rms radii, it should depend not only on the symmetry energy but also on the Coulomb interaction. One should take into account the nuclear charge dependence of the neutron skin thickness $\Delta R_{\rm np}$ as well as the isospin asymmetry I. In this work, we firstly propose an empirical formula to relate $\Delta R_{\rm np}$ to I and charge number Z. Then we suggest an alternative way to constrain the density dependence of the symmetry energy by the nuclear charge and isospin asymmetry dependent neutron skin thickness, based on the 26 experimental data from the antiprotonic atoms obtained up to now. In this way, only the tendency of $\Delta R_{\rm np}$ changing with the isospin asymmetry I is required to get the information about the symmetry energy. The systematic uncertainty due to the experimental method itself is therefore expected to be largely reduced. The more data of $\Delta R_{\rm np}$ for different nuclei with the same experimental method, the more accurate constraint on the density dependence of the symmetry energy that can be obtained.

2 Charge and isospin dependence of neutron skin thickness

Let us first study the systematic behavior of the nuclear charge dependence of $\Delta R_{\rm np}$. An energy density functional, which includes the standard Skyrme energy density and the Coulomb energy density with the Coulomb exchange term and the kinetic energy density, is applied in the calculations, in which the nuclear surface diffuseness is self-consistently taken into account. For the kinetic energy density, we adopt the approximation of semi-classical extended Thomas-Fermi expansion [21] up to second order (ETF2). We calculate the proton and neutron density distributions of nuclei (See Ref. [22] for details) with a spherical symmetric Fermi functions by means of restricted density variational method [21, 23, 24]. With the density distributions determined in this way, the ground state properties, such as the energy and the nuclear charge radii of a series of nuclei, have been calculated. The corresponding experimental data can be reasonably well reproduced [25]. Based on the calculated rms radii of protons and neutrons, we get the neutron skin thickness $\Delta R_{\rm np}$. The effective Skyrme interaction SLy4 is adopted in this work since SLy4 is very successful in describing the bulk properties and surface properties of nuclei.

With this approach, we calculate the $\Delta R_{\rm np}$ for the isotopes with charge number in the range of $20\leqslant Z\leqslant 92$, and only the even-even nuclei are taken into account. Fig. 1 shows the correlations between $\Delta R_{\rm np}$ and I for Ca, Ni, Zr, Sn, Yb, Pb and U isotopes. It is seen from the figure that the neutron skin thickness $\Delta R_{\rm np}$ and the isospin asymmetry I of nuclei are linearly correlated for Ca, Ni, Zr, Sn, Yb, Pb and U isotopes, respectively. The fitting lines are nearly parallel with each other and the slopes of $\Delta R_{\rm np}$ as a function of I for all selected elements is about 1.10 fm per unit I within the range of $0\leqslant I\leqslant 0.24$. The intercepts of the lines depend on the nuclear charge and roughly have a form of $\propto Z^{2/3}$. We have varied

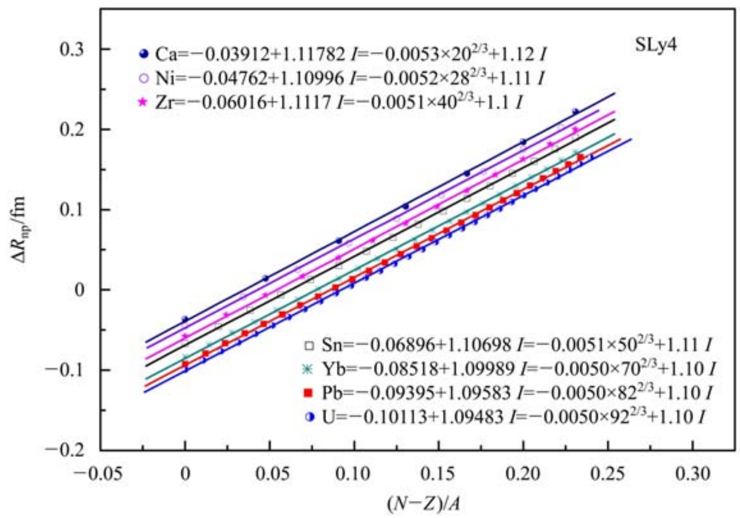

Fig. 1. Linear correlation between the neutron skin thickness and the isospin asymmetry for Ca, Ni, Zr, Sn, Yb, Pb and U elements. Scattered symbols denote the calculation results, and solid lines denote the linear fitting results with a form of $C_Z Z^{2/3} + C_I I$.

the form of the charge number dependence from $Z^{1/3}$ to $Z^{4/3}$ and find that only the form of $\propto Z^{2/3}$ can describe the charge dependence of $\Delta R_{\rm np}$. Obviously, increasing the charge number for the nuclei with the same isospin asymmetry, the neutron skin thickness will be reduced due to the enhanced Coulomb interaction. The nuclear charge dependent parameter of $\Delta R_{\rm np}$ is negative.

According to the above investigation, we propose an empirical expression to describe the neutron skin thickness of a nucleus as

$$\Delta R_{\rm np} = C_Z Z^{2/3} + C_I I. \tag{3}$$

We have performed an χ^2 analysis in order to obtain the optimized C_Z and C_I from the experimental data of $\Delta R_{\rm np}$ from antiprotonic atom measurement. The fitting values of $C_I = 1.00 \pm 0.21$ fm and $C_Z = -0.0036 \pm 0.0024$ fm are obtained, within the $\pm\sigma_q$ $(q = Z, I)$ confidence intervals. The comparison between the $\Delta R_{\rm np}$ obtained with the fitted empirical expression and experimental data is given in Fig. 2. The results of the droplet model (DM) [20] are also shown in this figure with the blue triangles for comparison. One can see that the results of Eq. (3) calculated with $C_I = 1.00$ fm and $C_Z = -0.0036$ fm can reproduce most of the experimental data well and also are in good agreement with the DM calculations. The dashed and solid gray curves in Fig. 2 correspond to the upper and lower boundary of the obtained confidence intervals of parameters C_I and C_Z, respectively. In Fig. 3, we pick out Sn, Te, Zr, Cd isotope chains from the available 26 experimental neutron skin thickness data and compare them with the results of Eq. (3). The gray regions reflect the confidence intervals limited by $C_I = 1.00 \pm 0.21$ fm and $C_Z = -0.0036 \pm 0.0024$ fm. One can see that the experimental data are reproduced reasonably well.

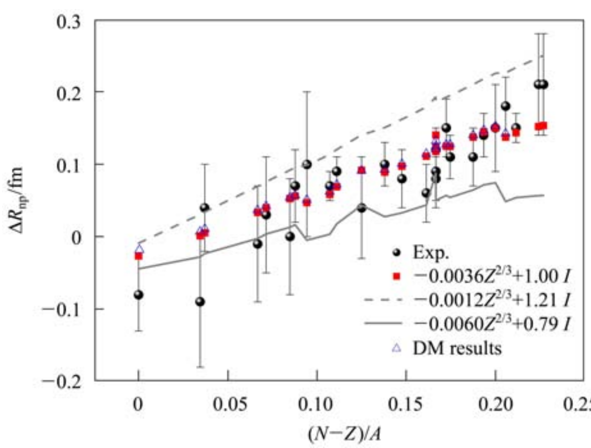

Fig. 2. Comparison of the results with $(-0.0036 \pm 0.0024) Z^{2/3} + (1.00 \pm 0.21) I$ to the experimental neutron skins from antiprotonic measurements [13]. The results of DM are also shown with blue triangles.

3 Density dependence of nuclear symmetry energy

Now let us explore the density dependence of the nuclear symmetry energy from the neutron skin thickness of nuclei $\Delta R_{\rm np}$ correlated with both nuclear charge Z and isospin asymmetry I of nuclei. Preparatorily, we select 9 sets of Skyrme interactions as SV, SkA, SkMP, SLy4, Skz1, MSk6, SkSC1, SVII and SkM1, whose slope parameter L of nuclear symmetry energy are different from each other in a range of

-35 MeV $\lesssim L \lesssim 96$ MeV, to investigate the relation between the nuclear symmetry energy and the neutron skin thickness of nuclei within the ETF2 approach. For example, we show the linear correlation between the neutron skin thickness and isospin asymmetry for a Pb isotope chain, calculated with the 9 sets of Skyrme interactions. It is clear that the slope of $\Delta R_{\rm np}$ varying with I (i.e. the parameter of C_I) is sensitive to the chosen Skyrme interactions, corresponding to the different slope parameters L of nuclear symmetry energy. By adopting L as the y-coordinate and C_I as the x-coordinate, we find a clear linear correlation between this two parameters, as shown in Fig. 4(b). We have checked this result for the other medium-heavy and heavy nuclei and get the same linear correlation.

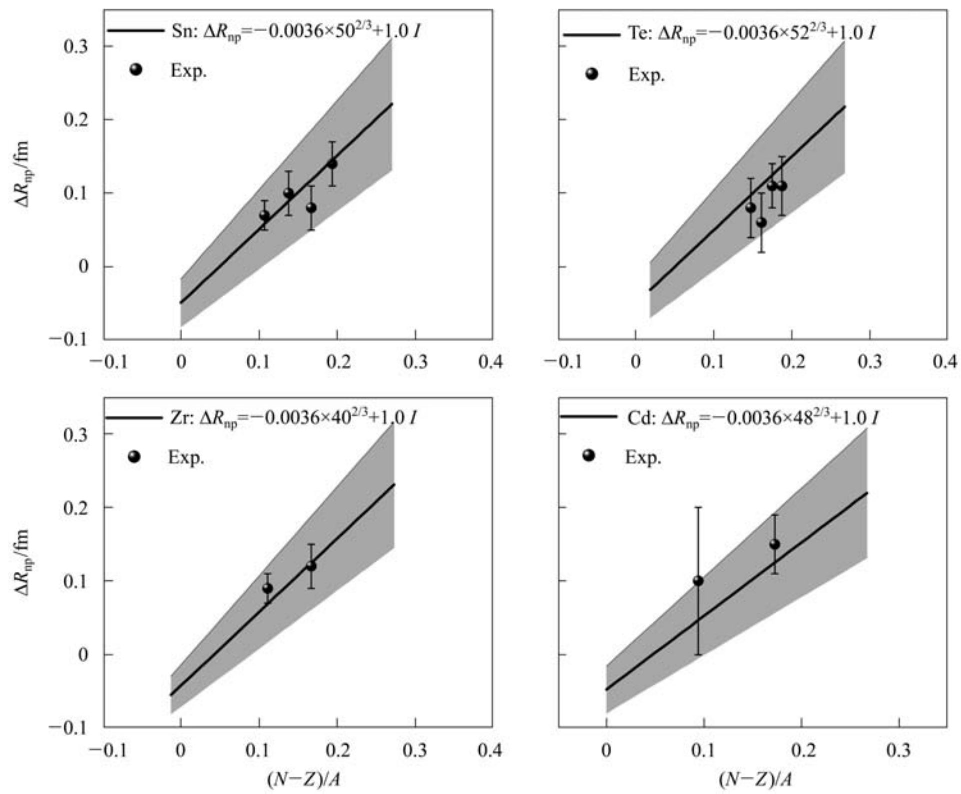

Fig. 3. Comparison of the results with the fitted neutron skin thickness to the experimental data from antiprotonic measurements for Sn, Zr, Te and Cd isotope chains.

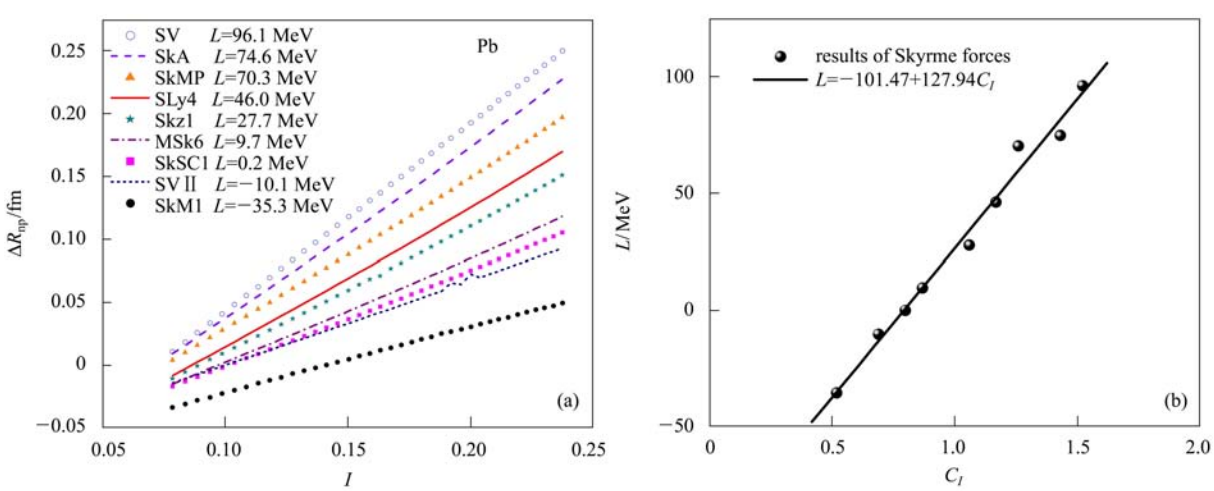

Fig. 4. (a) Linear relation of neutron skin thickness to the isospin asymmetry for the Pb isotope chain, calculated with 9 sets of Skyrme interactions. (b) Correlation between the slope parameter L of nuclear symmetry energy and the parameter C_I of neutron skin thickness for the Pb isotope chain.

Further, we perform systematic calculations within the ETF2 approach by using 70 sets of Skyrme interactions like SLy series, SkT series, v series, Skz-1∼4, MSk1∼6, SkSC1∼4, SkM, SkM*, SkM1, SkX, SkXm, SkXce, SkMP, SkI6, SI ∼ SVII, SIII*, SGI, SGII, Zs, Es, Gs, Rs, BSk1, RATP, SKRA. The slope parameter L of the nuclear symmetry energy covers a wide range from nearly −35 MeV to 98 MeV for all of the 70 Skyrme interactions. For each Skyrme interaction, we calculate the neutron skin thickness of the nuclei for which the neutron skin thickness data from antiprotonic atom measurement are available. Then we get the parameters C_Z and C_I in Eq. (3) for each Skyrme interaction. Fig. 5 displays the relations of the slope parameter L to the parameters C_Z and C_I, respectively. The full circles are the calculated C_Z and C_I for each Skyrme interaction. From Fig. 5(a) we can see that the calculated parameter C_Z changes from −0.007 to −0.003 fm, which is basically in the range of $[-0.006, -0.0012]$ fm extracted from the experimental data. Fig. 5(b) shows us the observed linear increasing correlation between L and C_I. The slope parameter L for different Skyrme interactions varies with C_I in a band area limited by two lines $L=(-103.53\pm23.00)+134.19C_I$, which are plotted in Fig. 5(b) with dashed red lines. With the confidence interval of $C_I=(1.00\pm0.21)$ fm extracted from the experimental data, the slope parameter L is constrained in $-20 \lesssim L \lesssim 82$ MeV. Within the 70 Skyrme interactions we adopted, 53 Skyrme interactions satisfy this relation. These forces are SLy series, v-series, MSk series, SkT1∼3, SkT6∼9, Skz0∼3, SkSC1,2,4, SkM, SkM*, SII, SIII*, SIV, SGI, SGII, RATP, SkP, BSk1, SkX, SkXce, SkXm and SkMP, respectively. The actual range of slope parameter L of these Skyrme interactions is -3.5 MeV $\leqslant L \leqslant$ 70.3 MeV. The corresponding symmetry energy coefficients at the saturation density ρ_0 for these Skyrme interactions are from 28 MeV to 34 MeV except for the case of SkSC2 and SGII. The uncertainty of L obtained from the extracted range of C_I is much smaller than that from C_Z. This means that the correlation between $\Delta R_{\rm np}$ and the isospin asymmetry I of nuclei is more sensitive to the density dependence of the symmetry energy than that between $\Delta R_{\rm np}$ and the nuclear charge. The obtained constraint on the slope parameter L of the symmetry energy is slightly wider than that obtained from the double neutron-proton ratios and isospin diffusion in heavy ion collisions [1] and from the DM model. The possible reasons are that 1) the precision of the data for the neutron skin thickness is not good enough and 2) the number of the data are still few. It needs more accurate data for neutron skin thickness to obtain more a precise value for C_I.

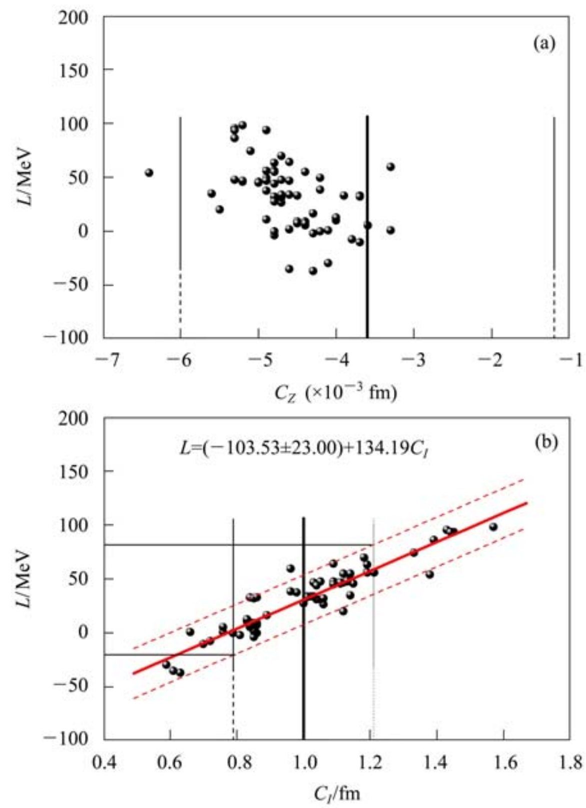

Fig. 5. Relations of the slope parameter L of nuclear symmetry energy to the nuclear charge and isospin symmetry dependent parameters C_Z and C_I of neutron skin thickness, respectively. The full circles are the calculated C_Z and C_I for each Skyrme interaction. The solid black vertical lines denote the extracted values of C_Z and C_I from experiment. The dashed black vertical lines denote the extracted upper and lower limits of their confidence intervals.

4 Summary

In summary, we propose an alternative way to constrain the density dependence of the symmetry energy by means of the relation of the neutron skin thickness to the nuclear charge and isospin asymmetry of nuclei. We show that the neutron skin thickness depends linearly on the isospin asymmetry I and the nuclear charge with the form $Z^{2/3}$. By fitting the available data of neutron skin thickness for 26 nuclei obtained from antiprotonic atom measurement, the nuclear charge and isospin asymmetry dependent parameters for the neutron skin thickness $\Delta R_{\rm np}$ can be extracted within the $\pm\sigma_q$ uncertainty. With the framework of the extended Thomas-Fermi approximation together with the Skyrme energy density functional and Coulomb energy density, we systematically calculate the neutron skin thickness with 70 Skyrme interactions for the 26 measured nuclei in the antiprotonic method, and get the parameters C_Z and C_I for each Skyrme interaction, respectively. Based on the clear linear correlation between the slope parameter L of the symmetry energy and the isospin dependent parameter C_I of the neutron skin thickness, we obtained the constraint on the slope parameter of the symmetry energy, i.e. -20 MeV $\lesssim L \lesssim$ 82 MeV, by using the extracted confidence interval of C_I from the experimental data. The Skyrme interactions satisfying this constraint are selected. It should be stressed that the spherical symmetry Fermi distribution for proton and neutron density in our calculation is in accordance with that in the analysis in the antiprotonic measurement. It is more suitable and consistent to extract the parameters C_Z and C_I with the data from antiprotonic measurement. We suggest more neutron skin thickness for different elements with the same isospin asymmetry to be measured experimentally, so as to extract a more accurate dependence of the neutron skin thickness on the nuclear charge and the isospin asymmetry. With the increase in the number and the accuracy of data for neutron skin thickness of nuclei based on the antiprotonic atom measurement in future, we believe that a more accurate constraint on the density dependence of the symmetry energy can be obtained with this alternative method.

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