The decoherence of quantum entanglement and teleportation in Bell-diagonal states*

QIN Meng(秦猛)[1)] LI Yan-Biao(李延标) WANG Xiao(王晓) BAI Zhong(白忠)

Institute of Science, PLA University of Science and Technology, Nanjing 211101, China

Abstract: We study the dynamics of entanglement and teleportation in Bell-diagonal states. Using the concepts of concurrence and fidelity, the analytical expressions of the entanglement, the output entanglement and the average fidelity with decoherence are obtained for this model. We discover a class of initial states in which the output entanglement and the average fidelity are destroyed by decoherence. The quality of teleportation depends on the system parameters and time.

Key words: Bell-diagonal states, entanglement, teleportation

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1 Introduction

Entanglement corresponds to global states of two or more quantum systems that cannot be separated into direct product states of individual subsystems [1]. People did not realize that entanglement was not merely a philosophical question until Bell proposed the Bell inequality. We now know that any pure entangled quantum state violates one of the Bell-like inequalities [2]. Some PHC and realignment criteria for pure states in infinite-dimensional bipartite quantum systems are also given [3].

As is mentioned, the phenomenon of quantum entanglement lies at the heart of quantum mechanics. And what lies at the heart of quantum mechanics may lie at the heart of future technology [4]. It is well known that quantum entanglement plays an important role in quantum computation and communications. Quantum entanglement distribution is an essential part of quantum communication and computation protocols [5]. Although entanglement is not necessary in both quantum cryptography [6] and quantum secure direct communication protocols using single photons [7], it is helpful, such as in the theoretical quantum key distribution scheme using Einstein-Podolsky-Rosen pairs [8] to enhance the capacity of the quantum communication protocol. The dynamics of entanglement and quantum phase transition (QPT) have been extensively studied. For instance, Han [9] indicates that the pairwise entanglement between two independent locations may be transferred into other multipartite forms which account for the correlations between the two independent locations. AI et al. [10] find that the occurrence of the QPT is reflected by the quantum characteristics of the photonic fields.

As an important source of entanglement, thermal entanglement and teleportation behaviors via thermal entangled states have been a subject of very active research over the past decade [11–14]. The Bell-diagonal states are a three-parameter set, whose geometry, including the separable and classical subsets, can be depicted in three dimensions. Level surfaces of entanglement and nonclassical measures can be plotted directly on this three-dimensional geometry [15]. Bell-diagonal states have been of interest in a variety of contexts in the field. However, the quantification of the entanglement, the output entanglement and the teleportation of Bell-diagonal states under the phase damping channel is still missing. That is the motivation of this paper.

In this paper, we focus on the dynamics of the

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1) E-mail: qrainm@gmail.com

output entanglement and the average fidelity for Bell-diagonal states by teleporting a two-qubit in an arbitrary pure state. In Section 2, the time-dependent behaviors of the quantum output entanglement are analytically and numerically investigated. In Section 3, the property of the average fidelity with phase flip channel is analyzed. Finally, we give a brief conclusion in Section 4.

2 The dynamics of the output entanglement for Bell-diagonal states

The Bell-diagonal states of two qubits, A and B, have density operators of the form [15]

$$\rho_{\rm AB}=\frac{1}{4}\left(I+\sum_{j}^{3}c_j\sigma_j^{\rm A}\otimes\sigma_j^{\rm B}\right), \quad (1)$$

where c_j (j=1, 2, 3) is a real number and $0\leqslant|c_j|\leqslant1$. We take this class of states with a maximally mixed marginal as the initial state. This class of states includes the Werner states ($|c_1|=|c_2|=|c_3|=c$), and the Bell states $|c_1|=|c_2|=|c_3|=1$. The eigenstates are the four Bell states $|\beta_{\rm ab}\rangle=(|0,b\rangle+(-1)^a|1,1\oplus b\rangle)/\sqrt{2}$, with eigenvalues

$$\lambda_{ab}=\frac{1}{4}\left[1+(-1)^ac_1-(-1)^{a+b}c_2+(-1)^bc_3\right]. \quad (2)$$

We consider the case of two qubits under the phase flip (phase damping) channels. For the initial state of Eq. (1), the time evolution of the total system is given by [13]

$$\rho_{\rm AB}(t)=\lambda_\Psi^+(t)|\Psi^+\rangle\langle\Psi^+|+\lambda_\Psi^-(t)|\Psi^-\rangle\langle\Psi^-| +\lambda^+{}_\Phi(t)|\Phi^+\rangle\langle\Phi^+|+\lambda_\Phi^-(t)|\Phi^-\rangle\langle\Phi^-|, \quad (3)$$

where

$$\lambda_\Psi^\pm(t)=\frac{1}{4}[1\pm c_1(t)\mp c_2(t)+c_3(t)], \quad (4)$$

$$\lambda_\Phi^\pm(t)=\frac{1}{4}[1\pm c_1(t)\pm c_2(t)-c_3(t)], \quad (5)$$

and $|\Psi^\pm\rangle=(|00\rangle\pm|11\rangle)/\sqrt{2}$, $|\Phi^\pm\rangle=(|01\rangle\pm|10\rangle)/\sqrt{2}$ are the four Bell states. The time dependent coefficients in Eqs. (4)–(5) are $c_1(t)=c_1(0){\rm e}^{-2\gamma t}$, $c_2(t)=c_2(0){\rm e}^{-2\gamma t}$, $c_3(t)=c_3(0)\equiv c_3$, with γ the phase damping rate [16, 17].

$$\rho_{\rm AB}(t)=\frac{1}{4}\begin{pmatrix}1+c_3 & 0 & 0 & c_1(t)-c_2(t)\\ 0 & 1-c_3 & c_1(t)+c_2(t) & 0\\ 0 & c_1(t)+c_2(t) & 1-c_3 & 0\\ c_1(t)-c_2(t) & 0 & 0 & 1+c_3\end{pmatrix}. \quad (6)$$

Then by the standard procedure, the corresponding concurrence [18] quantifying the entanglement of these two qubits is readily obtained as

$$C_{\rm AB}(t)=\frac{1}{2}\max\{0,|c_1(t)-c_2(t)|-1+c_3, |c_1(t)+c_2(t)|-1-c_3\}. \quad (7)$$

Now we take Lee and Kim's [19] two-qubit teleportation protocol using two copies of the above state $\rho_{\rm AB}(t)\otimes\rho_{\rm A'B'}(t)$ as the resource. We consider inputting a two-qubit state in the special pure state $|\psi\rangle_{\rm in}=\cos(\theta/2)|10\rangle+{\rm e}^{{\rm i}\varphi}\sin(\theta/2)|01\rangle(0\leqslant\theta\leqslant\pi,\ 0\leqslant\varphi\leqslant2\pi)$.

The output replica state can be obtained by applying a joint measurement and local unitary transformation to the input state [20]

$$\rho_{\rm out}=\sum_{i,j}p_{ij}(\sigma_i\otimes\sigma_j)\rho_{\rm in}(\sigma_i\otimes\sigma_j), \quad (8)$$

where σ_i (i=0, x, y, z) signify the unit matrix I and three components of the Pauli matrix, respectively, $p_{ij}={\rm tr}[E^i\rho(T)]{\rm tr}[E^j\rho(T)]$, $\sum p_{ij}=1$ and $\rho_{\rm in}=|\psi\rangle_{\rm in}\langle\psi|$. Here $E^0=|\Phi^-\rangle\langle\Phi^-|$, $E^1=|\Psi^-\rangle\langle\Psi^-|$, $E^2=|\Psi^+\rangle\langle\Psi^+|$, $E^3=|\Phi^+\rangle\langle\Phi^+|$.

And thus one can obtain $\rho_{\rm out}(t)$ as

$$\rho_{\rm out}({\rm t})=\frac{1}{4}\begin{bmatrix}1-c_3^2 & 0 & 0 & C_{\rm in}(c_1^2(t)-c_2^2(t))\\ 0 & 1+c_3^2+4c_3\left(\cos^2\frac{\arcsin(C_{\rm in})}{2}-\frac{1}{2}\right) & C_{\rm in}[c_1^2(t)+c_2^2(t)] & 0\\ 0 & C_{\rm in}[c_1^2(t)+c_2^2(t)] & 1+c_3^2-4c_3\left(\cos^2\frac{\arcsin(C_{\rm in})}{2}-\frac{1}{2}\right) & 0\\ C_{\rm in}(c_1^2(t)-c_2^2(t)) & 0 & 0 & 1-c_3^2\end{bmatrix}. \quad (9)$$

Then by the standard procedure, the corresponding concurrence [18] quantifying the entanglement of two qubits is readily obtained as

$$C_{\text{out}}(t)=\frac{1}{2}\max\left\{0,|C_{\text{in}}[c_1^2(t)-c_2^2(t)]|\right.$$
$$-\sqrt{(1+c_3^2)^2-16c_3^2\left(\frac{1}{2}-\cos^2\frac{\arcsin(C_{\text{in}})}{2}\right)^2}$$
$$\left.|C_{\text{in}}[c_1^2(t)+c_2^2(t)]|-(1-c_3^2)\right\}. \tag{10}$$

The properties of the output entanglement are shown in Fig. 1. In Fig. 1(a), it is shown that the values of the output entanglement decrease and are infinitely close to 0 with time. There exists a critical point γt_c, above which the entanglement vanishes. Different values of c_j have different γt_c, for which the $\gamma t_c \approx 0.4$ in this case. The linear increase of C_{out} with C_{in} also exists in this model, corresponding to Ref. [12] in the system they considered. In Fig. 1(b) we plot the time evolution of the output entanglement with the different parameter c_3. The maximum values of C_{out} exist in $c_1(0)=1$, $c_3=\pm1$, $c_2(0)=-c_3$, which is a Bell state. We find that the entanglement decreases with the increase in the parameter c_3, and the entanglement is symmetrical concerning $c_3=0$. The concurrence of the initial input state is $C_{\text{in}}=2|\mathrm{e}^{\mathrm{i}\varphi}\cos(\theta/2)\sin(\theta/2)|=\sin(\theta)$.

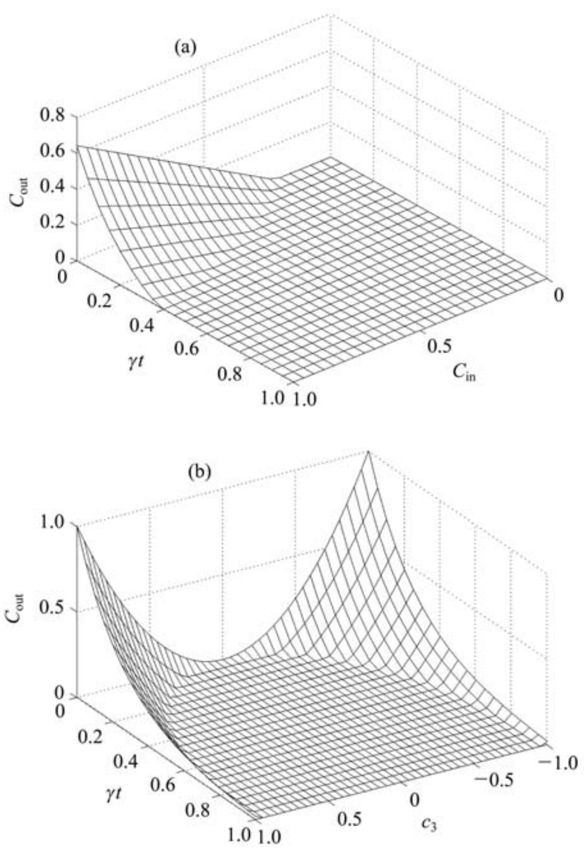

Fig. 1. (a) The dynamics of the output entanglement C_{out} as a function of γt and C_{in} for $c_1(0)=1$, $c_2(0)=-c_3$, $c_3=0.8$; (b) the dynamics of the output entanglement C_{out} as a function of γt and c_3 for $c_1(0)=1$, $c_2(0)=-c_3$.

3 The fidelity dynamics for the Bell-diagonal states

The fidelity between ρ_{in} and ρ_{out} characterizes the quality of the teleported state ρ_{out}. When the input is a pure state, we can apply the concept of fidelity as a useful indicator of the teleportation performance of a quantum channel. The fidelity of ρ_{in} and ρ_{out} is defined to be [21]

$$F(\rho_{\text{in}},\rho_{\text{out}})=\left\{\mathrm{tr}\left[\sqrt{(\rho_{\text{in}})^{1/2}\rho_{\text{out}}(\rho_{\text{in}})^{1/2}}\,\right]\right\}^2. \tag{11}$$

The average fidelity F_{A} is another useful concept for characterizing the quality of teleportation. The average fidelity F_{A} can be obtained by averaging F over all possible input states

$$F_{\text{A}}=\frac{\int_0^{2\pi}\mathrm{d}\varphi\int_0^{\pi}F\sin\theta\mathrm{d}\theta}{4\pi}. \tag{12}$$

For this model, F_{A} can be written as

$$F_{\text{A}}=\frac{(c_1(t)+c_2(t))^2-2c_3}{12}+\frac{1+c_3^2}{4}. \tag{13}$$

The dependence of the average fidelity on the time γt is shown in Fig. 2. We choose the parameters as $c_1(0)=1$, $c_2(0)=1$, $c_3=-1$. The decay in average fidelity occurs when the time γt increases. In order to

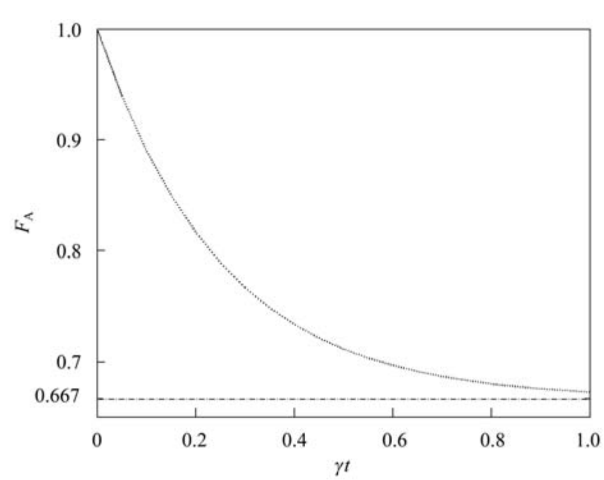

Fig. 2. The dynamics of the average fidelity F_{A} as a function of γt for $c_1(0)=1$, $c_2(0)=1$, $c_3=-1$.

transmit a quantum state better than any classical communication protocol, F_A must be greater than 2/3, which is the best fidelity in the classical world [22].

The average fidelity F_A as a function of parameter c_3 and γt is plotted in Fig. 3. The average fidelity increases linearly as c_3 increases. For finite time, the values of average fidelity are always nonzero. We find that the minimum value of the average fidelity is obtained at $c_3 \approx 0.32$, but not at the zero point. In order to get an average fidelity better than 0.667, we must set proper parameters for this model.

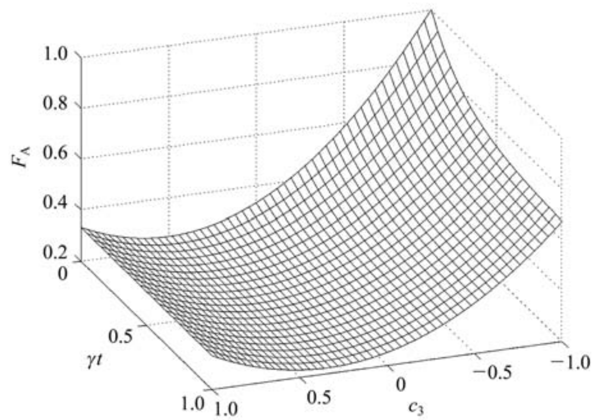

Fig. 3. The dynamics of the average fidelity F_A as a function of γt and c_3 for $c_1(0)=1$, $c_2(0)=-c_3$.

4 Conclusions

In summary, we evaluated quantum entanglement and teleportation for a class of two-qubit Bell-diagonal states with decoherence effect. The analytical formulas for the entanglement, the output entanglement and the average fidelity in this model were obtained. The influences of parameters and decoherence on the entanglement dynamics were addressed in detail. These generalized results will be useful for a large class of two-qubit states.

References

1 Fanchini F F, Castelano L K, Caldeira A O. New J. Phys., 2010, **12**: 073009
2 LI M, FEI S M, LI-JOST X Q. Chin. Sci. Bull., 2011, **56**(10): 945–954
3 GUO Y, QI X F, HOU J C. Chin. Sci. Bull., 2011, **56**(9): 840–846
4 Terhal B M. Theoretical Computer Science, 2002, **287**: 313–335
5 Shamsolah S, Shahram M. Chin. Sci. Bull., 2011, **56**(7): 618–625
6 Bennett C H, Brassard G. Proceedings of IEEE International Conference on Computers Systems and Signal Processing, 1984. 175–179
7 DENG F G, LONG G L. Phys. Rev. A, 2004, **69**: 052319
8 LONG G L, LIU X S. Phys. Rev. A, 2002, **65**: 032302
9 HAN F. Chin. Sci. Bull., 2010, **55**(17): 1758–1762
10 AI Q, WANG Y D, LONG G L, SUN C P. Science in China Series G, 2009, **52**(12): 1898–1905
11 QIN M, LI Y B. Chin. Phys. C (HEP & NP), 2010, **34**(4): 448–451
12 ZHANG G. F. Phys. Rev. A, 2007, **75**: 034304
13 Kheirandish F, Akhtarshenas S J, Mohammadi H. Phys. Rev. A, 2008, **77**: 042309
14 GUO J L, SONG H S. Eur. Phys. J. D, 2010, **56**: 265–269
15 LANG M D, Caves C M. Phys. Rev. Lett., 2010, **105**: 150501
16 Mazzola L, Piilo J, Maniscalco A. Phys. Rev. Lett., 2010, **104**: 200401
17 WANG Z J, ZHANG K. Commun. Theor. Phys., 2009, **52**: 832
18 Wotters W. K. Phys. Rev. Lett., 1998, **80**: 2245
19 Lee J, Kim M S. Phys. Rev. Lett., 2000, **84**: 4236
20 Bowen G, Bose S. Phys. Rev. Lett., 2001, **87**: 267901
21 Jozsa R. J. Mod. Opt., 1994, **41**: 315
22 CAI J T, ABLIZ A, ZHANG G F. Opt. Commun, 2010, **283**: 4415